MSC 35Q93, 34H05 DOI: 10.14529/jcem1602007

OPTIMAL CONTROL OF SOLUTIONS TO THE INITIAL-FINAL PROBLEM FOR THE SOBOLEV TYPE EQUATION OF HIGHER ORDER

A. A. Zamyshlyaeva[1], zamyshliaevaaa@susu.ru,
O. N. Tsyplenkova[1], tcyplenkovaon@susu.ru,
E. V. Bychkov[1], bychkovev@susu.ru.
[1] South Ural State University, Chelyabinsk, Russian Federation.

Of concern is the optimal control problem for the Sobolev type higher order equation with relatively polynomially bounded operator pencil. The theorem of existence and uniqueness of strong solutions to the initial-final problem for abstract equation is proved. The sufficient conditions for optimal control existence and uniqueness of such solutions are found. We use the ideas and methods developed by G.A. Sviridyuk and his disciples.

Keywords: Sobolev type equations, relatively polynomially bounded operator pencil, strong solutions, optimal control.

Introduction

Let $\mathfrak{X}$, $\mathfrak{Y}$ and $\mathfrak{U}$ be Hilbert spaces. Consider a complete Sobolev type equation of higher order

$$Ax^{(n)} = B_{n-1}x^{(n-1)} + \ldots + B_0 x + y + Cu, \tag{1}$$

where $A,\ B_{n-1}, \ldots, B_0$ are linear and continuous operators acting from $\mathfrak{X}$ to $\mathfrak{Y}$, C is a continuous linear operator acting from $\mathfrak{U}$ to $\mathfrak{Y}$, functions $u : [0, \tau] \subset R_+ \to \mathfrak{U}$, $y : [0, \tau] \subset R_+ \to \mathfrak{Y}\,(\tau < \infty)$.

Consider an initial-final problem [1]

$$\begin{matrix} P_{in}\left(x^{(k)}(0) - x_k^0\right) = 0, \\ P_{fin}\left(x^{(k)}(\tau) - x_k^\tau\right) = 0, \end{matrix} \qquad k = \overline{0, n-1}, \tag{2}$$

where $P_{in(fin)}$ are some projectors in the space $\mathfrak{X}$.

We are interested in an optimal control problem of finding a pair $(\hat{x}, \hat{u})$, where $\hat{x}$ is a solution of (1), (2), and the control $\hat{u}$ belongs to $\mathfrak{U}_{ad}$ and satisfies a relation

$$J\left(\hat{x}, \hat{u}\right) = \min_{(x,u) \in X \times \mathfrak{U}_{ad}} J(x, u). \tag{3}$$

Here $J(x, u)$ is some specially constructed penalty functional, and $\mathfrak{U}_{ad}$ is a closed convex set in the space $\mathfrak{U}$ of controls.

Sobolev type equations form a broad range of nonclassical equations of mathematical physics. A theory of linear first-order Sobolev type equations was constructed in [2]. A numerical algorithm for finding a solution of an optimal control problem for a system of Leontief type equations was suggested in [3 – 5]. The optimal control for semilinear first-order Sobolev type equations was considered in [6 – 8]. Zagrebina S.A. [9 – 10] considered an initial-final problem generalizing the Showalter – Sidorov problem for the first order Sobolev type equation. Stochastic Sobolev type equations were studied in [11 – 19]. The initial-final problem for a linear second-order Sobolev type equation was considered in [20] in the case of relative polynomial boundedness of the operator pencil. In the present paper, we consider the Cauchy problem and the initial-final problem, which is more convenient for the numerical solution of specific problems of mathematical physics [21 – 24].

1. Polynomially A-bounded operator pencils and projectors

By $\vec{B}$ denote an pencil formed by operators $B_{n-1},\ldots,B_0$. The sets

$$\rho^A(\vec{B})=\left\{\mu\in C:\left(\mu^nA-\mu^{n-1}B_{n-1}-\ldots-\mu B_1-B_0\right)^{-1}\in\mathcal{L}(\mathfrak{Y},\mathfrak{X})\right\}$$

and $\sigma^A(\vec{B})=\overline{C}\backslash\rho^A(\vec{B})$ are called an A-resolvent set and an A-spectrum of pencil $\vec{B}$, respectively. The operator function $R_\mu^A(\vec{B})=(\mu^nA-\mu^{n-1}B_{n-1}-\ldots-\mu B_1-B_0)^{-1}$ of a complex variable with domain $\rho^A(\vec{B})$ is called an A-resolvent of the pencil $\vec{B}$.

