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ONE NONCLASSICAL HIGHER ORDER MATHEMATICAL MODEL WITH ADDITIVE "WHITE NOISE"

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Sobolev type equations theory experiences an epoch of blossoming. The majority of researches is devoted to the determined equations and systems. However in natural experiments there are the mathematical models containing accidental indignation, for example, white noise. Therefore recently even more often there arise the researches devoted to the stochastic differential equations. A new conception of "white noise", originally constructed for finite dimensional spaces, is spread here to the case of infinite dimensional spaces. The main purpose is to develop stochastic higher order Sobolev type equations theory and practical applications. The main idea is in construction of "noise" spaces using the Nelson – Gliklikh derivative. Abstract results are applied for the investigation of the Boussinesq – Lòve model with additive "white noise" within the Sobolev type equations theory. At studying the methods and results of theory of Sobolev type equations with relatively p-sectorial operators are very useful. We use already well proved at the investigation of Sobolev type equations the phase space method consisting in a reduction of singular equation to regular one, defined on some subspace of initial space. In the first part of article the spaces of noises are constructed. In the second – the Cauchy problem for the stochastic Sobolev type equation of higher order is investigated. As an example the stochastic Boussinesq – Lòve model is considered.

Keywords: Sobolev type equation, propagator, "white noise", K-Wiener process.

Introduction

The Sobolev type equations make up a vast area of nonclassical equations of mathematical physics. Their systematic study started in the middle of last century after the fundamental works of S.L. Sobolev, although a lot of representatives of this class were obtained and studied earlier, in particular, the famous system of Navier – Stokes equations (see excellent review in [2]). Nowadays the investigations of the Sobolev type equations are increasing avalanche-like, we should mention several monographs adjoining our problems [16, 3, 17, 19, 1, 8]. The noncomplete Sobolev type equations of higher order

$$Lv^{(n)} = Mv + g \tag{1}$$

with the assumption $\ker L \neq \{0\}$ have been studied in different aspects [18, 20, 21]. Here the operators $L, M \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ (i.e. linear and continuous), $\mathfrak{U}$ and $\mathfrak{F}$ are Banach spaces, absolute term $g = g(t)$ models the external force, natural number $n \geq 2$. One of the prototypes of the equation (0.1) is the equation

$$(\lambda - \Delta)v_{tt} = \alpha\Delta v + f, \tag{2}$$

modelling the incompressible fluid free surface perturbation under the assumption of motion potentiality and conservation of mass in a layer [23], longitudinal vibrations of an elastic rod [22], wave processes in smectic and plasma [9].

The shortcoming of the model (2) with the deterministic absolute term consists in the fact that in natural experiments the system is exposed to random perturbation, for example in the form of white noise. The stochastic ordinary differential equations with different additive random processes (i.e. not only white noise, but more general Markov and diffusion processes) are now actively studied [5]. The traditional Ito – Stratonovich – Skorohod approach takes the priority, although new very promising directions of the research have recently appeared [10, 12].

The first results concerning the stochastic Sobolev type equations of the first order can be found in [25]. They are based on propagation of Ito – Stratonovich – Skorokhod method to the partial differential equations (see, for example, [7]). In this paper the stochastic Sobolev type equation of higher order

$$L\eta^{(n)} = M\eta + Nw \tag{3}$$

is considered. Here, in the right side, the term w denotes the random process. It is required to find the random process $\eta(t)$, satisfying (in some sense) equation (3) and the initial conditions

$$\eta^{(m)}(0) = \xi_m, \ m = 0, 1, \dots, n-1, \tag{4}$$

where ξ_m are given random variables.

