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COMPUTATIONAL EXPERIMENTS FOR ONE CLASS OF MATHEMATICAL MODELS IN THERMODYNAMICS AND HYDRODYNAMICS

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The article contains the results of computational experiments for the Dzeczer mathematical model and the generalized Fisher – Kolmogorov mathematical model. Information on the solvability of the studied models is given. We describe both an algorithm to find an approximate solution of mathematical models of thermodynamics and hydrodynamics, and implementation of the algorithm as a program in the computer mathematics system Maple. The results of computational experiments for the studied models are presented.

Keywords: evolution equation; Dzektser mathematical model; generalized Fisher – Kolmogorov mathematical model; numerical solution; projection method.

Introduction

Let Ω be a bounded domain in $\mathbb{R}^d$ with infinitely smooth boundary $\partial\Omega$. Consider the Cauchy problem

$$w(x,0) = w_0(x), \quad x \in \Omega \tag{1}$$

for *Dzektser mathematical model*

$$(\lambda - \Delta)w_t = (\beta\Delta - \alpha\Delta^2)w, \qquad \lambda, \beta \in \mathbb{R}, \alpha \in \mathbb{R}_+, \tag{2}$$

$$w = \Delta w = 0 \quad \text{on} \quad \partial\Omega. \tag{3}$$

Note that the equation (2) is a generalization of the equation of groundwater movement with a free surface [1] and simulates an evolution of the filtering liquid free surface. The equation (2) is called the Dzekzer equation [2]. Point out that the operator on the left in the equation (2) can be degenerate for some values of the parameter λ. Therefore the equation belongs to the large class of nonclassical equations of mathematical physics[3].

We also consider the generalized linearized *Fisher – Kolmogorov mathematical model*

$$\frac{\partial w}{\partial t} = -\alpha\Delta^2 w + \beta\Delta w + \gamma w, \qquad (x,t) \in \Omega \times (0,T], \tag{4}$$

$$\frac{\partial w}{\partial n} = \frac{\partial \Delta w}{\partial n} = 0 \quad \text{on} \quad \partial\Omega \tag{5}$$

with the initial condition (1), where Ω is a bounded domain in $\mathbb{R}^d, d \leq 2$ with boundary $\partial\Omega$, and $\alpha > 0$ is a hyperdiffusion coefficient. The equation (4) is a generalization of the classical Fisher –Kolmogorov equation for $\alpha = 0$, which appeared recently in the study of phase transitions of critical points (Lifshitz points) [4] and was studied as a high-order model equation for the bistable systems [5].

Both models can be considered within the abstract Sobolev type equation [6]

$$L\dot{u} = Mu, \tag{6}$$

with operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ and $M \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$ in the Banach spaces $\mathfrak{U}$ и $\mathfrak{F}$. The vector function $u \in C^\infty(\mathbb{R}_+;\mathfrak{U})$ is said to be the solution of the equation (6), if we get an identity after substituting $u \in C^\infty(\mathbb{R}_+;\mathfrak{U})$ in (6). The solution $u = u(t)$ of such equation is called the solution of the Cauchy problem

$$u(0) = u_0, \tag{7}$$

if in addition it satisfies the Cauchy condition (7) for some $u_0 \in \mathfrak{U}$. Several researchers (see, for example, [7, 8]) note that the Cauchy problem (7) for the equation (6) is unsolvable for arbitrary initial data. To solve this problem, we apply the phase space method [6, 7]. Note that for the first time for numerical study the method was applied in the finite-dimensional case in [9].

The article is devoted to analytical and numerical study of the models (1)–(3) and (1),(4),(5) for numerical study we apply a method developed for quasi-Sobolev spaces [10]. The results confirm an effectiveness of the developed numerical method to find an approximate solution for the studied models.

