

# COMPUTATIONAL MATHEMATICS

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## SOLUTION TO THE PROBLEM ON IDENTIFICATION OF PARAMETERS OF UNIFORM MULTI-LEVEL PROCESS OF INFORMATION ACCUMULATION BY ROBOT

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The theory of emotional robots proposes a mathematical model that describes a uniform multi-level process of information accumulation by a robot. The process is uniquely defined by the following parameters: a piece of information that the robot receives at each event, and a coefficient of the information short-term memory of the robot. If the parameters are not known, then there is the problem of forecasting the piece of information accumulated by the robot for a certain time tact. We are the first to solve the problem on identification of the parameters of a uniform multi-level process of information accumulation by a robot on the basis of the experimental pieces of the accumulated information, which are measured in the robot at certain moments of time. The article introduces necessary restrictions. The problem is divided into two subproblems. The first is to generate all possible pairs of sequences of information accumulation levels and time steps at the levels. The second is to solve the optimization problem by the Lagrange multiplier method for each pair of sequences, which are obtained as a result of solving the first subproblem.

*Keywords: robot, theory of emotional robots, information accumulation, optimization problem, Lagrange multiplier method.*

### Introduction

One of the topical problems of modern robot technology is to construct the personal robots. According to the forecasts of the International federation of robotics, sales of personal robots in 2018-2020 will be 10.5 million units, that is about 7.5 billion US dollars [1]. Therefore, on average for the year, sales will grow by 40% compared to 2017. It is expected that this market will grow significantly over the next 20 years.

The personal robots are based on many theories and models, which allow to model the cognitive capabilities of memory, thinking and cognitive activity of a person. In the paper [2] the elements of forgetting information are added in the robot memory. Also, in the article [3] the forgetting and generalisation of information is modeled on the basis of the DataMining classification methods. In order to describe the memory, the methods from the hierarchical classification is used in [4]. The paper [5] models the memory of a robot, which recognizes people faces, extracts distinctive features from the images of faces and stores the obtained information in its memory. In order to ensure the work of such memory, the problem of memory overflow is solved by applying a "self-organizing feature card" [6] and reasoning methods in the context [7]. The papers [7, 8] use combinations of sub-symbol and symbol representation of data in order to store the information in memory. The works [9, 10] use X-clustering in order to split the memorized information into parts, and then apply the method of reasoning at a high level.

The emotional robot theory applies principles of the person emotion functioning [11, 12, 13] for modeling the accumulation and forgetting of information by a robot. Such mathematical models use parameters, which are analogues to the psychological characteristics of a person, for example, coefficients of short-term and long-term memory. We can obtain the robot behavior, which in some sense is similar to the behavior of a person, i.e. create a personalized robot, in the following way. Measure the parameters of a person and use the obtained values in constructing a robot. This approach distinguishes the theory of emotional robots from the papers mentioned above.

In the theory of emotional robots, a robot is an automatic machine, which can come to a conclusion by itself. Mathematical modeling of the information accumulation takes place under conditions of time discretization. This time is measured in tacts. At each tact, a robot receives a piece of information  $s_i$ , which is measured in bits. The qualitative side of information in current models is not considered. In the process of information accumulation, the modelled robots are able to forget a piece of previously received information. A share of information that the robot "remembers" from the information accumulated to the previous tact is called the coefficient of short-term information memory  $\lambda_i$ , where  $i$  is a number of the current tact.

According to the paper [12], an *information pseudo-setting of the robot  $h$*  is a stationary piece of information, i.e.  $h = s_0 = s_1 = \dots = s_i = \dots$ . However,  $h$  can change its value at some tact and remain constant for the sequence of the following tacts. Therefore, the paper [12] proposes to call a number of changes in the values of the information pseudo-setting of the robot  $h$  to the current time of the information accumulation by *the level of information accumulation by a robot  $l$* .

The equation of the uniform process of information accumulation by a robot for the level  $l$  is described by the formula

$$S_{i+1}^{[l]} = h^{[l]} + \lambda^{[l]} S_i^{[l]},$$

where  $\lambda^{[l]} = \lambda_0^{[l]} = \lambda_1^{[l]} = \dots = \lambda_i^{[l]} = \dots$ .