Initially w was understood as white noise which is a generalized derivative of the Wiener process. Meanwhile there have appeared [12] and is actively developing [13, 6] a new approach in the investigation of equation (3), where "white noise" means the Nelson – Gliklikh derivative of the Wiener process. (Note that this "white noise" is more adequate to the theory of Brownian motion by Einstein – Smolukhovsky in comparison to traditional white noise [12, 13]). Initially the "white noise" was used in the theory of optimal measurement theory [14, 11], where the special space of "noises" was constructed [15]. The concept of "white noise" in this theory (that is only in the finite dimensional spaces) showed its high efficiency so there have appeared the idea of extending of this concept to the infinite-dimensional spaces. The main goal of this extending is the development of the theory of stochastic Sobolev type equations and elaboration of the applications of this theory to nonclassical models of mathematical physics of practical importance.

The paper in addition to the introduction includes three sections. The first one introduces the space of "noises" which is fundamental for the further constructions. The theory of abstract stochastic Sobolev type equations of higher order with relatively p-bounded operators is presented in the second section. In the third section the results obtained for abstract problem are applied to the investigation of the initial-boundary problem for the stochastic Boussinesq – Lòve equation with additive "white noise".

1. Deterministic equations with (n,p)-sectorial operators

Fundamentals of the relatively p-sectorial operators theory were laid by G.A.Sviridyuk and were developed by his disciples. We extend these ideas and methods to the case of equations of arbitrary order. Let $\mathfrak{U}$ and $\mathfrak{F}$ be separable reflexive Banach spaces, the operator $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ (linear and bounded), the operator $M \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$ (linear, closed and densely defined in $\mathfrak{U}$).

Consider the relative spectrum set $\sigma^L(M)$ and build the sets

$$\sigma_n^L(M) = \{\mu^n : \mu \in \sigma^L(M)\}, \ \rho_n^L(M) = \mathbf{C}\backslash\sigma_n^L(M)$$

and the operator-functions

$$R^L_{(\mu,p)}(M) = \prod_{k=0}^{p}(\mu_k L - M)^{-1}L,\ \ L^L_{(\mu,p)}(M) = \prod_{k=0}^{p} L(\mu_k L - M)^{-1},$$

called the right and he left (L,p)-resolvents of the operator M.

Definition 1. *The operator M is called (n,p)-sectorial with respect to operator L or (L,n,p)-sectorial if there are constants $K > 0$, $\theta \in (\pi/2, \pi)$, such that the set*

$$S^L_{\theta,n}(M) = \{\mu \in C : |\arg(\mu^n)| < \theta, \quad \mu \neq 0\} \subset \rho^L_n(M), \tag{5}$$

and

$$\max\left\{\left\|R^L_{(\mu^n,p)}(M)\right\|_{L(\mathfrak{U})}, \left\|L^L_{(\mu^n,p)}(M)\right\|_{L(\mathfrak{F})}\right\} \leq \frac{K}{\prod_{k=0}^{p}|\mu_k^n|}. \tag{6}$$

Consider the Cauchy problem

$$\lim_{t\to 0+} v^{(m)}(t) = v_m,\ \ m = 0, 1, ..., n-1 \tag{7}$$

for the deterministic Sobolev type equation (1). Take $\alpha \in \rho^l(M)$ and consider equivalent equations

$$R^L_\alpha(M)v^{(n)} = (\alpha L - M)^{-1}Mv + h_1, \tag{8}$$

$$L^L_\alpha(M)f^{(n)} = M(\alpha L - M)^{-1}f + h_2, \tag{9}$$

defined on $\mathfrak{U}$ and $\mathfrak{F}$ respectively.

Definition 2. The operator-function $V^\bullet \in C^\infty(\mathbb{R}_+; \mathcal{L}(\mathfrak{U}))$ is called a propagator of the homogenious equation (8), if for all $v \in \mathfrak{U}$ the vector-function $v(t) = V^t v$ is the solution of this equation.

The propagator of (9) is defined analogously.

Lemma 1. *Let the operator M be (L,n,p)-sectorial. Then the integrals of Dunford–Schwartz type*

$$U^t_m = \frac{1}{2\pi i}\int_\gamma \mu^{n-m-1}(\mu^n L - M)^{-1}Le^{\mu t}d\mu, \tag{10}$$

$$F^t_m = \frac{1}{2\pi i}\int_\gamma \mu^{n-m-1}L(\mu^n L - M)^{-1}e^{\mu t}d\mu, \tag{11}$$

where $t \in R_+$, $m = 0, 1, ..., n-1$, and $\gamma \subset \rho^L_n(M)$ is the contour formed by rays emanating from the origin at angles θ and $-\theta$, determine the propagators of the homogeneous equations (8) and (9).