1. Solvability of the Dzektser and Fisher – Kolmogorov mathematical models

Consider *the Dzektser mathematical model* (2), (3) in spaces $\mathfrak{U}$=$W_q^{m+2}(\Omega)$ and $\mathfrak{F}$=$W_q^m(\Omega)$, $m \in \mathbb{R}$ for $q = 2$. As is well known, in this case the Laplace operator spectrum is real-valued, nonpositive, discrete, finite-fold, and condensed only to the point $-\infty$. Denote the spectrum of the operator Δ by $\{\nu_k\} \subset \mathbb{R}_-$, and denote the corresponding eigenfunctions by $\{\varphi_k\} \subset W_q^m(\Omega)$. Here the spectrum points are numbered according to nondecreasing, taking into account their multiplicity. Since the eigenfunctions of the Laplace operator form a basis, then any function from $W_q^m(\Omega)$ can be represented as $w = \sum\limits_{k=1}^{\infty} u_k\varphi_k$.

We set a definitional domain dom$(- \alpha\Delta^2 + \beta\Delta)$ =$W_q^{m+4}(\Omega)$. Then the operators are $L = (\lambda - \Delta) \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$, $M = (- \alpha\Delta^2 + \beta\Delta) \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$.

The relative spectrum for the equation (2) has the form

$$\sigma^L(M) = \left\{\mu_k : \mu_k = \frac{\beta\nu_k - \alpha\nu_k^2}{\lambda - \nu_k} \quad \text{for} \quad k \in \mathbb{N} : \nu_k \neq \lambda \right\}. \tag{8}$$

It is clear that $\mu_k \to -\infty$ for $\nu_k \to -\infty$. Hence, in view of [6], there exists a resolving semigroup for the equation (2) and it has the form

$$U^t = \begin{cases} \sum_{k=1}^{\infty} e^{\mu_k t} <., \varphi_k > \varphi_k, & \text{if } \lambda_k \neq \lambda \text{ for all } k \in \mathbb{N}; \\ \sum_{k\in\mathbb{N}:k\neq\ell} e^{\mu_k t} <., \varphi_k > \varphi_k, & \text{if there exists } \quad \ell \in \mathbb{N}:\lambda_\ell = \lambda. \end{cases}$$

In view of [6], there are the following theorems.

Theorem 1. *For any* $m, \lambda, \beta \in \mathbb{R}$, $q, \alpha \in \mathbb{R}_+$ *a phase space of the model* (2), (3) *is a set*

$$\mathfrak{U}^1 = \begin{cases} \mathfrak{U}, & \text{if } \nu_k \neq \lambda \quad \text{for} \quad \text{all } k \in \mathbb{N}; \\ \{u \in \mathfrak{U}\colon <u, \varphi_k> = 0, & \text{if } k \in \mathbb{N}: \ \nu_k = \lambda\}. \end{cases}$$

Theorem 2. *For any* $m, \lambda, \beta \in \mathbb{R}$, $\tau, \alpha \in \mathbb{R}_+$ *and for any* $w_0 \in \mathfrak{U}^1$ *there exists a unique solution* $w \in C^1((0, \tau); \mathfrak{U})$ *of the problem* (1) *for the model* (2), (3) *of the form*

$$w(t) = \sum_{k=1}^{\infty} e^{\mu_k t} <w_0, \varphi_k> \varphi_k, \qquad \text{where } \mu_k \text{ from } (8).$$

Now consider *the Fisher – Kolmogorov mathematical model* (4), (5) in spaces $\mathfrak{U} = W_q^{m+2}(\Omega)$ and $\mathfrak{F} = W_q^m(\Omega)$, $m \in \mathbb{R}$, $q = 2$.

We set a definitional domain $\mathrm{dom}(-\alpha\Delta^2 + \beta\Delta + \gamma) = W_q^{m+4}(\Omega)$. Then the operators are $L = I \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$, $M = (-\alpha\Delta^2 + \beta\Delta + \gamma) \in \mathcal{C}l(\mathfrak{U}; \mathfrak{F})$.

In view of [6], there are the following theorems.

Theorem 3. *For any* $m, \beta, \gamma \in \mathbb{R}$, $\alpha \in \mathbb{R}_+$ *a phase space of the model* (4), (5) *is a set* $\mathfrak{U}$.