The condition for a transition from the level  $l$  to the level  $l+1$  is the requirement that the following inequality is satisfied:

$$\left| S_i^{[l]} - U^{[l]} \right| < \sigma^{[l]},$$

where  $S_i^{[l]}$  is a piece of the accumulated information at the level  $l$  to the step  $i$ ,  $U^{[l]}$  is a limit of the information accumulation by the robot on the level  $l$ ,  $\sigma^{[l]}$  is a deviation of the current piece of the accumulated information from the limit piece of the accumulated information at the level  $l$ .

The value  $\sigma^{[l]}$  is given from the outside, measured in bits and must satisfy the condition  $0 < \sigma^{[l]} < U^{[l]}$ .

The paper [11] introduces the condition of connection between the information pseudo-settings of neighboring levels. The information pseudo-setting  $h^{[l+1]}$  of a robot is equal to the piece of information accumulated at the tact of transition from the level  $l$  to the level  $l+1$ :

$$h^{[l+1]} = S^{[l]} = \lim_{i \rightarrow \infty} S_i^{[l]} - \sigma^{[l]} = \frac{h^{[l]}}{1 - \lambda} - \sigma^{[l]}, \quad (1)$$

where  $S^{[l]}$  is the piece of information accumulated at the tact of transition from the level  $l$  to the level  $l+1$ .

Convert the equation (1) and fix  $h = h^{[1]}$ . We have the following equation describing the uniform multi-level process of information accumulation by a robot:

$$S^{[l]} = \frac{h}{(1-\lambda)^{l-1}} - \sigma \frac{1-(1-\lambda)^{l-1}}{\lambda(1-\lambda)^{l-2}}, \quad l = 1, 2, \dots \quad (2)$$

Therefore, the uniform multi-level process of information accumulation by a robot, proceeding at several levels of information accumulation, is determined by the following characteristics:

1.  $\lambda$  is a coefficient of short-term information memory;
2.  $h$  is an information pseudo-setting;
3.  $\sigma$  is a value of deviation of the current accumulated information piece from the limit information piece for all levels.

Assume that  $\sigma$  is a known constant, and  $h, \lambda$  are unknown parameters of the problem.

## 1. Statement of the Parameter Identification Problem

The solution to the inverse problem is **actual**. Consider a robot, which accumulates the information by a uniform multi-level process. Suppose that the parameters  $\lambda$  and  $h$  that define this process are not known. At some points in time, we measure pieces of the information accumulated by the robot  $\bar{S}_1, \bar{S}_2, \dots, \bar{S}_n$ . It is necessary to determine the parameters  $\lambda$  and  $h$  based on the obtained experimental data  $\bar{S}_i$ , where  $i = \overline{1, n}$ .

**The purpose** of the paper is to solve the problem on identification of the parameters of a uniform multi-level process of information accumulation by a robot at several levels.

Minimize the squared deviation of the uniform multi-level process of information accumulation by the robot (2) at the node points from the set of experimental pieces of the information accumulated by the robot. We take this function as an objective function. A value of deviation of the current accumulated information piece from the limit information piece at all levels is fixed to be equal to  $\sigma$ .

Write the objective function in the following form:

$$J_{l,i}(\lambda, h) = \sum_{j=1}^n \left( \bar{S}_j - S_{i_j}^{[l_j]} \right)^2. \quad (3)$$

Divide the problem of parameter identification into the following subproblems:

1. Generate all possible sequences  $\{l\} = l_1, l_2, \dots, l_n$  and  $\{i\} = i_1, i_2, \dots, i_n$ .
2. Solve the optimization problem on finding the minimum of the objective function  $J_{l,i}(\lambda, h)$  for each generated pair of sequences  $\{l\}$  and  $\{i\}$  by the Lagrange multiplier method.

The solution to the problem of parameter identification is the point  $(\lambda^\bullet, h^\bullet) = \arg \min_{\lambda, h} \{ \min_{l, i} (J_{l, i}(\lambda, h)) \}$ .

The paper [14] gives a solution to the optimization problem on the identification of the parameters of a uniform multi-level process of information accumulation by a robot. In order to find the minimum of the objective function, the Lagrange multiplier method is used. In this article we give only the main results described in the paper [14].