Set

$$\mathfrak{U}^0 = \bigcap_{m=0}^{n-1} \ker U^\bullet_m = \bigcap_{m=0}^{n-1}\{\varphi \in \mathfrak{U} : U^t_0\varphi = 0\ \exists t \in \mathbb{R}_+\},$$

$$\mathfrak{F}^0 = \bigcap_{m=0}^{n-1} \ker F^\bullet_0 = \bigcap_{m=0}^{n-1}\{\psi \in \mathfrak{F} : F^t_0\psi = 0\ \exists t \in \mathbb{R}_+\}.$$

By $L_0(M_0)$ denote the restriction of the operator $L(M)$ to the subspace $\mathfrak{U}^0$.

Corollary 1. *Under the conditions of lemma 1 the operators* $L_0 \in \mathcal{L}(\mathfrak{U}^0;\mathfrak{F}^0)$, $M_0 \in \mathcal{C}l(\mathfrak{U}^0;\mathfrak{F}^0)$, *and there exists the operator* $M_0^{-1} \in \mathcal{L}(\mathfrak{F}^0;\mathfrak{U}^0)$.

Set $\mathfrak{U}^1 = \text{im}\, U_0^{\cdot} = \{u \in \mathfrak{U} : \lim\limits_{t\to 0+} U_0^t u = u\}$, $\mathfrak{F}^1 = \text{im}\, F_0^{\cdot} = \{f \in \mathfrak{F} : \lim\limits_{t\to 0+} F_0^t f = f\}$. By $L_1(M_1)$ denote the restriction of the operator $L(M)$ to the subspace $\mathfrak{U}^1$.

Corollary 2. *Under the conditions of lemma 1 the operators* $L_1 \in \mathcal{L}(\mathfrak{U}^1;\mathfrak{F}^1)$, $M_1 \in \mathcal{C}l(\mathfrak{U}^1;\mathfrak{F}^1)$.

Obviously, $\mathfrak{U}^0 \oplus \mathfrak{U}^1 \subset \mathfrak{U}$ and $\mathfrak{F}^0 \oplus \mathfrak{F}^1 \subset \mathfrak{F}$. Further we need the following assumptions:

$$\mathfrak{U}^0 \oplus \mathfrak{U}^1 = \mathfrak{U}\ (\mathfrak{F}^0 \oplus \mathfrak{F}^1 = \mathfrak{F}), \tag{12}$$

$$\text{there exists the operator} \quad L_1^{-1} \in \mathcal{L}(\mathfrak{F}^1;\mathfrak{U}^1). \tag{13}$$

The assumption(12) takes place in the case of reflexivity of the spaces $\mathfrak{U}$ ($\mathfrak{F}$) (the Yagi – Fedorov theorem [4]). The assumption (13) is true if (12) is fulfilled and im $L_1 = \mathfrak{F}^1$ (the Banach theorem). Note that (12) leads to the existence of the projectors $P = s - \lim\limits_{t\to 0+} U_0^t$ and $Q = s - \lim\limits_{t\to 0+} F_0^t$ in the spaces $\mathfrak{U}$, $\mathfrak{F}$ respectively.

Corollary 3. *Let the operator M be (L,n,p)-sectorial and (12), (13) be fulfilled. The operator $H = M_0^{-1}L_0 \in \mathcal{L}(\mathfrak{U}^0)$ is nilpotent of a degree p.*

Due to the (L,n,p)-sectoriality of the operator M and (12), (13) the equation (1) can be reduced to the form

$$H(v^0)^{(n)} = v^0 + M_0^{-1} f^0, \tag{14}$$

$$(v^1)^{(n)} = Sv^1 + L_1^{-1} f^1, \tag{15}$$

where operator $S = L_1^{-1}M_1 \in \mathcal{C}l(\mathfrak{U}^1)$, functions $f^0 = (\mathbb{I} - Q)f$, $f^1 = Qf$, $v^0 = (\mathbb{I} - P)v$, $v^1 = Pv$.