Definition 1. *The operator pencil $\vec{B}$ is said to be polynomially bounded with respect to operator A (or simply polynomially A-bounded) if there exists a constant $a\in R_+$ such that for each $\mu\in C$ an inequality $|\mu|>a$ implies an inclusion $R_\mu^A(\vec{B})\in\mathcal{L}(\mathfrak{Y},\mathfrak{X})$.*

Introduce an additional condition

$$\int_\gamma\mu^kR_\mu^A(\vec{B})d\mu\equiv0,\quad k=\overline{0,n-2}.\tag{A}$$

Lemma 1. *Let the pencil $\vec{B}$ be polynomially A-bounded, and let condition (A) be satisfied. Then operators*

$$P=\frac{1}{2\pi i}\int_\gamma R_\mu^A(\vec{B})\mu^{n-1}Ad\mu,\quad Q=\frac{1}{2\pi i}\int_\gamma\mu^{n-1}AR_\mu^A(\vec{B})d\mu$$

are projectors in the spaces $\mathfrak{X}$ and $\mathfrak{Y}$, respectively.

Set $\mathfrak{X}_0=\ker P,\ \mathfrak{Y}_0=\ker Q,\ \mathfrak{X}_1=\mathrm{Im}P,\ \mathfrak{Y}_1=\mathrm{Im}Q$. It follows from Lemma 1 that $\mathfrak{X}=\mathfrak{X}_0\oplus\mathfrak{X}_1$ and $\mathfrak{Y}=\mathfrak{Y}_0\oplus\mathfrak{Y}_1$. By A^k (respectively, B_l^k) denote a restriction of the operator A (respectively, B_l) to $\mathfrak{X}^k$, $k=0,1$, $l=\overline{0,n-1}$.

The following assertion was proved in [25].

Theorem 1. [25] *Let the assumptions of Lemma 1 be satisfied. Then*

(i) $A^k\in\mathcal{L}(\mathfrak{X}^k;\mathfrak{Y}^k)$, $k=0,1$;

(ii) $B_l^k\in\mathcal{L}(\mathfrak{X}^k;\mathfrak{Y}^k)$, $k=0,1$, $l=0,1,\ldots,n-1$;

(iii) there exists an operator $(A^1)^{-1}\in\mathcal{L}(\mathfrak{Y}^1;\mathfrak{X}^1)$;

(iv) there exists an operator $(B_0^0)^{-1}\in\mathcal{L}(\mathfrak{Y}^0;\mathfrak{X}^0)$.

Let us construct the operators $H_0=(B_0^0)^{-1}A^0$, $H_k=(B_0^0)^{-1}B_{n-k}^0$, $k=\overline{1,n-1}$, $S_k=(A^1)^{-1}B_k^1$, $k=\overline{0,n-1}$.

Definition 2. *Introduce a family of operators $\{K_q^1,K_q^2,\ldots,K_q^n\}$ as follows:*

$$K_0^s=\mathbb{O},s\neq n,K_0^n=\mathbb{I},\ K_1^1=H_0,\ K_1^2=-H_{n-1},\ldots,K_1^s=-H_{n+1-s},\ldots,K_1^n=-H_1,$$
$$K_q^1=K_{q-1}^nH_0,\ K_q^2=K_{q-1}^1-K_{q-1}^nH_{n-1},\ldots,\ K_q^s=K_{q-1}^{s-1}-K_{q-1}^nH_{n+1-s},\ldots,$$
$$K_q^n=K_{q-1}^{n-1}-K_{q-1}^nH_1,q=2,3,\ldots$$

Definition 3. *The point ∞ is called:*

(i) a removable singularity of A-resolvent of pencil $\vec{B}$, if $K_1^1 = K_1^2 = \ldots = K_1^n \equiv \mathbb{O}$;

(ii) a pole of order $p \in \mathbb{N}$ of A-resolvent of pencil $\vec{B}$, if $K_p^s \not\equiv \mathbb{O}$, for some s, but $K_{p+1}^s \equiv \mathbb{O}$ for any s;

(iii) an essentially singular point of A-resolvent of pencil $\vec{B}$ if $K_p^n \not\equiv \mathbb{O}$ for any $p \in \mathbb{N}$.