The restrictions on the coefficient of short-term information memory are given by a chain of the following inequalities:

$$\delta < \lambda \leq 1 - \delta, \tag{4}$$

where  $0 < \delta \ll 1$ ,  $\delta = const$ .

The restrictions on the information pseudo-setting  $h$  are given by a chain of the following inequalities:

$$\varepsilon \leq h \leq M, \tag{5}$$

where  $0 < \varepsilon \ll 1$ ,  $\varepsilon = const$ ,  $M$  is a fixed large number.

We obtain a solution as a system of equations for finding critical points in which the objective function can have a minimum.

$$h = \frac{\sum_{j=1}^n \left( \left( \bar{S}_j - \sigma \frac{1-\lambda^{i_j+1}}{\lambda} \right) \frac{1-\lambda^{i_j+1}}{(1-\lambda)^{l_j}} \right)}{\sum_{j=1}^n \left( \frac{1-\lambda^{i_j+1}}{(1-\lambda)^{l_j}} \right)^2} + \frac{\sigma(1-\lambda)}{\lambda}.$$

$$\sum_{j=1}^n \left( \bar{S}_j \lambda - A \cdot \frac{1-\lambda^{i_j+1}}{(1-\lambda)^{l_j}} \right) \times \left( l_j h \lambda^2 \frac{1-\lambda^{i_j+1}}{(1-\lambda)^{l_j+1}} - \frac{(i_j+1) \lambda^{i_j+1}}{(1-\lambda)^{l_j}} \cdot A - \sigma \frac{1-\lambda^{i_j+1}}{(1-\lambda)^{l_j}} \cdot B \right) = 0, \tag{6}$$

where  $A = h\lambda - \sigma(1-\lambda) \left( 1 - (1-\lambda)^{l_j-1} \right)$  and  $B = (1-\lambda)^{l_j} + l_j\lambda - 1$ .

Also, we obtain another equation for finding the critical points in which the objective function can have a minimum. This equation is (6), where  $A = \varepsilon\lambda - \sigma(1-\lambda) \left( 1 - (1-\lambda)^{l_j-1} \right)$  and  $B = (1-\lambda)^{l_j} + l_j\lambda - 1$ .

According to the Weierstrass theorem (for example, see [15]), a continuous function on a non-empty bounded closed subset of a finite-dimensional space has absolute minimum. By construction, the objective function  $J_{l, i}(\lambda, h)$  is continuous on the closed domain of definition, which is formed by the restrictions (4) and (5). Therefore, the objective function  $J_{l, i}(\lambda, h)$  has a minimum, and the optimization problem has at least one solution. This means that the problem on identification of the parameters  $h$  and  $\lambda$  as a solution to the optimization problem on the set of pairs of sequences  $\{l\}$  and  $\{i\}$  also has a solution.

## 2. Solution to the Problem on Generation of all Possible Pairs of Sequences of Information Accumulation Levels and Tacts at the Levels

Without loss of generality, we assume that all pieces  $\bar{S}$  are different and numbered in ascending order, i.e.  $\bar{S}_p < \bar{S}_q$  for  $1 \leq p < q \leq n$ . If the pieces  $\bar{S}$  are not sorted, then sort them and rename the sequence. If the pieces  $\bar{S}$  are repeated, then remove the duplicates,

reduce the value  $n$  and also rename the sequence. This procedure is necessary to reduce the number of possibilities to go through the elements of the sequences  $\{l\}$  and  $\{i\}$ .

Assume that the process of information accumulation by the robot can be presented on the coordinate two-dimensional plane  $P$ , whose axes correspond to the time of the process and the accumulated information piece. Without loss of generality, we take  $\langle\langle l_u, i_u \rangle, S_u \rangle$  as coordinates of the point  $u$  on the plane  $P$ . The point  $u$  corresponds to the end of the tact  $i_u$  at the level  $l_u$  and the accumulated information piece  $S_u$ . Therefore, the coordinates  $l_u$  and  $i_u$  are the time coordinates (level number and time tact number at this level), and  $S_u$  is a coordinate of the point  $u$  along the axis of the accumulated information.