Theorem 4. *For any* $m, \beta, \gamma \in \mathbb{R}$, $\tau, \alpha \in \mathbb{R}_+$, $w_0 \in \mathfrak{U}$ *there exists a unique solution* $w \in C^1((0, \tau); \mathfrak{U})$ *of the problem* (1) *for the model* (4), (5), *which has the form*

$$w(t) = \sum_{k=1}^{\infty} e^{\mu_k t} <w_0, \varphi_k> \varphi_k, \qquad \text{where } \mu_k = -\alpha\nu_k^2 + \beta\nu_k + \gamma.$$

2. Numerical study of models

Let $\mathfrak{U} = W_q^{m+2n}(\Omega)$, $\mathfrak{F} = W_q^m(\Omega)$, $m \in \{0\} \cup \mathbb{N}$, $q \geq 2$, $Q_n(\lambda) = \sum\limits_{i=0}^{n} c_i \lambda^i$ and $R_s(\lambda) = \sum\limits_{j=0}^{s} d_j \lambda^j$ be polynomials with real coefficients such that the polynomials do not have common roots and powers of the polynomials are n and s, respectively, where $n < s$ and $d_s c_n < 0$, the operators are $L = Q_n(\Delta)$, $M = R_s(\Delta)$. Let $\{\lambda_k\}$ be a set of eigenvalues of the operator $-\Delta$ with a homogeneous Dirichlet condition, numbered in nondecreasing order taking into account the multiplicity, and let $\{\varphi_k\}$ be a family of corresponding eigenfunctions, orthonormalized with respect to the scalar product $< \cdot, \cdot >$ from $L^2(\Omega)$. Since the family $\{\varphi_k\}$ forms a basis in the space $L^2(\Omega)$, then the initial function can be expanded in a Fourier series $w_0(x) = \sum\limits_{k=1}^{\infty} < w_0, \varphi_k > \varphi_k(x)$, $x \in \Omega$.In order to find the approximate solution $\tilde{w}(x,t)$ we use the representation $\tilde{w}(x,t) = w^N(x,t) = \sum\limits_{k=1}^{N} \tilde{u}_k(t)\varphi_k(x)$, where both the number $N \in \mathbb{N}$ and the approximate solution in quasi-Sobolev spaces $\tilde{u}(t) = \{\tilde{u}_k(t)\}$ are found by the previously developed numerical method [10] for a given accuracy ε.

The developed algorithm of the numerical method to study this class of evolutionary mathematical models is implemented in a complex of programs in the computer mathematics system Maple 15.0.

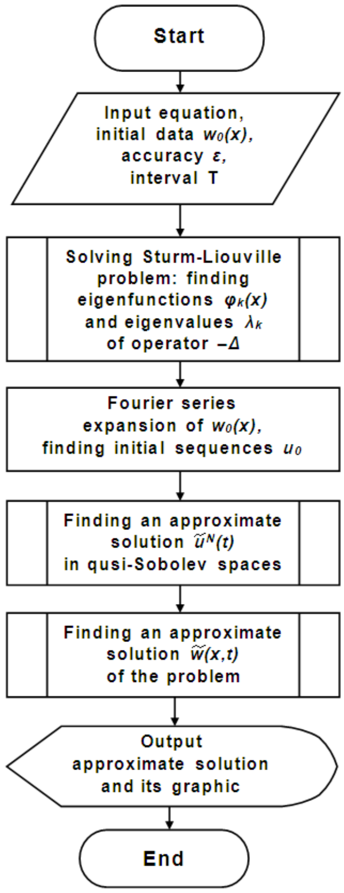

Fig. 1. A block diagram of the algorithm

Functionality of the program, range of the program application

The complex consists of two modules. First module allows to obtain a numerical solution of the Cauchy problem for the class of equations in quasi-Sobolev spaces. Second module allows to to solve the Cauchy problem for the Dzekzer and Fisher –Kolmogorov models on a closed interval or in a rectangle, depending on the given parameters and the initial data for the quasinorm parameter $q \geq 2$. The phase space method and the modified projection method are implemented in the program complex. The program allows to draw graphs of components of the numerical solution in quasi-Sobolev spaces and graphs of the solutions of the considered models as a function of time and spatial variables.