2. Strong solutions

Consider a linear homogeneous Sobolev type equation

$$Ax^{(n)} = B_{n-1}x^{(n-1)} + \ldots + B_0 x. \tag{4}$$

Introduce an additional condition

$$\begin{array}{c}\sigma^A(\vec{B}) = \sigma_0^A(\vec{B}) \cup \sigma_1^A(\vec{B}), \sigma_k^A(\vec{B}) \neq \emptyset, k = \overline{0,1};\\ \text{and contour } \gamma_0 \text{ is the boundary of domain } \Omega \subset \mathbb{C} \text{ such that}\\ \Omega \cap \sigma_0^A(\vec{B}) = \sigma_0^A(\vec{B}), \bar{\Omega} \cap \sigma_1^A(\vec{B}) = \emptyset.\end{array} \tag{B}$$

Then there exists an operator

$$P_{fin} = \frac{1}{2\pi i}\int_{\gamma_0} R_\mu^A(\vec{B})\mu^{n-1}A d\mu \in \mathcal{L}(\mathfrak{X}).$$

More over let

$$\int_{\gamma_0} \mu^m R_\mu^A(\vec{B}) d\mu \equiv 0, m = \overline{0, n-2}, \tag{A_0}$$

be fulfilled.

Definition 4. *The mapping $V^\bullet \in C^\infty(\mathbb{R}; \mathcal{L}(\mathfrak{X}))$ is called a propagator of equation (4), if for all $x \in \mathfrak{X}$ the vector-function $x(t) = V^t x$ is a solution of (4).*

Let the pencil $\vec{B}$ be polynomially A-bounded, and let condition (A) be satisfied. Fix a contour $\gamma = \{\mu \in \mathbb{C} : |\mu| = r > a\}$ and for all $t \in R$ consider a family of operators

$$X_k^t = \frac{1}{2\pi i}\int_\gamma R_\mu^A(\vec{B})(\mu^{n-k-1}A - \mu^{n-k-2}B_{n-1} - \ldots - B_{k+1})e^{\mu t} d\mu,$$

where $k = \overline{0, n-1}$.

Lemma 2. [26] *(i) For every $k = \overline{0, n-1}$ the operator function X_k^t is a propagator of (4).*

(ii) For every $k = \overline{0, n-1}$ the operator function X_k^t is an entire function.

(iii) $\frac{d^l}{dt^l} X_k^t \Big|_{t=0} = \begin{cases} P, \ l = k; \\ \mathbb{O}, \ l \neq k; \end{cases}$ *for all* $k = \overline{0, n-1}, \ l = 0, 1, \ldots$

Lemma 3. *Let the pencil $\vec{B}$ be polynomially A-bounded, let conditions (A), (B), (A_0) be satisfied. Then P_{fin} is a projector, and $P_{fin}P = PP_{fin} = P_{fin}$.*

Construct an operator $P_{in} = P - P_{fin} \in \mathcal{L}(\mathfrak{X})$. By Lemma 3 operator P_{in} is a projector.

Consider a family of operators

$$X^k_{fin}(t) = \frac{1}{2\pi i}\int_{\gamma_0} R^A_\mu(\vec{B})(\mu^{n-k-1}A - \mu^{n-k-2}B_{n-1} - ... - B_{k+1})e^{\mu t}d\mu.$$

Lemma 4. *Let the pencil $\vec{B}$ be polynomially A-bounded, let conditions (A), (B), (A_0) be satisfied. Then X^k_{fin} is a propagator of (4).*

Introduce a family of operators

$$X^k_{in}(t) = X^t_k - X^k_{fin}(t), \quad k = \overline{0, n-1}.$$

Proceed to linear inhomogeneous Sobolev type equation

$$Ax^{(n)} = B_{n-1}x^{(n-1)} + \ldots + B_0 x + y. \tag{5}$$

Theorem 2. *Let the pencil $\vec{B}$ be polynomially A-bounded, condition (A) be satisfied and ∞ be a pole of order $p \in \{0\} \cup N$ of A-resolvent. Let the vector function $y : [-\tau, \tau] \to \mathfrak{Y}$ be such that*

$$y^0 = (\mathbb{I} - Q)y \in C^{p+n}([0, \tau]; \mathfrak{Y}^0)$$

and $y^1 = Qy \in C([-\tau, \tau]; \mathfrak{Y}^1)$. Then for arbitrary $x^0_k, x^\tau_k \in X,\ k = \overline{0, n-1}$ there exists a unique solution to problem (2), (5) for $t \in [0, \tau]$ given by

$$\begin{gathered} x(t) = -\sum_{q=0}^{p} K^n_q (B^0_0)^{-1}\tfrac{d^q}{dt^q}y^0(t) + \sum_{k=0}^{n-1} X^k_{in}(t)P_{in}x^0_k + \sum_{k=0}^{n-1} X^k_{fin}(t)P_{fin}x^\tau_k + \\ + \int_0^t X^{n-1}_{in}(t-s)(A^1)^{-1}y^{in}(s)ds - \int_t^\tau X^{n-1}_{fin}(t-s)(A^1)^{-1}y^{fin}(s)ds. \end{gathered} \tag{6}$$

