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RESEARCH OF THE MATHEMATICAL MODEL FOR CONTINUOUS PSEUDO-EDUCATION OF ROBOTS

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We consider mathematical models describing the pseudo-education of robots and the emotions of robots arising as a result of continuous impact. The article investigates the hypothesis of possibility to decompose the total pseudo-education of robots into a sum of positive and negative components. We formulate and prove the theorem on the necessary and sufficient conditions to decompose the total positive pseudo-education of an unforgiving robot into the sum of positive pseudo-education and negative pseudo-education. We show that the considered hypothesis is not true for evenly-forgetful robots. The article describes ways to determine unforgiving and not unforgiving robots.

Keywords: robot, education, emotions, mathematical modeling.

Introduction

The papers [1, 2, 3, 4] describe the mathematical models to calculate the pseudo-education of a robot, formed by both continuous impact on the robot by events and emotions of the robot arising as a result of the impact:

$$R_i = r_i + \Theta_i \cdot R_{i-1}, \tag{1}$$

where i is a number of the event that impacts on the robot and generates an elementary pseudo-education r_i of the robot, R_i is the total pseudo-education of the robot obtained as a result of the impact on the robot of i-th event, Θ_i is a memory coefficient characterizing the fraction of the previous total pseudo-education that the robot remembers at the time when the i-th event impacts on the robot, $\Theta_i \in [0; 1)$.

Let us formulate the hypotheses that the pseudo-education of a robot can be represented in the form $R_i = R_i^+ + R_i^-$, i.e. any pseudo-education is a sum of positive part R_i^+ and negative part R_i^-, where $R_i^+ > 0$, $R_i^- < 0$, and R_i^+ and R_i^- satisfy the following formulas

$$R_i^+ = r_i^+ + \Theta^+ \cdot R_{i-1}^+, \quad R_i^- = r_i^- + \Theta^- \cdot R_{i-1}^-, \tag{2}$$

where $r_i^+ > 0$ is a positive perception, $r_i^- < 0$ is a negative perception, $\Theta^+ \in [0; 1)$ is a memory coefficient of the positive perception, $\Theta^- \in [0; 1)$ is a memory coefficient of the negative perception.

According to the paper [3], a robot is called unforgiving, if the condition $\Theta^- > \Theta^+$ holds, and a robot is called not unforgiving, if $\Theta^- < \Theta^+$.

1. Model Description

Let us describe the model to determine whether a robot is unforgiving or not. It is easy to see that the condition $r_i^+ > 0$ implies $R_i^+ > 0$, and the condition $r_i^- < 0$ implies $R_i^- < 0$, where $i = \overline{1, n}$.

Let $r_i^+ = q^+ = const$, $r_i^- = q^- = const$. In this case

$$R_i^+ = q^+ \cdot \frac{1-(\Theta^+)^i}{1-\Theta^+}, \quad R_i^- = q^- \cdot \frac{1-(\Theta^-)^i}{1-\Theta^-}, \tag{3}$$

and R_i can be represented in the form

$$R_i = q^+ \cdot \frac{1-(\Theta^+)^i}{1-\Theta^+} + q^- \cdot \frac{1-(\Theta^-)^i}{1-\Theta^-}. \tag{4}$$

Let us give the definitions. A positive perception part of pseudo-education of the robot is a parameter Θ^+ satisfying the following formula:

$$\frac{1-(\Theta^+)^i}{1-\Theta^+},$$

and a negative perception part of pseudo-education of the robot is a parameter Θ^+ satisfying the following formula:

$$\frac{1-(\Theta^-)^i}{1-\Theta^-},$$

where $q^+ > 0$, $\Theta^+ \in [0;\, 1)$, $q^- < 0$, $\Theta^- \in [0;\, 1)$.

2. Research of the Model

Consider the following case. The total perception is positive, but the robot still pays attention to negative perception.

Prove the theorem that connects the parameters q^+, q^-, when $\Theta^- > \Theta^+$ and the inequality $R_i > 0$ holds.

Theorem 1. *Let the total pseudo-education be greater than zero. Then the modulus of negative pseudo-education part is less than the modulus of positive pseudo-education part if and only if the following inequality holds:*

$$\frac{q^+}{1-\Theta^+} > -\frac{q^-}{1-\Theta^-}.$$

Proof.