Description of the logical structure

The first module of the program complex is described in [10].

Fig. 1 shows a scheme for the algorithm of the second module of the program complex.

The developed program allows:

1. Enter the parameters of the equation and the initial function.
2. Take into account the degeneracy of the mathematical model and apply the phase space method.
3. Find and show an approximate solution of the problem.
4. Get a graphical representation of the solution as a function of time and spatial variables.

A detailed description of the algorithm (here each block of the diagram corresponds to one step) is the following:

Step 1. Start the program and require to enter parameters of the Dzektser or Fisher-Kolmogorov equation, an initial function $w_0(x)$, a time interval $T : t \in [0, T]$ and an accuracy of the approximate solution ε.

Step 2. Construct polynomials $Q_n(x) = \sum_{k=1}^{n} c_k x^k$, $R_s(x) = \sum_{k=1}^{s} d_k x^k$ by the equation parameters.

Step 3. Solve Sturm-Liouville problem: find eigenfunctions and eigenvalues of the operator $-\Delta$ in the corresponding domain. That is, obtain a monotonically increasing sequence $\{\lambda_k\}$ and a sequence of eigenfunctions $\{\varphi_k\}$.

Step 4. Expand the initial function in a Fourier series of eigenfunctions. Obtain the initial sequence u_0 from its coefficients.

Step 5. Start the first module [10], and find an approximate solution in quasi-Sobolev spaces by data obtained in the previous steps.

Step 6. Obtain an approximate solution of the initial model in the form $\sum_{k=1}^{N} \tilde{u}_k(t)\varphi_k(x)$.

Step 7. Display the obtained solution both as a function and as a graph.

Technical means used

In order to implement the computational algorithms, we use the built-in functions and standard operators of the computer mathematics system Maple 15.0. In order to create dialog boxes, we connect the package called the Maplets[Elements]. In order to obtain a graphic image, we connect the package called plots. The author of the program created a M-file to study (to find a solution) one class of evolutionary models in quasi-Sobolev spaces, as well as the Dzektser and Fisher – Kolmogorov mathematical models on a closed interval or in a rectangle. The program is operated on a personal computer having Intel (80×86) platform, running under Microsoft Windows.

Output data

The output data is an on-screen display of both the solution components $u_k(t)$ and a graph of the solution $w(x, t)$ at certain time intervals.

3. Computational experiments for the Dzektser and Fisher – Kolmogorov mathematical models

Let us give the results of computational experiments for *the Dzektser mathematical model*

$$(\lambda - \Delta)w_t = (\beta\Delta - \alpha\Delta^2)w, \qquad \lambda, \beta \in \mathbb{R}, \alpha \in \mathbb{R}_+ \tag{9}$$

$$w(x, 0) = w_0(x), \quad x \in [0, l] \tag{10}$$

$$w(0, t) = w(l, t) = w_{xx}(0, t) = w_{xx}(l, t) = 0, \quad t \in [0, T] \tag{11}$$

in the Banach spaces $\mathfrak{U} = W_2^{m+2}(0, l)$ and $\mathfrak{F} = W_2^m(0, l)$, $m \in \{0\} \cup \mathbb{N}$.

Example 1. The problem is to find the numerical solution of the mathematical model (9) – (11), where $\lambda = -4$, $\beta = 0$, $\alpha = 1, m = 1, l = \pi, T = 0.4$. Obviously, $\lambda_k = k^2$ and $\varphi_k = \sin kx$ are eigenvalues and eigenfunctions of the operator $-\Delta$ with homogeneous

Dirichlet boundary condition. Assume

$$w_0(x) = \sum_{k\neq 2} \frac{1}{k^2} \sin kx.$$

The mathematical model (9) – (11) is degenerate, but the initial function belongs to the phase space of the equation (9). Then for a given accuracy $\varepsilon = 0.1$, as a result of the program, we get

$\widetilde{u}(t)$=($e^{0.33t}$, 0, $0.11e^{-16.2t}$, 0, 0,..., 0,...) , which are coefficients of the approximate solution. Fig. 2 presents their graphs. Since $\mu_1 = 0.33 > 0$, then, in view of [11], the equation (9) has an exponential dichotomy. It can be seen from Fig. 2 that the first component of the solution grows and the others decrease.