Definition 5. *The vector-function $x \in H^n(\mathfrak{X}) = \{x \in L_2(0, \tau; \mathfrak{X}) :\ x^{(n)} \in L_2(0, \tau; \mathfrak{X})\}$ is called a strong solution of (5), if it turns the equation to an identity almost everywhere on interval $(0, \tau)$. The strong solution $x = x(t)$ of (5) is called a strong solution to (2), (5) if condition (2) holds.*

This is correctly defined by virtue of the continuity of the embedding $H^n(\mathfrak{X}) \hookrightarrow C^{n-1}([0, \tau]; \mathfrak{X})$. The term "strong solution" has been introduced to distinguish a solution of (5) in this sense from solution (6), which is usually said to be classical. Note that classical solution (6) is also a strong solution to problem (2), (5).

Let us construct the spaces

$$H^{p+n}(\mathfrak{Y}) = \{v \in L_2(0, \tau; \mathfrak{Y}) : v^{(p+n)} \in L_2(0, \tau; \mathfrak{Y}), p \in \{0\} \cup \mathbb{N}\}.$$

The space $H^{p+n}(\mathfrak{Y})$ is a Hilbert space with inner product

$$[v, w] = \sum_{q=0}^{p+n} \int_0^\tau \left\langle v^{(q)}, w^{(q)} \right\rangle_{\mathfrak{Y}} dt.$$

Let $y \in H^{p+n}(\mathfrak{Y})$. Introduce the operators

$$A_1 y(t) = -\sum_{q=0}^{p} K^n_q (B^0_0)^{-1}\frac{d^q}{dt^q}y^0(t),$$

$$A_2y(t) = \int_0^t X_{in}^{n-1}(t-s)(A^1)^{-1}y^{in}(s)ds, \quad A_3y(t) = -\int_t^\tau X_{fin}^{n-1}(t-s)(A^1)^{-1}y^{fin}(s)ds,$$

and the functions

$$k_1(t) = \sum_{k=0}^{n-1} X_{in}^k(t)P_{in}x_k^0, \quad k_2(t) = \sum_{k=0}^{n-1} X_{fin}^k(t)P_{fin}x_k^\tau.$$

Lemma 5. [27] *Let the pencil $\vec{B}$ be polynomially A-bounded, and condition (A) be satisfied. Then*

(i) $A_1 \in \mathcal{L}(H^{p+n}(\mathfrak{Y}); H^n(\mathfrak{X}))$;
(ii) for arbitrary $x_k^0 \in \mathfrak{X}$, *the vector function* $k_1 \in C^n([0,\tau);\mathfrak{X})$;
(iii) $A_2 \in \mathcal{L}(H^{p+n}(\mathfrak{Y}); H^n(\mathfrak{X}))$;
(iv) for arbitrary $x_k^\tau \in \mathfrak{X}$, *the vector function* $k_2 \in C^n([0,\tau);\mathfrak{X})$;
(v) $A_1 \in \mathcal{L}(H^{p+n}(\mathfrak{Y}); H^n(\mathfrak{X}))$.

Theorem 3. *Let the pencil $\vec{B}$ be polynomially A-bounded, let conditions (A), (B), (A_0) be satisfied. Then, for arbitrary x_k^0, $x_k^\tau \in \mathfrak{X}$, $k = \overline{0, n-1}$ and $y \in H^{p+n}(\mathfrak{Y})$ there exists a unique strong solution to (2), (5).*

3. Optimal control

Consider the initial-final problem (2) for linear inhomogeneous Sobolev type equation (1), where the functions x, y, u lie in the Hilbert spaces $\mathfrak{X}$, $\mathfrak{Y}$ and $\mathfrak{U}$, respectively.