Denote a sequence of  $n$  points on the plane  $P$  by  $\langle\langle l, i \rangle, S \rangle$ , where  $l = l_1, \dots, l_n$ ,  $i = i_1, \dots, i_n$ ,  $S = S_1, \dots, S_n$ .

Reformulate the problem on identification of the parameters of a uniform multi-level process of information accumulation by a robot: find a pair of  $h$  and  $\lambda$  such that the objective function (3) has a minimum on the set of all possible sequences of points  $\langle\langle l, i \rangle, \bar{S} \rangle_\gamma$ . Here  $\gamma$  is a serial number of the generated sequence of points on the plane  $P$ , and  $\bar{S} = \bar{S}_1, \dots, \bar{S}_n$  is a set of experimental pieces of the information accumulated by the robot.

Let  $T$  be a time of the real process of information accumulation, i.e. the pieces of accumulated information are taken from the robot during the time  $T$  of the robot operation,  $\tau$  be a time of one tact of information accumulation by a robot.  $T$  and  $\tau$  are measured in seconds. Let  $L_{MAX}$  be the maximum number of information accumulation levels that a robot can pass for a time  $T$ ,  $J^{[l]}$  be a number of tacts of information accumulation at the level  $l$ .

According to the above assumptions, we obtain the equality:

$$T = \sum_{l=1}^{L_{MAX}} (\tau \cdot J^{[l]}).$$

Let  $J_{MAX} = \max_l \{J^{[l]}\}$ , where  $l = \overline{1, L_{MAX}}$ . The following equality is true:

$$\tilde{T} = \tau \cdot L_{MAX} \cdot J_{MAX}, \tag{7}$$

i.e.  $T \leq \tilde{T}$ . We convert the equality (7) and obtain the following identity:

$$J_{MAX} = \frac{\tilde{T}}{\tau \cdot L_{MAX}}.$$

Introduce the following restrictions. We go through the values  $i$  and  $l$  under the condition that  $0 \leq i \leq J_{MAX}$  and  $1 \leq l \leq L_{MAX}$ .

In order to solve the problem on parameter identification, we use the time of modelling  $\tilde{T}$  as one of the parameters. This means that for each information accumulation level  $l$  we go through  $J_{MAX}$  tacts. Therefore, the set of solutions found for the time of modelling  $\tilde{T}$  includes the set of solutions for the time  $T$  of the real process of information accumulation.

Therefore, the input data of the algorithm are the following:

1.  $\sigma$  is a value of deviation of the current accumulated information piece from the accumulated information limit piece for all levels.

2.  $\overline{S}_1, \overline{S}_2, \dots, \overline{S}_n$  is a sequence of the pieces of information accumulated by a robot (experimental values of the real process of information accumulation by a robot).
3.  $\tilde{T}$  is a time of modelling.
4.  $\tau$  is a time of one tact of information accumulation by a robot.
5.  $L_{MAX}$  is a maximum number of information accumulation levels that a robot can pass for a time  $\tilde{T}$ .

The output data of the algorithm are the following parameters:

1.  $J_{MAX}$  is a maximum number of tacts at one level of information accumulation.
2. There is a set of all sequences of points  $\langle\langle l, i \rangle, \overline{S} \rangle$  on the plane P, where  $1 \leq l_j \leq L_{MAX}$ ,  $0 \leq i_j \leq J_{MAX}$  and  $1 \leq j \leq n$ .
3.  $\lambda$  is a coefficient of the short-term information memory.
4.  $h$  is an information pseudo-setting.
5. There is a sequence of points  $\langle\langle l^*, i^* \rangle, S \rangle_{\gamma^*}$  on the plane P, where each point  $\langle\langle l_j^*, i_j^* \rangle, S_j \rangle$  corresponds to the value of the constructed uniform multi-level process of information accumulation by a robot on the tact  $i_j^*$  of the level  $l_j^*$ ,  $j = \overline{0, \tilde{t}}$ , and  $\tilde{t}$  corresponds to the last tact of the time of modelling  $\tilde{T}$ .