The approximate solution has the form

$\tilde{w}(x,t)$=$e^{0.33t} \sin x + 0.11e^{-16.2t} \sin 3x$.

The graph of the approximate solution is shown in Fig. 3.

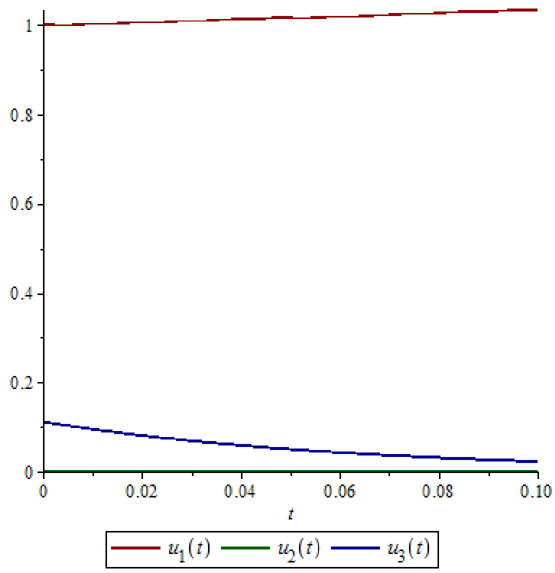

Fig. 2. Components of the solution from the example 1

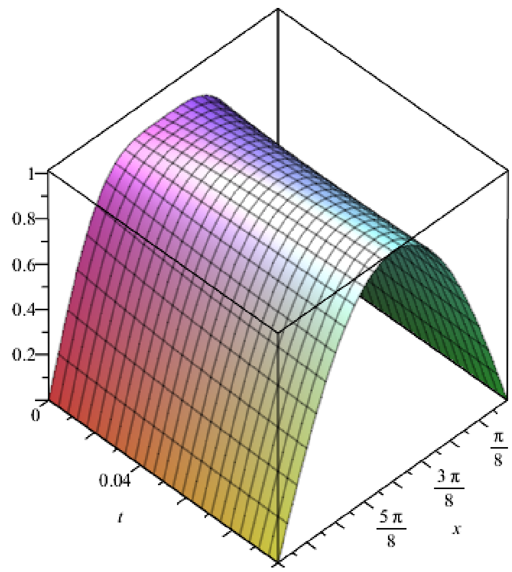

Fig. 3. Solution graph from the example 1

Example 2. The problem is to find the numerical solution of the mathematical model (9) – (11), where $\lambda = -4$, $\beta = 0$, $\alpha = 1, m = 1, l = \pi, T = 0.4$. Obviously, $\lambda_k = k^2$ and $\varphi_k = \sin kx$ are eigenvalues and eigenfunctions of the operator $-\Delta$ with the homogeneous Dirichlet boundary condition, respectively. Assume

$$w_0(x) = \sin x + 3 \sin 3x.$$

The mathematical model (9) – (11) is degenerate. In this case, the initial function does not belong to the phase space of the equation (9) and the program gives the message: "No solutions".

Example 3. The problem is to find the numerical solution of the mathematical model (9) – (11), where $\lambda = -4,\ \beta = 0,\ \alpha = 1, m = 1, l = \pi, T = 0.4$. Assume

$$w_0(x) = \sin x + 3\sin 3x.$$

The mathematical model (9) – (11) is degenerate, but the initial function belongs to the phase space of the equation (9). Then for a given accuracy $\varepsilon = 0.1$, as a result of the program, we get

$\widetilde{u}(t)$=($e^{0.33t}$, 0, $3e^{-16.2t}$, 0, 0,..., 0,...), which are the coefficients of the approximate solution. Fig. 4 shows their graphs. Since $\mu_1 = 0.33 > 0$, then, in view of [11], the equation (9) has an exponential dichotomy. Fig. 4 shows that the first component of the solution grows and the others decrease.