Introduce a control space

$$\overset{\circ}{H}{}^{p+n}(\mathfrak{U}) = \{u \in L_2(0,\tau;\mathfrak{U}) : u^{(p+n)} \in L_2(0,\tau;\mathfrak{U}), u^{(q)}(0) = 0, q = \overline{0,p}\},$$

$p \in \{0\} \cup \mathbb{N}$. It is a Hilbert space with inner product

$$[v,w] = \sum_{q=0}^{p+n} \int_0^\tau \left\langle v^{(q)}, w^{(q)} \right\rangle_{\mathfrak{Y}} dt.$$

In the space $\overset{\circ}{H}{}^{p+n}(\mathfrak{U})$ we single out a closed convex subset $\overset{\circ}{H}{}^{p+n}_\partial(\mathfrak{U})$, which will be called the set of admissible controls.

The vector function $\hat{u} \in \overset{\circ}{H}{}^{p+n}_\partial(\mathfrak{U})$ is called an optimal control of solutions to (1), (2), if relation (3) holds.

Our goal is to prove the existence of a unique control $\hat{u} \in \overset{\circ}{H}{}^{p+n}_\partial(\mathfrak{U})$, minimizing the penalty functional

$$J(x,u) = \mu \sum_{q=0}^{n} \int_0^\tau ||x^{(q)} - \tilde{x}^{(q)}||^2 dt + \nu \sum_{q=0}^{p+n} \int_0^\tau \left\langle N_q u^{(q)}, u^{(q)} \right\rangle_{\mathfrak{U}} dt. \tag{7}$$

Here $\mu, \nu > 0$, $\mu + \nu = 1$, $N_q \in \mathcal{L}(\mathfrak{U})$, $q = 0, 1, \ldots, p+n$, are self-adjoint positively defined operators, and $\tilde{x}(t)$ is the target state of the system.

Theorem 4. *Let the pencil $\vec{B}$ be polynomially A-bounded, let conditions (A), (B), (A_0) be satisfied. Then for arbitrary x_k^0, $x_k^\tau \in \mathfrak{X}$, $k = \overline{0, n-1}$ and $y \in H^{p+n}(\mathfrak{Y})$ there exists a unique optimal control of solutions to problem (1), (2).*

References

1. Zagrebina S.A. The Initial-Finish Problem for the Navier – Stokes Linear System. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2011, no. 4(221), issue 7, pp. 35 – 39. (in Russian)

2. Sviridyuk G.A., Fedorov V.E. *Linear Sobolev Type Equations and Degenerate Semigroups of Operators*. Utrecht, Boston, Köln, Tokyo, VSP, 2003. doi: 10.1515/9783110915501

3. Keller A.V. Numerical Solution of the Problem of Starting Control for a System of Equations of Leont'ev Type. *Obozrenie prikladnoi i promyshlennoi matematiki – Review Industrial and Applied mathematics*, 2009, vol. 16, issue 2, pp. 345–346. (in Russian)

4. Keller A.V. The Algorithm for Solution of the Showalter – Sidorov Problem for Leontief Type Models. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2011, no. 4(221), issue 7, pp. 40–46. (in Russian)

5. Keller A.V., Nazarova E.I. Optimal Measuring Problem: the Computation Solution, the Program Algorithm. *The Bulletin of Irkutsk State University. Series: Mathematics*, 2011, no. 3, pp. 74–82. (in Russian)

6. Sviridyuk G.A., Efremov A.A. Optimal Control of Sobolev Type Linear Equations with Relatively p-Sectorial Operators. *Differential Equations*, 1995, vol. 31, no. 11, pp. 1882–1890. (in Russian)

7. Manakova N.A. An Optimal Control Problem for the Oskolkov Nonlinear Filtration Equation. *Differential Equations*, 2007, vol. 43, no. 9, pp. 1185–1192. (in Russian)

8. Manakova N.A., Dylkov A.G. Optimal Control of Solutions of Initial-Finish Problem for the Linear Sobolev Type Equations. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2011, no. 17(234), issue 8, pp. 113–114. (in Russian)

9. Zagrebina S.A. The Initial-Finish Problems for Nonclassical Models of Mathematical Physics. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2013, vol. 6, no. 2, pp. 5–24. (in Russian)

10. Zagrebina S.A., Sagadeeva M.A. Generalized Showalter – Sidorov Problem for Sobolev Type Equations with Strong (L,p)-Radial Operator. *Bulletin of the Magnitogorsk State University. Series: Mathematics*, 2006, issue 9, pp. 17–27. (in Russian)

11. Sviridyuk G. A., Manakova N.A. The Dynamical Models of Sobolev Type with Showalter – Sidorov Condition and Additive "Noise". *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2014, vol. 7, no. 1, pp. 90–103. doi: 10.14529/mmp140108 (in Russian)

