ENGINEERING MATHEMATICS

SIMULATION MODELING OF RADIATION FIELD IN BIOLOGICAL TISSUE OF MAMMARY GLAND

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In this paper we develop a mathematical model of the distribution of microwave electric field in heterogenous biological tissue of mammary gland. We use this model to investigate the efficiency of the medical diagnostic method based on microwave thermometry. Also we run a numerical modeling of electromagnetic field in biotissue of mammary gland for various sets of the spatial structure of biotissue. The small-scale structure is caused by a complex combination of several components: blood flows, fat tissue, muscle tissue, milk lobules, skin. Next we vary in the model the spatial structure of the tissue to evaluate the effect of the heterogeneous structure of the tissue on the distribution of the electromagnetic field in the volume of mammary gland.

Keywords: mathematical modeling, numerical methods, biological tissues, oncology, microwave radiothermometry, heat transfer, radiation fields.

Introduction

Among the various methods for cancer detection, radiothermometry is extremely important, because it allows rapid mass screening [1, 2, 3, 4]. The method is based on the experimental measurement of the temperature inside the biological tissue at various points. We will discuss the results of a series of simulation modeling of radiation fields in the microwave range for the agenda to increase the efficiency of radio thermometry method of cancer detection [5].

One of the problems is the variability of measurements of the temperature field and individual physiological characteristics of biological tissue in different people [6, 7]. We base our model on numerical integration of the Maxwell equations. The complex spatial structure of the tissue and its heterogeneity on small scales requires the use of unstructured numerical grids to calculate the electric field [8].

We compare the distribution of the radiation field $\vec{E}(\vec{r})$ and its power P_d for different models against each other.

1. Basic Models

1.1. Formulation of the Problem

We consider a model of the mammary gland in the form of a hemisphere with an adjacent cylinder (fig. 1). The main internal components are muscle and fat tissue, breast lobules, blood, and skin.
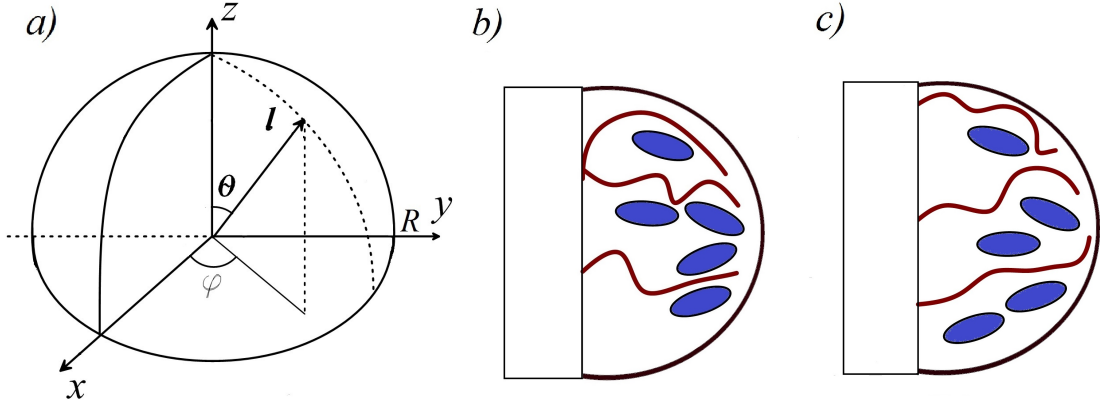
Fig. 1. The geometry of the breast measurement model (a), Schematic representation of two different internal structures of the tissue (b, c)

There is a natural variation of this internal structure in women. Individual differences can be significant, and this leads to differences in spatial distributions of physical parameters of biological tissues. We use a microwave antenna in the range 1–1.5 GHz to measure the internal temperature. Formation of electromagnetic field inside the tissue depends on its conductivity σ, permittivity ε and resistivity $\mathcal{R}$. Various biological components have different values of σ, ε, $\mathcal{R}$. We build sets of models with different spatial structure within the limits of natural variability and we calculate the electric field for each model. The simulation modeling allows us to estimate an error of internal temperature measurement by the method of microwave thermometry [3, 9].

Each experiment in the series is different in number and location of the main components (milk lobules, connective and fat tissues, blood flows). The radius of the breast is the same in all models, the antenna parameters are fixed. Each component has the same set of characteristics σ, ε, $\mathcal{R}$ (Table 1).