Lemma 2. *Let the operator M be (L,n,p)-sectorial and (12), (13) be fulfilled. For any vector-function $f^0 \in C^{n(p+1)}([0,T];\mathfrak{F}^0)$ there exists a unique solution of the equation (14), which is represented in the form*

$$v^0(t) = -\sum_{q=0}^{p} H^q M_0^{-1} f^{0(nq)}(t).$$

*Proof.*Substituting the vector-function $v^0 = v^0(t)$ into (14) one can verify the existence of the solution. Uniqueness is obtained in a consistent derivation of the equation (14): $0 = H^p v^{0(np)} = \ldots = Hv^{0(n)} = v^0$.

Remark 1. From Lemma 2 it directly follows that all initial values v_k need to belong to the sets

$$\mathcal{M}_f^k = \{v \in \mathfrak{U} : (\mathbb{I} - P)u = -\sum_{q=0}^{p} H^q M_0^{-1} f^{0(nq+k)}(0)\}, k = 0, ..., n-1. \tag{16}$$

Lemma 3. *Under the conditions of Lemma 2 for any $v_m \in \mathfrak{U}^1$, $m = 0, ..., n-1$ u $f^1 \in C([0,T];\mathfrak{F}^1)$ there exists a unique solution of the Cauchy problem (7) for the equation (1), which is represented in the form*

$$v^1(t) = \sum_{m=0}^{n-1} V_m^t v_m + \int_0^t V_{n-1}^{t-s} L_1^{-1} f^1(s) ds.$$

So, we have proved

Theorem 1. *Let the operator M be (L, n, p)-sectorial and (12), (13) be fulfilled. For any $u_k \in \mathcal{M}_f^k, k = 0, ..., n-1$ and vector-function $f = f(t)$, $t \in [0,T]$, satisfying the conditions of Lemmas 2, 3, there exists a unique solution of the problem (14), (15), which can be represented as $v(t) = v^0(t) + v^1(t)$.*

2. The spaces of "noises"

Let $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ be a complete probability space, $\mathbb{R}$ be a set of real numbers endowed with Boreal σ-algebra. The measurable mapping $\xi : \Omega \to \mathbb{R}$ is called *a random variable.* The set of random variables forms a Hilbert space with the scalar product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\xi_2$. This Hilbert space will be denoted by $\mathbf{L_2}$. The random variables $\xi \in \mathbf{L_2}$, with normal (Gaussian) distribution will be very important later on; they are called Gaussian random variables. Let $\mathcal{A}_0$ be σ-subalgebra of σ-algebra $\mathcal{A}$. Construct the space $\mathbf{L_2^0}$ of random variables, measurable with respect to $\mathcal{A}_0$. Obviously, $\mathbf{L_2^0}$ is a subset of $\mathbf{L_2}$; denote by $\Pi : \mathbf{L_2} \to \mathbf{L_2^0}$ the orthoprojector. Let $\xi \in \mathbf{L_2}$, then $\Pi\xi$ is called *conditional expectation* of the random variable ξ and is denoted by $\mathbf{E}(\xi|\mathcal{A}_0)$. It is easy to see that $\mathbf{E}(\xi|\mathcal{A}_0) = \mathbf{E}\xi$, if $\mathcal{A}_0 = \{\emptyset, \Omega\}$; and $\mathbf{E}(\xi|\mathcal{A}_0) = \xi$, if $\mathcal{A}_0 = \mathcal{A}$. Finally, the minimal σ-subalgebra $\mathcal{A}_0 \subset \mathcal{A}$, regarding which the random variable ξ is measurable, is called *the σ-algebra generated by ξ.*