12. Favini A., Sviridyuk G.A., Zamyshlyaeva A.A. One Class of Sobolev Type Equations of Higher Order with Additive "White Noise". *Communications on Pure and Applied Analysis*, 2016, vol. 15, no. 1, pp. 185–196. doi: 10.3934/cpaa.2016.15.185

13. Favini A., Sviridyuk G.A., Manakova N.A. Linear Sobolev Type Equations with Relatively p-Sectorial Operators in Space of "Noises". *Abstract and Applied Analysis*, 2015, vol. 2015(2015), article ID 697410, 8 pages. doi: 10.1155/2015/697410

14. Keller A.V., Shestakov A.L., Sviridyuk G.A., Khudyakov Y.V. The Numerical Algorithms for the Measurement of the Deterministic and Stochastic Signals. *Springer Proceedings in Mathematics and Statistics*, 2015, pp. 183–195. doi: 10.1007/978-3-319-12145-1_11

15. Sviridyuk G.A., Manakova N.A. An Optimal Control of the Solutions of the Initial-Final Problem for Linear Sobolev Type Equations with Strongly Relatively p-Radial Operator. *Springer Proceedings in Mathematics and Statistics*, 2015, pp. 213–224. doi: 10.1007/978-3-319-12145-1_13

16. Sagadeeva M.A., Sviridyuk G.A. The Nonautonomous Linear Oskolkov Model on a Geometrical Graph: the Stability of Solutions and Optimal Control Problem. *Springer Proceedings in Mathematics and Statistics*, 2015, pp. 257–271. doi: 10.1007/978-3-319-12145-1_16

17. Shestakov A.L., Sviridyuk G.A., Khudyakov Y.V. Dynamical Measurements in the View of the Group Operators Theory. *Springer Proceedings in Mathematics and Statistics*, 2015, pp. 273–286. doi: 10.1007/978-3-319-12145-1_17

18. Zagrebina S.A., Soldatova E.A., Sviridyuk G.A. The Stochastic Linear Oskolkov Model of the Oil Transportation by the Pipeline. *Springer Proceedings in Mathematics and Statistics*, 2015, pp. 317–326. doi: 10.1007/978-3-319-12145-1_20

19. Zamyshlyaeva A.A., Sviridyuk G.A. The Linearized Benney–Luke Mathematical Model with Additive White Noise. *Springer Proceedings in Mathematics and Statistics*, 2015, pp. 327–336. doi: 10.1007/978-3-319-12145-1_21

20. Zamyshlyaeva A.A. State Spaces of a Class of Linear Second-Order Equations of Sobolev Type. *Computational Technologies*, 2003, vol. 8, no. 4, pp. 45–54. (in Russian)

21. Zamyshlyaeva A.A., Bychkov E.V. The Phase Space of the Modified Boussinesq Equation. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2012, no. 18(277), issue 12, pp. 13–19. (in Russian)

22. Zagrebina S.A., Konkina A.S. The Multipoint Initial-Final Value Condition for the Navier – Stokes Linear Model. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2015, vol. 8, no. 1, pp. 132–136. doi: 10.14529/mmp150111

23. Bychkov E.V. On a Semilinear Sobolev Type Mathematical Model. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2014, vol. 7, no. 2, pp. 111– 117. doi: 10.14529/mmp140210 (in Russian)

24. Zamyshlyaeva A. A., Bychkov E.V., Tsyplenkova O.N. Mathematical Models Based on Boussinesq – Love Equation. *Applied Mathematical Sciences*, 2014, vol. 8, no. 110, pp. 5477–5483. doi: 10.12988/ams.2014.47546

25. Zamyshlyaeva A. A. The Phase Space of a High Order Sobolev type Equation. *The Bulletin of Irkutsk State University. Series: Mathematics*, 2011, no. 4, pp. 45–57. (in Russian)

26. Zamyshlyaeva A. A. *Linear Sobolev Type Equations of Higher Order*. Chelyabinsk, Publishing center of SUSU, 2012. (in Russian)

27. Zamyshlyaeva A. A. The Higher-Order Sobolev Type Models. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2014, vol. 7, no. 2, pp. 5–28. (in Russian)

28. Tsyplenkova O.N. Optimal Control of Solutions to the Cauchy Problem for the Sobolev Type Equation of Higher Order. *Journal of Computational and Engineering Mathematics*, 2014, vol. 1, no. 2, pp. 62–67.