Necessity. Consider a robot with the general positive pseudo-education, i.e. $R_i > 0$, then the formula (4) implies that

$$q^+ \cdot \frac{1-(\Theta^+)^i}{1-\Theta^+} + q^- \cdot \frac{1-(\Theta^-)^i}{1-\Theta^-} > 0.$$

For $i \to \infty$, taking into account that $\Theta^+ \in [0;\, 1)$ and $\Theta^- \in [0;\, 1)$, we have the inequality

$$\frac{q^+}{1-\Theta^+} > -\frac{q^-}{1-\Theta^-}.$$

Therefore, if the total pseudo-education of a robot is greater than zero, then the modulus of the negative pseudo-education part is less than the modulus of the positive pseudo-education part.

Sufficiency. Consider decomposition of the pseudo-education of a robot, described by the formula (4), which implies that the parameter q^+ is the following:

$$q^+ = \left(R_i - q^- \cdot \frac{1-(\Theta^-)^i}{1-\Theta^-} \right) \cdot \frac{1-\Theta^+}{1-(\Theta^+)^i}.$$

Note that the following inequality holds: $\dfrac{q^+}{1-\Theta^+} > -\dfrac{q^-}{1-\Theta^-}$. Then, taking into account that $1-\Theta^+ > 0$, we have $q^+ > -q^- \cdot \dfrac{1-\Theta^+}{1-\Theta^-}$.

Therefore,

$$\left(R_i - q^- \cdot \frac{1-(\Theta^-)^i}{1-\Theta^-} \right) \cdot \frac{1-\Theta^+}{1-(\Theta^+)^i} > -q^- \cdot \frac{1-\Theta^+}{1-\Theta^-}.$$

This inequality implies that R_i can be represented as $R_i > q^- \cdot \dfrac{(\Theta^+)^i - (\Theta^-)^i}{1-\Theta^-}$, then for $i \to \infty$ the numerator of the fraction tends to zero, because $\Theta^+ \in [0;\,1)$ and $\Theta^- \in [0;\,1)$, i.e. $R_i > 0$. This completes the proof.

□

Consider evenly-forgetful robots [1], i.e. $R_i = q \cdot \dfrac{1-\Theta^i}{1-\Theta}$.

It is easy to see that within the framework of the hypothesis for several pseudo-educational events, starting with the first one, the following equations are true:

$$q \cdot \frac{1-\Theta^i}{1-\Theta} = q^+ \cdot \frac{1-(\Theta^+)^i}{1-\Theta^+} + R_i^- + q^- \cdot \frac{1-(\Theta^-)^i}{1-\Theta^-}, \quad i = \overline{1,n}. \tag{5}$$

Unknown parameters in the system are q, Θ, q^+, Θ^+, q^-, Θ^-. For $n=6$ the system takes the form:

$$\begin{cases} q = q^+ + q^-, \\ q \cdot \dfrac{1-\Theta^2}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^2}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^2}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^3}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^3}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^3}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^4}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^4}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^4}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^5}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^5}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^5}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^6}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^6}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^6}{1-\Theta^-}. \end{cases} \tag{6}$$

Let us investigate the solution of this system. Obviously, unknown parameters must satisfy the following conditions:

$$q > 0, \quad q^+ > 0, \quad q^- < 0, \quad \Theta \in [0;\,1)\,, \quad \Theta^+ \in [0;\,1)\,, \quad \Theta^- \in [0;\,1)\,. \tag{7}$$

The solution of the system found by Mathematica [6] does not satisfy the conditions (7). Therefore, the hypothesis for six events is not true.

Similarly, the solutions of the system for $n = 5$

$$\begin{cases} q = q^+ + q^-, \\ q \cdot \dfrac{1-\Theta^2}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^2}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^2}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^3}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^3}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^3}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^4}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^4}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^4}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^5}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^5}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^5}{1-\Theta^-} \end{cases} \tag{8}$$

and the coefficients Θ, which take values from the set $\{0, 0.05, 0.1, \ldots, 0.95\}$, also do not satisfy the conditions (7).

Solutions of the system for $n = 4$

$$\begin{cases} q = q^+ + q^-, \\ q \cdot \dfrac{1-\Theta^2}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^2}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^2}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^3}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^3}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^3}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^4}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^4}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^4}{1-\Theta^-} \end{cases} \tag{9}$$

and the coefficients Θ and Θ^+, which take values from the set $\{0, 0.05, 0.1, \ldots, 0.95\}$, also do not satisfy the conditions (7).