Let us propose **an algorithm for solving the problem on the parameters identification**:

1. Get the input parameters (see above).
2. Sort the experimental pieces of the accumulated information  $\overline{S}_1, \overline{S}_2, \dots, \overline{S}_n$  in ascending order.
3. Initialize the sequence of points  $\langle\langle l, i \rangle, \overline{S} \rangle_{\gamma}$ , where  $l_j = 1$ ,  $i_j = j - 1$ ,  $j = \overline{1, n}$ .
4. Determine a value of the left-hand side of the equation (6) based on the generated sequence of points  $\langle\langle l, i \rangle, \overline{S} \rangle_{\gamma}$ .
5. Separate roots of the non-linear equation (6).
6. Clarify each separated root by the dichotomy method.
7. Save the obtained solution in the "journal".
8. If new sequence of points  $\langle\langle l, i \rangle, \overline{S} \rangle_{\gamma}$  can be generated (see in detail below), then generate the sequence and go to Step 4, otherwise go to Step 9.
9. Among the solutions recorded in the journal, determine the "best" solution to the problem on parameter identification, i.e. a solution that corresponds to the point  $(\lambda^*, h^*) = \arg \min_{\lambda, h} \{ \min_{l, i} (J_{l, i}(\lambda, h)) \}$ .

10. Based on the "best" solution, restore the uniform multi-level process of information accumulation by the robot.
11. Visualize the obtained process of information accumulation by the robot on a diagram.

In more detail, consider Step 8 of the algorithm, which is associated with the generation of a sequence of the points  $\langle \langle l, i \rangle, \overline{S} \rangle_\gamma$  on the plane  $P$ .

The experimental piece  $\overline{S}_j$  corresponds to the information accumulation level and the tact at this level, which are determined by the values  $l_j$  and  $i_j$ .

The algorithm to generate the sequence of points  $\langle \langle l, i \rangle, \overline{S} \rangle_\gamma$  is divided into two parts.

1. Generate the sequence  $\{l\}$ . From the point of view of combinatorics, the sequence  $\{l\}$  is a permutation with repetitions and satisfies the following conditions.
  - (a)  $l_j \in [1; L_{MAX}]$ , where  $j = \overline{1, n}$ ;
  - (b)  $\sum_{d=1}^{L_{MAX}} p_d = n$ , where  $p_d$  is a number of repetitions of each element from the set  $[1; L_{MAX}]$  in the permutation  $\{l\}$ , i.e. a sum of the repetitions of each element is equal to the number of elements. Note that  $p_d$  can be zero for some elements;
  - (c)  $l_k \leq l_m$ , where  $1 \leq k < m \leq n$ , i.e. the permutation elements are sorted in ascending order.
2. Generate the sequence  $\{i\}$ , which satisfies the following conditions.
  - (a)  $i_j \in [0; J_{MAX}]$ , where  $j = \overline{1, n}$ ;
  - (b)  $i = \tilde{i}^1 \oplus \tilde{i}^2 \oplus \dots \oplus \tilde{i}^{L_{MAX}}$ , where  $\oplus$  is a sign of the concatenation of permutations without repetitions  $\tilde{i}^d$ ,  $d \in [1; L_{MAX}]$ ;
  - (c)  $\tilde{i}^d = \{i_1, \dots, i_{p_d}\}$ , where  $p_d$  is a length of the permutation  $\tilde{i}^d$  and is equal to the number of repetitions of an element  $d$  from the set  $[1; L_{MAX}]$  in the permutation  $\{l\}$  (see 1.b of the conditions for generation of the sequence  $\{l\}$ );
  - (d) if  $p_d = 0$ , then  $\tilde{i}^d = \emptyset$ ;
  - (e)  $\tilde{i}_u^d < \tilde{i}_v^d$ , where  $1 \leq u < v \leq p_d$  for each permutation  $\tilde{i}^d$ , i.e. the elements of each permutation  $\tilde{i}^d$  are sorted in ascending order.

In other words, elements of the permutation  $\tilde{i}^d$  correspond to the information tacts at the information accumulation level  $d$ . The number of permutations  $\tilde{i}^d$  for the information accumulation level  $d$  is equal to the number of combinations that result when choose  $p_d$  elements from the number of elements of the set  $[0; J_{MAX}]$ , i.e.  $C_{J_{MAX}+1}^{p_d}$ . An exception is the case when  $p_d = 0$ . In this case, the number of permutations  $\tilde{i}^d$  is zero.