We fix the number of breast lobules and total length of the blood streams, and we vary only the spatial location.

1.2. Small-scale Internal Structure of Biotissue

Here we consider two series of numerical experiments: 1) the antenna is located in the nipple area (let's call it the point "0"); 2) the antenna is dislocated from the point "0" by 45° (fig. 1).

We restrict ourselves to the set of models, where among other structural components just milk lobules (mg) and blood streams (bl) can vary their position. They are surrounded by connective and fat tissue (mus). Skin parameters are fixed (sk). The relative volumes of all four components mg, bl, mus, sk in all models are the same:

$$\gamma_{mg} = \frac{V_{mg}}{V_0} = \frac{402 \text{ sm}^3}{1436 \text{ sm}^3} \approx 0.28, \gamma_{sk} = \frac{V_{sk}}{V_0} = \frac{143 \text{ sm}^3}{1436 \text{ sm}^3} \approx 0.1,$$

$$\gamma_{mus} = \frac{V_{mus}}{V_0} = \frac{877 \text{ sm}^3}{1436 \text{ sm}^3} \approx 0.61, \gamma_{bl} = \frac{V_{bl}}{V_0} = \frac{14 \text{ sm}^3}{1436 \text{ sm}^3} \approx 0.01, \tag{1}$$

where $\gamma_{mg} + \gamma_{mus} + \gamma_{sk} + \gamma_{bl} = 1$.

We calculate the electric fields $E(x, y, z)$ for 9 different models. They depend on the spatial distributions of the permittivity $\varepsilon(x, y, z)$, resistivity $\mathcal{R}(x, y, z)$ and conductivity $\sigma(x, y, z)$ (Table 1). For i-th model ($i = 1, ..., 9$) we have the power density $P_d = \sigma E^2/2$. Coefficients

$$\beta_{ij} = \sqrt{\int \left[\frac{\sigma_j E_j^2 - \sigma_i E_i^2}{\sigma_j E_j^2} \right]^2 dV} \tag{2}$$

reflect the influence of the internal structure (components position relative to each other) on the power density distribution of the electric field and the internal temperature.
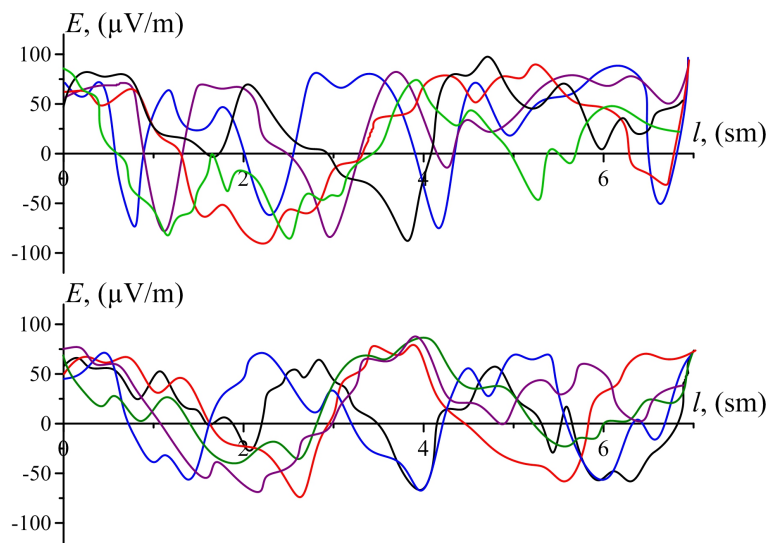

Fig. 2. Values of E along the line l for two different models

Table 1

Physical parameters of components

	Skin	Mammary gland	Connective tissue	Bloodstream
Dielectric permeability, ε	55.4	5.5	46	1.87
Electric conduction, σ (1/Ohm)	1.08	0.06	2.55	48
Resistivity, $\mathcal{R}$ (Ohm·m)	55	15	1.6	1.5