Alyona A. Zamyshlyaeva, Doctor of Physico-Mathematical Sciences, Docent, Department of Equations of Mathematical Physics, South Ural State University (Chelyabinsk, Russian Federation), zamyshlyaevaaa@susu.ac.ru

Olga N. Tsyplenkova, Candidate of Physico-Mathematical Sciences, Department of Equations of Mathematical Physics, South Ural State University (Chelyabinsk, Russian Federation), tcyplenkovaon@susu.ac.ru

Evgeniy V. Bychkov, Candidate of Physico-Mathematical Sciences, Department of Equations of Mathematical Physics, South Ural State University (Chelyabinsk, Russian Federation), bychkovev@susu.ac.ru

Received June 3, 2016

УДК 517.9 **DOI: 10.14529/jcem1602007**

ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ РЕШЕНИЯМИ НАЧАЛЬНО-КОНЕЧНОЙ ЗАДАЧИ ДЛЯ УРАВНЕНИЯ СОБОЛЕВСКОГО ТИПА ВЫСОКОГО ПОРЯДКА

А. А. Замышляева, О. Н. Цыпленкова, Е. В. Бычков

В работе рассматривается задача оптимального управления для уравнения соболевского типа высокого порядка с относительно полиномиально ограниченным пучком операторов. Доказана теорема существования и единственности сильного решения начально-конечной задачи для абстрактного уравнения соболевского типа высокого порядка. Найдено достаточное условие существования таких решений. В работе использованы идеи и методы, разработанные Г.А. Свиридюком и его учениками.

Ключевые слова: уравнения соболевского типа, относительно ограниченный пучок операторов, сильные решения, оптимальное управление.

Литература

1. Загребина, С.А. Начально-конечная задача для линейной системы Навье – Стокса / С.А. Загребина // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2011. – № 4(221), вып. 7. – С. 35–39.
2. Sviridyuk, G.A. Linear Sobolev Type Equations and Degenerate Semigroups of Operators / G.A. Sviridyuk, V.E. Fedorov. – Utrecht, Boston, Köln, Tokyo: VSP, 2003.
3. Келлер, А.В. Численное решение задачи стартового управления для системы уравнений леонтьевского типа / А.В. Келлер // Обозрение прикладной и промышленной математики. – 2009. – Т. 16, вып. 2. – С. 345–346.
4. Келлер, А.В. Алгоритм решения задачи Шоуолтера – Сидорова для моделей леонтьевского типа / А.В. Келлер // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2011. – № 4(221), вып. 7. – С. 40–46.
5. Келлер, А.В. Задача оптимального измерения: численное решение, алгоритм программы / А.В. Келлер, Е.И. Назарова // Известия Иркутского государственного университета. Серия: Математика. – 2011. – № 3. – С. 74–82.
6. Свиридюк, Г.А. Оптимальное управление линейными уравнениями типа Соболева с относительно p-секториальными операторами / Г.А. Свиридюк, А.А. Ефремов // Дифференциальные уравнения. – 1995. – Т. 31, № 11. – С. 1882–1890.
7. Манакова, Н.А. Задача оптимального управления для уравнения Осколкова нелинейной фильтрации / Н.А. Манакова // Дифференциальные уравнения. – 2007. – Т. 43, № 9. – С. 1185–1192.
8. Манакова, Н.А. Оптимальное управление решениями начально-конечной задачи для линейных уравнений соболевского типа / Н.А. Манакова, А.Г. Дыльков // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2011. – № 17 (234), вып. 8. – С. 113–114.

9. Загребина, С.А. Начально-конечные задачи для неклассических моделей математической физики / С.А. Загребина // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2013. – Т. 6, № 2. – С. 5–24.

10. Загребина, С.А. Обобщенная задача Шоуолтера – Сидорова для уравнений соболевского типа с сильно (L,P)-радиальным оператором / С.А. Загребина, М.А. Сагадеева // Вестник Магнитогорского государственного университета. – 2006. – Вып. 9. – С. 17–27.

11. Свиридюк, Г.А. Динамические модели соболевского типа с условием Шоуолтера – Сидорова и аддитивными шумами / Г.А. Свиридюк, Н.А. Манакова // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2014. – Т. 7, № 1. – С. 90–103.

12. Favini, A. One Class of Sobolev Type Equations of Higher Order with Additive "White Noise" / A. Favini, G.A. Sviridyuk, A.A. Zamyshlyaeva // Communications on Pure and Applied Analysis. – 2016. – V. 15, № 1. – P. 185–196.