Therefore, a length of the sequence  $\{l\}$  is equal to the sum of lengths of the permutations  $\tilde{i}^d$ . Therefore, a length of the sequence  $\{l\}$  is equal to a length of the sequence  $\{i\}$  and a length of the sequence  $\overline{S}$ , that is  $n$ . This means that after generation of the sequences  $\{l\}$  and  $\{i\}$ , we can correspond coordinates of the point  $\langle \langle l_j, i_j \rangle, \overline{S}_j \rangle$  on

the plane  $P$  to each piece  $\overline{S}_j$ , where  $j = \overline{1, n}$ . All  $n$  points form a sequence of points  $\langle \langle l, i \rangle, \overline{S} \rangle_\gamma$ , where  $\gamma$  is a serial number of the generation of the sequences  $\{l\}$  and  $\{i\}$ .

An example of the sequences  $\{l\}$  and  $\{i\}$  after one of the generations ( $n = 9$ ,  $L_{MAX} = 7$ ,  $J_{MAX} = 4$ ) is given in Fig. 1.

	Level 1			Level 2		Level 5	Level 7		
$l$	1	1	1	2	2	5	7	7	7
$i$	0	1	2	1	2	0	0	3	4

Fig. 1. Sequences  $\{l\}$  and  $\{i\}$

**An algorithm to generate the sequence of information accumulation levels  $\{l\}$**  is described by the following steps.

1. Initialize the initial sequence  $\{l\}$  (Fig. 2 (a)).
2. If the sequence  $\{l\}$  can be generated, i.e. the previous sequence is not the final one (Fig. 2 (b)), then go to Step 3, otherwise go to Step 12.
3. Set  $j=n$ .
4. If  $l_j = L_{MAX}$ , then go to Step 5, otherwise go to Step 6.
5. Set  $j = j - 1$ . Go to Step 4.
6. Set  $l_j = l_j + 1$ .
7. If  $j \neq n$ , then go to Step 8, otherwise go to Step 11.
8. Set  $u=j + 1$ .
9. Set  $l_u = l_j$ .
10. If  $u < n$ , then set  $u = u + 1$  and go to Step 9, otherwise go to Step 11.
11. Use the permutation  $\{l\}$  to generate the sequence  $\{i\}$ . Go to Step 2.
12. End of the algorithm.

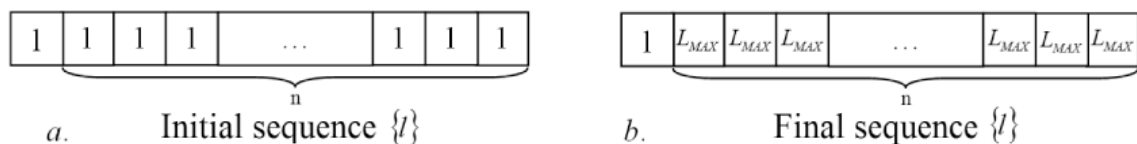


Fig. 2. Initial and final sequences  $\{l\}$



Note that steps 7–10 of the algorithm ensure that Condition 1.(c) for generation of the sequence  $\{l\}$  holds.

**An algorithm to generate the sequence of tacts  $\{i\}$**  is carried out for each generated permutation  $\{l\}$ , obtained in Step 11 of the algorithm for generation of the sequence  $\{l\}$ .