The approximate solution has the form

$\tilde{w}(x,t)$=$e^{0.33t}\sin x + 3e^{-16.2t}\sin 3x$.

The approximate solution graph is shown in Fig. 5.

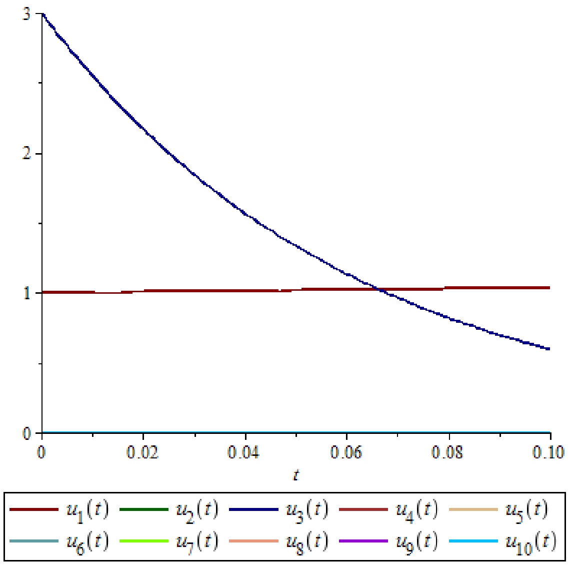

Fig. 4. The components of the solution from the example 3

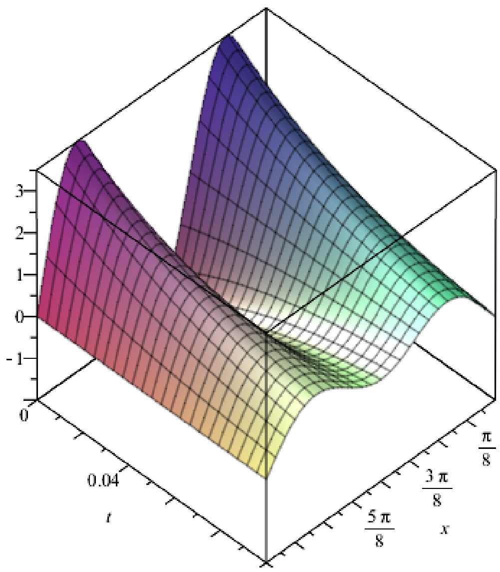

Fig. 5. Solution graph from the example 3

Let us give the results of computational experiments for the *Fisher – Kolmogorov mathematical model*

$$w_t = (-\alpha\Delta^2 - \beta\Delta + \gamma)w, \qquad \beta,\gamma \in \mathbb{R},\ \ \alpha \in \mathbb{R}_+, \tag{12}$$

$$w(x,0) = w_0(x), \quad x \in [0,l] \tag{13}$$

$$w_x(0,t) = w_x(l,t) = w_{xxx}(0,t) = w_{xxx}(l,t) = 0, \quad t \in [0,T] \tag{14}$$

in the Banach spaces $\mathfrak{U} = W_2^{m+2}(0,l)$ and $\mathfrak{F} = W_2^m(0,l)$, $m \in \{0\} \cup \mathbb{N}$.

Example 4. The problem is to find the numerical solution of the mathematical model (12) – (14), where $\alpha = 1,\ \beta = 1,\ \gamma = -15,\ m = 0, l = \pi, T = 0.4$. Obviously, $\lambda_k = k^2$

and $\varphi_k = \cos kx$ are eigenvalues and eigenfunctions of the operator $-\Delta$ with homogeneous Dirichlet boundary condition. Assume

$$w_0(x) = \sum_{k=1}^{\infty} \frac{1}{k^2} \cos kx.$$

Then for a given accuracy $\varepsilon = 0.001$, as a result of the program, we get

$\widetilde{u}(t)$=(e^{15t}, $0.25e^{3t}$, $0.11e^{-57t}$, $0.06e^{-225t}$, $0.04e^{-585t}$, $0.03e^{-1245t}$, $0.02e^{-2337t}$, 0, 0,..., 0,...) which are coefficients of the approximate solution. Fig. 6 presents their graphs.