Let $\mathfrak{I} \subset \mathbb{R}$ be some interval. Consider two mappings: $f : \mathfrak{I} \to \mathbf{L_2}$, which maps each $t \in \mathfrak{I}$ to a random variable $\xi \in \mathbf{L_2}$, and $g : \mathbf{L_2} \times \Omega \to \mathbb{R}$, which maps every pair (ξ, ω) to the point $\xi(\omega) \in \mathbb{R}$. The mapping $\eta : \mathfrak{I} \times \Omega \to \mathbb{R}$ of the form $\eta = \eta(t, \omega) = g(f(t), \omega)$ is called a *(one-dimensional) random process.* Thus, for every fixed $t \in \mathfrak{I}$ the random process $\eta = \eta(t, \cdot)$ is the random variable, i.e. $\eta(t, \cdot) \in \mathbf{L_2}$, and for every fixed $\omega \in \Omega$ the random process $\eta = \eta(\cdot, \omega)$ is called *the (sample) trajectory.* The random process η is called *continuous* if almost surely (a.s.) all its trajectories are continuous, that is, for almost every (a.e.) $\omega \in \Omega$ the trajectories $\eta(\cdot, \omega)$ are continuous. The set of continuous random processes form a Banach space, which will be denoted by $\mathbf{CL_2}$. The continuous random process, representing different t independent Gaussian random variables, is called *Gaussian.*

The (one-dimensional) Wiener process $\beta = \beta(t)$, modeling Brownian motion on the line in Einstein – Smolukhovsky theory, is one of the most important examples of the continuous Gaussian random processes. It has the following properties:

(W1) a.s. $\beta(0) = 0$, a.s. all its trajectories $\beta(t)$ are continuous, and for all $t \in \overline{\mathbb{R}}_+ (= \{0\} \cup \mathbb{R}_+)$ the random variable $\beta(t)$ is Gaussian;

(W2) the mathematical expectation $\mathbf{E}\left(\beta\left(t\right)\right) = 0$ and autocorrelation function $\mathbf{E}\left(\left(\beta\left(t\right) - \beta\left(s\right)\right)^2\right) = |t - s|$ for all $s, t \in \overline{\mathbb{R}}_+$;

(W3) the trajectories $\beta(t)$ are nondifferentiable at any point $t \in \overline{\mathbb{R}}_+$ and have unlimited variation at an arbitrarily small interval.

Theorem 2. *There exists a random process β, satisfying properties (W1), (W2); moreover, it can be represented in the form*

$$\beta(t) = \sum_{k=0}^{\infty} \xi_k \sin \frac{\pi}{2}(2k+1)t,$$

where ξ_k are independent Gaussian variables, $\mathbf{E}\xi_k = 0$, $\mathbf{D}\xi_k = [\frac{\pi}{2}(2k+1)]^{-2}$.

The random process β, satisfying properties (W1) – (W3), will be called *Brownian motion.*

Now fix $\eta \in \mathbf{CL_2}$ and $t \in \mathfrak{I}(= (\varepsilon, \tau) \subset \mathbb{R})$ and by $\mathcal{N}_t^\eta$ denote the σ-algebra generated by the random variable $\eta(t)$. For the sake of brevity, we introduce the notation $\mathbf{E}_t^\eta = \mathbf{E}(\cdot|\mathcal{N}_t^\eta)$.

Definition 3. Let $\eta \in \mathbf{CL_2}$, the random variable

$$D\eta\,(t,\cdot) = \lim_{\triangle t \to 0+} \mathbf{E}_t^\eta \left(\frac{\eta\,(t+\triangle t,\cdot) - \eta(t,\cdot)}{\triangle t} \right)$$

$$\left(D_*\eta\,(t,\cdot) = \lim_{\triangle t \to 0-} \mathbf{E}_t^\eta \left(\frac{\eta\,(t,\cdot) - \eta\,(t-\triangle t,\cdot)}{\triangle t} \right) \right),$$

is called *a forward $D\eta(t,\cdot)$ (a backward $D_*\eta(t,\cdot)$) mean derivative of the random process η at the point* $t \in (\varepsilon, \tau)$ if the limit exists in the sense of uniform metric on $\mathbb{R}$.