1.3. Mathematical and Numerical Models

Antenna with a frequency of a few GHz allows to measure thermal radiation from biological tissues in the frequency range $\omega_{\min} \leq \omega \leq \omega_{\max}$ [5, 10]. The biological tissue

has an inhomogeneous temperature distribution, that's why the method gives us just the weighted average temperature of some inner region $T_b^{(exp)}$. The error in the RTM method is also caused by the noise temperature of the receiver T_{REC}, the effects of the antenna mismatch ($S_{11}(\omega)$ coefficient), and environmental influence (T_{EMI}). As the result, we have to determinate the brightness temperature by the integral representation:

$$T_B^{(exp)} = \int\limits_{\omega_{\min}}^{\omega_{\max}} \left\{ s_{11} \left[T_{EMI} + \int_{V_0} W(x,y,z;\omega)\, T(x,y,z)\, dV \right] + |S_{11}(\omega)|^2 T_{REC} \right\} d\omega\,, \quad (3)$$

where $s_{11} = 1 - |S_{11}|^2$ takes into account the antenna mismatch,

$$W = \frac{P_d(x,y,z;\omega)}{\int_{V_b} P_d\, dV} \quad (4)$$

is weight function with normalization,

$$\int_{V_0} W\, dV = 1, \quad (5)$$

$$P_d = \frac{1}{2}\sigma(x,y,z;\omega) \cdot |\vec{E}(x,y,z;\omega)|^2 \quad (6)$$

is power density, and σ is electric conduction.

To construct a stationary electric field distribution, it is convenient to solve the time-dependent Maxwell equations and as the result to obtain the stationary-state:

$$\frac{\partial \vec{B}}{\partial t} + rot(\vec{E}) = 0\,, \quad \frac{\partial \vec{D}}{\partial t} - rot(\vec{H}) = 0\,, \quad \vec{B} = \mu\vec{H}\,, \quad \vec{D} = \varepsilon\vec{E}\,. \quad (7)$$

It is convenient to use a uniform grid numerical scheme $x_{i+1} = x_i + \Delta x$, $y_{j+1} = x_j + \Delta y$, $z_{k+1} = x_k + \Delta z$:

$$\frac{B_x^{n+1/2}(x_i, y_{j+1/2}, z_{k+1/2}) - B_x^{n-1/2}(x_i, y_{j+1/2}, z_{k+1/2})}{\Delta t} = \quad (8)$$

$$= \frac{E_y^n(x_i, y_{j+1/2}, z_{k+1}) - E_y^n(x_i, y_{j+1/2}, z_k)}{\Delta z} -$$

$$\frac{E_z^n(x_i, y_{j+1}, z_{k+1/2}) - E_z^n(x_i, y_j, z_{k+1/2})}{\Delta y}\,,$$

$$\frac{D_x^n(x_{i+1/2}, y_j, z_k) - D_x^{n-1}(x_{i+1/2}, y_j, z_k)}{\Delta t} = \quad (9)$$

$$= \frac{H_z^{n-1/2}(x_{i+1/2}, y_{j+1/2}, z_k) - H_z^{n-1/2}(x_{i+1/2}, y_{j-1/2}, z_k)}{\Delta y} -$$

$$-\frac{H_y^{n-1/2}(x_{i+1/2}, y_j, z_{k+1/2}) - H_y^{n-1/2}(x_{i+1/2}, y_j, z_{k-1/2})}{\Delta z}\,,$$

$$\frac{B_y^{n+1/2}(x_{i+1/2}, y_j, z_{k+1/2}) - B_y^{n-1/2}(x_{i+1/2}, y_j, z_{k+1/2})}{\Delta t} = \quad (10)$$

$$= -\frac{E_x^n(x_{i+1/2}, y_j, z_{k+1}) - E_x^n(x_{i+1/2}, y_j, z_k)}{\Delta z} +$$

$$+ \frac{E_z^n(x_{i+1}, y_j, z_{k+1/2}) - E_z^n(x_i, y_j, z_{k+1/2})}{\Delta x}.$$

The rest of equations with $\partial B_z/\partial t$, $\partial D_y/\partial t$, $\partial D_z/\partial t$ we approximate by similar finite-difference expressions. And we impose the standard constraints on Δt, Δx, Δy, and Δz to ensure the stability of an explicit numerical scheme.

$$\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} > c_{\max}\Delta t, \tag{11}$$

where c_{max} is the maximum speed of light $c_v = c/\sqrt{\varepsilon\mu}$ in the volume V_0 (it is convenient to take as $c_{\max}$ the speed of light in a vacuum).