Since $\mu_1 = 15 > 0$ and $\mu_2 = 3 > 0$, then, in view of [11], the equation (12) has an exponential dichotomy. It can be seen from Fig. 6 that the first and the second components of the solution grows and the others decrease.

The approximate solution has the form

$\widetilde{w}(x,t) = e^{15t}\cos x + 0.25e^{3t}\cos 2x + 0.11e^{-57t}\cos 3x + 0.06e^{-225t}\cos 4x + 0.04e^{-585t}\cos 5x + 0.03e^{-1245t}\cos 6x + 0.02e^{-2337t}\cos 7x$.

The approximate solution graph is shown in Fig. 7. The solution is unstable, because there are exponential dichotomies [11].

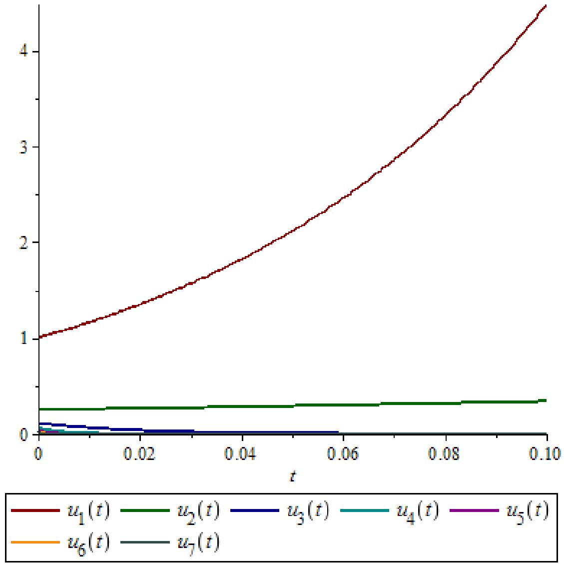

Fig. 6. Solution components from the example 4

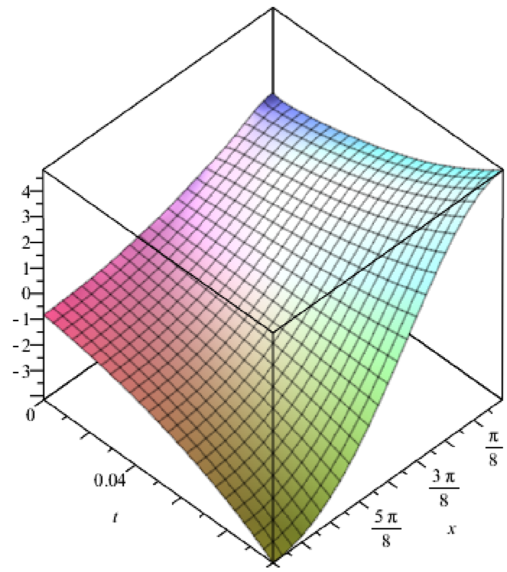

Fig. 7. Solution graph from the example 4

Example 5. The problem is to find the numerical solution of the mathematical model (12) – (14), where $\alpha = 1$, $\beta = 12$, $\gamma = -12$, $m = 0, l = \pi, T = 0.4$, with the initial function

$$w_0(x) = \sum_{k=1}^{\infty} \frac{1}{k^2} \cos kx.$$

Then for a given accuracy $\varepsilon = 0.001$, as a result of the program, we get

$\widetilde{u}(t)$=(e^{-t}, $0.25e^{20t}$, $0.11e^{15t}$, $0.06e^{-76t}$, $0.04e^{-337t}$, $0.03e^{-876t}$, $0.02e^{-1825t}$, 0, 0,..., 0,...) which are coefficients of the approximate solution. Fig. 8 presents their graphs. Since

$\mu_2 = 20 > 0$ and $\mu_3 = 15 > 0$, then, in view of [11], the equation (12) has an exponential dichotomy. It can be seen from Fig. 8 that the second and the third components of the solution grow and the others decrease.