Also, the solutions of the system for $n = 3$

$$\begin{cases} q = q^+ + q^-, \\ q \cdot \dfrac{1-\Theta^2}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^2}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^2}{1-\Theta^-}, \\ q \cdot \dfrac{1-\Theta^3}{1-\Theta} = q^+ \cdot \dfrac{1-(\Theta^+)^3}{1-\Theta^+} + q^- \cdot \dfrac{1-(\Theta^-)^3}{1-\Theta^-} \end{cases} \tag{10}$$

and the coefficients Θ, Θ^+ and Θ^-, which take values from the set $\{0, 0.05, 0.1, \ldots, 0.95\}$, do not satisfy the conditions (7).

Therefore, for evenly-forgetting robots, the hypothesis of the decomposition of pseudo-education into a sum of the positive perception part and the negative perception part is not true.

However, for unevenly-forgetting robots, the hypothesis may be true. Consider examples of such robots.

Let the robot be not unforgiving with characteristics given in Table 1.

Table 1

Values of not unforgiving robot parameters

Parameters	q^+	Θ^+	q^-	Θ^-
Values	65	0,6	−45	0,4

The total pseudo-education R_i for values from the Table 1 is given in Fig. 1.

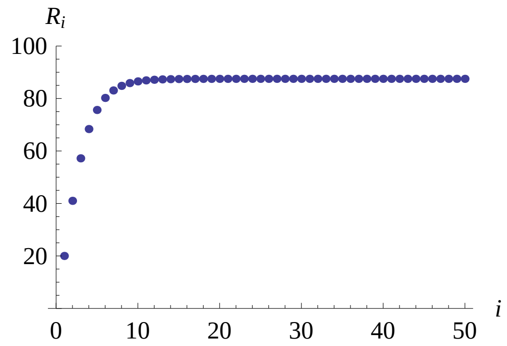

Fig. 1. Graph of pseudo-education of not unforgiving robot

Table 2 gives characteristics of the unforgiving robot.

Table 2

Values of the unforgiving robot parameters

Parameters	q^+	Θ^+	q^-	Θ^-
Values	80	0,4	-25	0,8

The total pseudo-education R_i for values from the Table 2 is given in Fig. 2.

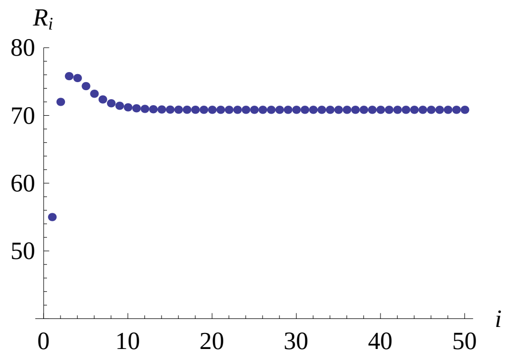

Fig. 2. Graph of pseudo-education of the unforgiving robot

Conclusion

The paper shows that the hypothesis of the decomposition of pseudo-education into positive and negative components is not true for evenly-forgetful robots, but for not evenly-forgetful robots this hypothesis is true.

Also, for not evenly-forgetful robots we can determine whether a robot is unforgiving or not, when the total pseudo-education is positive.

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ИССЛЕДОВАНИЕ МАТЕМАТИЧЕСКОЙ МОДЕЛИ НЕПРЕРЫВНОГО ПСЕВДОВОСПИТАНИЯ РОБОТОВ

Н.В. Ощепкова

В статье рассматриваются математические модели, описывающие псевдооспитание роботов и эмоции, возникающие у них в результате непрерывного воздействия. В статье проведены исследования гипотезы о возможности разложения общего псевдовоспитания роботов на сумму положительной и отрицательной компоненты. Сформулирована и доказана теорема о необходимых и достаточных условиях при разложении общего положительного псевдовоспитания злопамятного робота на сумму положительного псевдовоспитания и отрицательного псевдовоспитания. Автором показано, что рассматриваемая гипотеза не верна для равномернозабывчивых роботов.

Ключевые слова: робот, воспитание, эмоции, математическое моделирование.

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