1. Select all subsequences with repeating elements  $l_j$  in the permutation  $\{l\}$ . Remember the position numbers of the first and last element of each subsequence with respect to the entire sequence  $\{l\}$ . The position numbers define the length  $p_d$  of each permutation  $\tilde{i}^d$ . Also, these position numbers are equal to the position numbers of the first and last element of the corresponding permutation  $\tilde{i}^d$  with respect to the entire sequence  $\{i\}$ . Therefore, the concatenation of permutations  $\tilde{i}$  is performed by insertion of the elements of these permutations in the sequence  $\{i\}$  in accordance with the position numbers of the first and last element of the permutation  $\tilde{i}^d$ .
2. Set  $d=1$ .
3. If the permutation  $\tilde{i}^d$  can be generated (see in detail below), i.e. the previous permutation is not final (Fig. 3 (b)), then go to Step 4, otherwise go to Step 8.
4. Generate the permutation  $\tilde{i}^d$ . In accordance with the position numbers of the first and last element of the permutation  $\tilde{i}^d$ , fill the sequence  $\{i\}$ .
5. Set  $d = d + 1$ .
6. If  $d$  does not exceed the number of the last permutation that is included in the sequence  $\{i\}$ , then go to Step 3, otherwise go to Step 7.
7. All permutations  $\tilde{i}^d$  are generated and inserted in the sequence  $\{i\}$ . Perform the algorithm for solving the optimization problem, where the input data of the algorithm are the generated sequences  $\{l\}$  and  $\{i\}$ . Go to Step 3.
8. If  $d > 1$ , then set  $d = d - 1$  and go to Step 3, otherwise go to Step 9.
9. End of the algorithm.

Note that the algorithm for generation of the permutation  $\tilde{i}^d$  repeats the algorithm for generation of the sequence  $\{l\}$  up to the notation except for the four steps. Therefore, we give only steps that are different.

1. Initialize the initial permutation  $\tilde{i}^d$  (Fig. 3 (a)).
2. If the permutation  $\tilde{i}^d$  can be generated, i.e. the previous permutation is not final (Fig. 3 (b)), then go to Step 3, otherwise go to Step 12.
- ...
4. If  $i_j = J_{MAX} - p_d + j$ , then go to Step 5, otherwise go to Step 6.
- ...
8. Set  $\tilde{i}_u^d = \tilde{i}_{u-1}^d + 1$ .
- ...

Note that steps 7–10 of the algorithm ensure that the condition 2. (e) of generation of the sequence  $\{i\}$  holds.

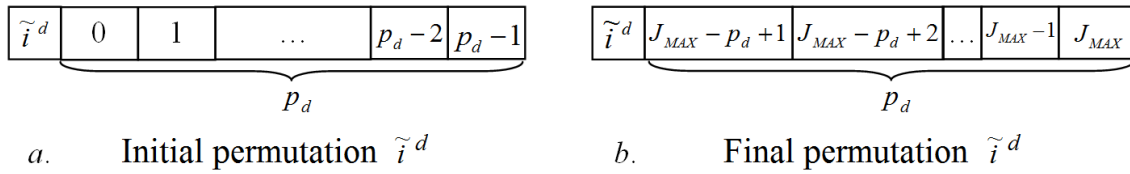


Fig. 3. Initial and final permutations  $\tilde{i}^d$

Consider an example. Let  $n = 2$ ,  $L_{MAX} = 2$ ,  $J_{MAX} = 2$ ,  $\bar{S} = \{\bar{S}_1, \bar{S}_2\}$ .

Initial sequence is  $\{l\} = \{1, 1\}$ . This means that  $\bar{S}_1$  and  $\bar{S}_2$  are at the first level of information accumulation. Therefore, for the first level  $\tilde{i}$ , the number of permutations is determined by the formula  $C_{2+1}^2 = 3$ . These permutations are used as the sequences  $\{i\}$ . All possible sequences  $\{i\}$ , which correspond to the conditions of generation, are  $\{i\} = \{0, 1\}$ ,  $\{i\} = \{0, 2\}$ ,  $\{i\} = \{1, 2\}$ .

Next sequence is  $\{l\} = \{1, 2\}$ . This means that  $\bar{S}_1$  and  $\bar{S}_2$  are at the first and second levels of information accumulation, respectively. Therefore, the sequence  $\{i\}$  is a result of concatenation of the permutations  $\tilde{i}^1$  and  $\tilde{i}^2$ , where each permutation contains one element of the set  $\{0, 1, 2\}$ . All combinations of permutations give the following sequences  $\{i\}$ , which correspond to the conditions of generation:  $\{i\} = \{0, 0\}$ ,  $\{i\} = \{0, 1\}$ ,  $\{i\} = \{0, 2\}$ ,  $\{i\} = \{1, 0\}$ ,  $\{i\} = \{1, 1\}$ ,  $\{i\} = \{1, 2\}$ ,  $\{i\} = \{2, 0\}$ ,  $\{i\} = \{2, 1\}$ ,  $\{i\} = \{2, 2\}$ .