The approximate solution has the form

$\tilde{w}(x,t) = e^{-t}\cos x + 0.25e^{20t}\cos 2x + 0.11e^{15t}\cos 3x + 0.06e^{-76t}\cos 4x + 0.04e^{-337t}\cos 5x + 0.03e^{-876t}\cos 6x + 0.02e^{-1825t}\cos 7x$.

The approximate solution graph is shown in Fig. 9. The solution is unstable, because there are exponential dichotomies [11].

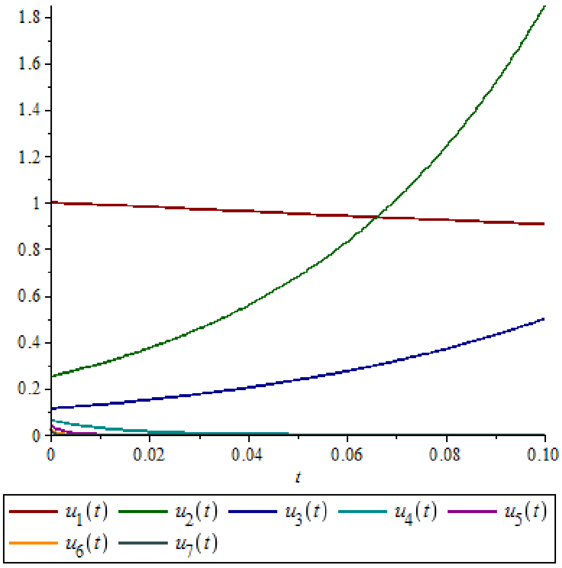

Fig. 8. Solution components from the example 5

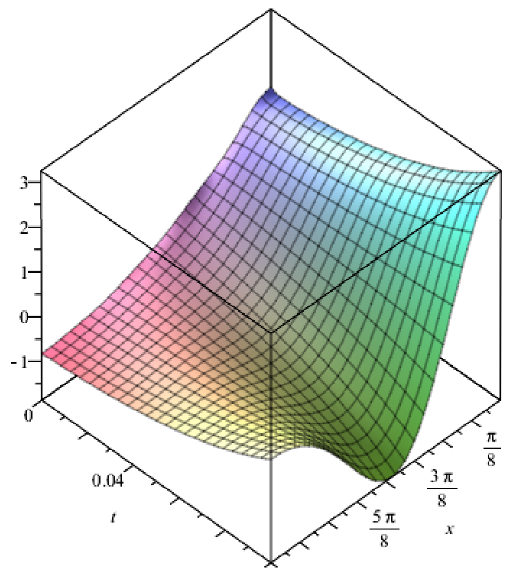

Fig. 9. Solution graph from the example 5

Remark 1. We note that the exponential dichotomies illustrated by the numerical experiments are in agreement with the results obtained earlier [12].

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ВЫЧИСЛИТЕЛЬНЫЕ ЭКСПЕРИМЕНТЫ ДЛЯ ОДНОГО КЛАССА МАТЕМАТИЧЕСКИХ МОДЕЛЕЙ ТЕРМО- И ГИДРОДИНАМИКИ

Д.К.Т. Аль Исави

Статья содержит результаты вычислительных экспериментов для математической модели Дзекцера и обобщенной математической модели Фишера – Колмогорова. Приводятся сведения о разрешимости исследуемых моделей. Описан алгоритм метода нахождения приближенного решения математических моделей термо- и гидродинамики и его реализация в виде программы в среде Maple. Представлены результаты вычислительных экспериментов для исследуемых моделей.

Ключевые слова: эволюционное уравнение; математическая модель Дзекцера; обобщенная математическая модель Фишера – Колмогорова; численное решение; проекционный метод.

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