2. Results of Simulation Modeling

<div align="right">Table 2</div>

Array β_{ij} for the angle $\alpha = 90°$

	Mod.1	Mod.2	Mod.3	Mod.4	Mod.5	Mod.6	Mod.7	Mod.8	Mod.9
Mod.1	0	0.4	0.49	0.44	0.35	0.45	0.5	0.41	0.37
Mod.2	0.4	0	0.31	0.62	0.51	0.39	0.56	0.53	0.41
Mod.3	0.49	0.31	0	0.6	0.53	0.56	0.46	0.48	0.42
Mod.4	0.44	0.62	0.6	0	0.49	0.54	0.39	0.52	0.51
Mod.5	0.35	0.51	0.53	0.49	0	0.47	0.44	0.4	0.46
Mod.6	0.45	0.39	0.56	0.54	0.47	0	0.57	0.52	0.46
Mod.7	0.5	0.56	0.46	0.39	0.44	0.57	0	0.4	0.47
Mod.8	0.41	0.53	0.48	0.52	0.4	0.52	0.4	0	0.41
Mod.9	0.37	0.41	0.42	0.51	0.46	0.46	0.47	0.41	0

<div align="right">Table 3</div>

Array β_{ij} for the angle $\alpha = 45°$

	Mod.1	Mod.2	Mod.3	Mod.4	Mod.5	Mod.6	Mod.7	Mod.8	Mod.9
Mod.1	0	0.44	0.53	0.42	0.37	0.53	0.45	0.47	0.33
Mod.2	0.44	0	0.35	0.3	0.47	0.44	0.32	0.3	0.4
Mod.3	0.53	0.35	0	0.36	0.53	0.36	0.35	0.4	0.46
Mod.4	0.42	0.3	0.36	0	0.47	0.34	0.41	0.36	0.53
Mod.5	0.37	0.47	0.53	0.47	0	0.52	0.47	0.36	0.5
Mod.6	0.53	0.44	0.36	0.34	0.52	0	0.3	0.47	0.49
Mod.7	0.45	0.32	0.35	0.41	0.47	0.3	0	0.42	0.44
Mod.8	0.47	0.3	0.4	0.36	0.36	0.47	0.42	0	0.48
Mod.9	0.33	0.4	0.46	0.53	0.5	0.49	0.44	0.48	0

The processing of results of the Maxwell equations numerical solution requires a transition to a grid in the spherical coordinate system.

Tables 2, 3 show the results of our calculations for two antenna positions on the breast surface. As we see, the relative variations in the electric field power can depend quite strongly on the spatial structure of the biotissue, and it can lead to significant individual deviations of the internal temperature in the model.

Conclusions

We create the software for numerical calculation of electric field in a mammary gland. It takes into account breast's multicomponence and heterogeneity in the distribution of physical parameters. Our model of the power distribution of the radiation field in the P_d inside the mammary gland shows strong relative variations, depending on the small-scale internal structure of the tissue. We calculate RMS deviations of the P_d for 9 different models of the internal structure. They are within $30 \div 50\%$. This result indicates the need to take into account the small-scale internal structure of the biotissue when processing and analyzing RTM measurement data.

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ИМИТАЦИОННОЕ МОДЕЛИРОВАНИЕ РАДИАЦИОННОГО ПОЛЯ В БИОТКАНИ МОЛОЧНОЙ ЖЕЛЕЗЫ

М.В. Поляков, А.В. Хоперсков, А.В. Светлов

Построена математическая модель распределения микроволнового электрического поля в неоднородной биоткани молочной железы для исследования эффективности метода медицинской диагностики на основе микроволновой термометрии. Проведено численное моделирование электромагнитного поля в молочной железе для различных наборов пространственной структуры биоткани. Мелкомасштабная структура обусловлена сложной комбинацией нескольких компонент: кровотоки, жировая ткань, мышечная ткань, молочные дольки, кожа. Варьируя пространственную структуру биоткани, мы оценили влияние этого фактора на распределение электромагнитного поля в объеме молочной железы, которое лежит в основе измерений внутренней температуры методом РТМ.

Ключевые слова: математическое моделирование, численные методы, биологические ткани, онкология, микроволновая радиотермометрия, перенос тепла, радиационные поля.

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