The last sequence of levels that can be generated is  $\{l\} = \{2, 2\}$ . This means that  $\bar{S}_1$  and  $\bar{S}_2$  are at the second level of information accumulation. As in the case of the sequence  $\{l\} = \{1, 1\}$ , the number of permutations is determined by the formula  $C_{2+1}^2 = 3$ . These permutations are used as the sequences  $\{i\}$ . Therefore, the sequences of tacts are  $\{i\} = \{0, 1\}$ ,  $\{i\} = \{0, 2\}$ ,  $\{i\} = \{1, 2\}$ .

Let us consider Step 9 of the algorithm for solving the problem on parameter identification in more detail. Namely, we consider finding the "better" solution. A set of solutions to the problem is finite under the condition that  $0 \leq i \leq J_{MAX}$  and  $1 \leq l \leq L_{MAX}$ . For each sequence of points  $\langle \langle \{l, i\}, \bar{S} \rangle \rangle_\gamma$  we solve the optimization problem and determine the minimal value of the objective function (3).

Therefore, the solution to the problem on identification of the parameters  $\lambda$  and  $h$  is the minimum point of the objective function  $J_{l,i}(\lambda, h)$ , which has the minimum value on the set of all possible sequences  $\{l\}$  and  $\{i\}$ , i.e. the solution to the problem of parameter identification is the point  $(\lambda^\bullet, h^\bullet) = \arg \min_{\lambda, h} \{ \min_{l, i} (J_{l,i}(\lambda, h)) \}$ .

## Conclusion

The problem on identification of the parameters of a uniform multi-level process of information accumulation by a robot is divided into two subproblems. We give the generation algorithms and restrictions, which are necessary to solve the subproblem on generation of sequences of information accumulation levels and tacts. We solve the optimization problem by the Lagrange multiplier method and get two sets of critical points in which the objective function can have a minimum value. The existence of a solution to the optimization problem within the considered restrictions is proved.

The obtained solution to the problem on identification of the parameters of a uniform multi-level process of information accumulation by a robot allows to forecast the piece of

information accumulated at a certain tact by a robot, if there is no a priori information about the following parameters of the information accumulation:  $h$ , that is a piece of information that the robot receives at each tact, and  $\lambda$ , that is a coefficient of information short-term memory of the robot.

In order to solve the problem on identification of parameters of a uniform multi-level process of information accumulation by a robot, we implemented a program, registered it in Rospatent under the title "Solution to the optimization problem on pseudo-education of a robot" and received a certificate of state registration of computer programs no. 2014660778.

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## РЕШЕНИЕ ЗАДАЧИ ИДЕНТИФИКАЦИИ ПАРАМЕТРОВ РАВНОМЕРНОГО МНОГОУРОВНЕВОГО ПРОЦЕССА НАКОПЛЕНИЯ ИНФОРМАЦИИ РОБОТОМ

*Ю.А. Шарпов*

Теория эмоциональных роботов предлагает математическую модель, описывающую равномерный многоуровневый процесс накопления информации роботом. Такой процесс однозначно задается параметрами: порцией информации, которую получает робот на каждом такте, и коэффициентом информационной кратковременной памяти робота. В том случае, если параметры не известны, возникает проблема прогнозирования величины накопленной информации роботом для некоторого такта времени. Научная новизна статьи заключается в решении задачи идентификации параметров равномерного многоуровневого процесса накопления информации роботом на основе экспериментальных значений накопленной информации, измеренных у робота в некоторые моменты времени. В статье вводятся необходимые ограничения. Задача разделяется на две подзадачи. Первая – подзадача заключается в генерации всех возможных пар последовательностей уровней накопления информации и тактов на этих уровнях. Вторая – решение оптимизационной задачи методом множителей Лагранжа для каждой пары последовательностей, полученных в результате решения первой подзадачи.

*Ключевые слова: робот, теория эмоциональных роботов, накопление информации, оптимизационная задача, метод множителей Лагранжа.*

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