13. Favini, A. Linear Sobolev Type Equations with Relatively p-Sectorial Operators in Space of "noises" / A. Favini, G.A. Sviridyuk, N.A. Manakova // Abstract and Applied Analysis. – 2015. – V. 2015(2015). – Article ID 697410, 8 pages.

14. Keller, A.V. The Numerical Algorithms for the Measurement of the Deterministic and Stochastic Signals / A.V. Keller, A.L. Shestakov, G.A. Sviridyuk, Y.V. Khudyakov // Springer Proceedings in Mathematics and Statistics. – 2015. – P. 183–195.

15. Sviridyuk, G.A. An Optimal Control of the Solutions of the Initial-Final Problem for Linear Sobolev Type Equations with Strongly Relatively p-Radial Operator / G.A. Sviridyuk, N.A. Manakova // Springer Proceedings in Mathematics and Statistics. – 2015. – P. 213–224.

16. Sagadeeva, M.A. The Nonautonomous Linear Oskolkov Model on a Geometrical Graph: the Stability of Solutions and Optimal Control Problem / M.A. Sagadeeva, G.A. Sviridyuk // Springer Proceedings in Mathematics and Statistics. – 2015. – P. 257–272.

17. Shestakov, A.L. Dynamical Measurements in the View of the Group Operators Theory / A.L. Shestakov, G.A. Sviridyuk, Y.V. Khudyakov // Springer Proceedings in Mathematics and Statistics. – 2015. – P. 273–286.

18. Zagrebina, S.A. The Stochastic Linear Oskolkov Model of the Oil Transportation by the Pipeline / S.A. Zagrebina, E.A. Soldatova, G.A. Sviridyuk // Springer Proceedings in Mathematics and Statistics. – 2015. – P. 317–326.

19. Zamyshlyaeva, A.A. The Linearized Benney–Luke Mathematical Model with Additive White Noise / A.A. Zamyshlyaeva, G.A. Sviridyuk // Springer Proceedings in Mathematics and Statistics. – 2015. – P. 327–336.

20. Замышляева, А.А. Фазовые пространства одного класса линейных уравнений соболевского типа второго порядка / А.А. Замышляева // Вычислительные технологии. – 2003. – Т. 8, № 4. – С. 45–54.

21. Замышляева, А.А. Фазовое пространство модифицированного уравнения Буссинеска / А.А. Замышляева, Е.В. Бычков // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2012. – №18 (277), вып. 12. – С. 13–19.
22. Загребина, С.А. Многоточечное начально-конечное условие для линейной модели Навье - Стокса / С.А. Загребина, А.С. Конкина // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2015. – Т. 8, № 1. – С. 132–136.
23. Бычков, Е.В. Об одной полулинейной математической модели соболевского типа высокого порядка / Е.В. Бычков // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2014. – Т. 7, № 2. – С. 111–117.
24. Zamyshlyaeva, A.A. Mathematical Models Based on Boussinesq – Love Equation / A.A. Zamyshlyaeva, E.V. Bychkov, O.N. Tsyplenkova // Applied Mathematical Sciences. – 2014. – V. 8, № 110. P. 5477–5483.
25. Замышляева, А.А. Фазовое пространство уравнения соболевского типа высокого порядка / А.А. Замышляева // Известия Иркутского государственного университета. Серия: Математика. – 2011. – № 4. – С. 45–57.
26. Замышляева, А.А. Линейные уравнения соболевского типа высокого порядка / А.А. Замышляева. – Челябинск: Издательский центр ЮУрГУ, 2012.
27. Замышляева, А.А. Математические модели соболевского типа высокого порядка / А.А. Замышляева // Вестник Южно-Уральского государственного университета. Серия: Математическое моделирование и программирование. – 2014. – Т. 7, № 2. – С. 5–28.
28. Tsyplenkova, O.N. Optimal Control of Solutions to the Cauchy Problem for the Sobolev Type Equation of Higher Order / O.N. Tsyplenkova // Journal of Computational and Engineering Mathematics. – 2014. – V. 1, № 2. – P. 62–67.

Замышляева Алена Александровна, доктор физико-математических наук, доцент, кафедра уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), zamyshliaevaaa@susu.ru.

Цыпленкова Ольга Николаевна, кандидат физико-математических наук, кафедра уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), tcyplenkovaon@susu.ru.

Бычков Евгений Викторович, кандидат физико-математических наук, кафедра уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), bychkovev@susu.ru.

Поступила в редакцию 3 июня 2016 г.