The random process η is called *forward (backward) mean differentiable on* (ε, τ), if for every point $t \in (\varepsilon, \tau)$ there exists the forward (backward) mean derivative.

Now let the random process $\eta \in \mathbf{CL_2}$ be forward (backward) mean differentiable on (ε, τ). Its forward (backward) mean derivative is also a random process; we denote it by $D\eta$ $(D_*\eta)$. If the random process $\eta \in \mathbf{CL_2}$ is forward (backward) mean differentiable on (ε, τ), then *the symmetric (antisymmetric) mean derivative* $D_S\eta = \frac{1}{2}\,(D + D_*)\,\eta$ $\left(D_A\eta = \frac{1}{2}\,(D_* - D)\,\eta\right)$ can be defined. Since the mean derivatives were introduced by E. Nelson [12], and the theory of these derivatives was developed by Yu.E. Gliklikh [16], the symmetric mean derivative D_S of the random process η will henceforth be called *the Nelson – Gliklikh derivative* for brevity and will be denoted by $\overset{o}{\eta}$, i.e. $D_S\eta \equiv \overset{o}{\eta}$. By $\overset{o}{\eta}^{(l)}$, $l \in \mathbb{N}$ denote the l-th Nelson – Gliklikh derivative of the random process η. Note that, if the trajectories of the random process η are a.s. continuously differentiable in a "common sense" on (ε, τ), then the Nelson – Gliklikh derivative of η coincides with the "regular"derivative. This happens, for example, in the case of the random process $\eta = \alpha \sin(\beta t)$, where α is a Gaussian random variable, $\beta \in \mathbb{R}_+$ is a fixed constant, and $t \in \mathbb{R}$ has the physical meaning of time.

Theorem 3. (Yu.E. Gliklikh) $\overset{o}{\beta}^{(l)}(t) = (-1)^{l+1}(2t)^{-l}\beta(t)$ *for all* $t \in \mathbb{R}_+$ *and* $l \in \mathbb{N}$.

Introduce the space $\mathbf{C}^l\mathbf{L}_2$, $l \in \mathbb{N}$ of random processes of $\mathbf{CL}_2$, whose trajectories are a.s. Nelson – Gliklikh differentiable on $\mathfrak{I}$ to order l inclusively. If $\mathfrak{I} \subset \mathbb{R}_+$, then due to theorem 3 there exists the derivative $\overset{o}{\beta} \in \mathbf{C}^1\mathbf{L}_2$, which will be called (*one-dimensional*) *"white noise"*. In [15] it is suggested that the spaces $\mathbf{C}^l\mathbf{L}_2$ be called *spaces of differentiable "noises"*.

Now let $\mathfrak{U} \equiv (\mathfrak{U}, \langle\cdot,\cdot\rangle)$ be a real separable Hilbert space; consider the operator $K \in \mathcal{L}(\mathfrak{U})$ with spectrum $\sigma(K)$ being nonnegative discrete with finite multiplicity tending only to zero. By $\{\nu_j\}$ denote the sequence of eigenvalues of operator K, numbered in decreasing order according to multiplicity. Note that the linear span of related orthonormal eigenfunctions $\{\varphi_j\}$ of operator K is dense in $\mathfrak{U}$. Suppose that the operator K is nuclear (i.e. its trace $\operatorname{Tr} K = \sum\limits_{j=1}^{\infty} \nu_j < +\infty$).

Take the sequence of independent random processes $\{\eta_j\}$ and define the K-random process

$$\Theta_K(t) = \sum_{j=1}^{\infty} \sqrt{\nu_j}\eta_j(t)\varphi_j, \tag{17}$$

provided that the series (2.2) converges uniformly on any compact from $\mathfrak{I}$. Note that, if $\{\eta_j\} \subset \mathbf{CL}_2$ and the K-random process Θ_K exists, then a.s. its trajectories are continuous. Denote the space of such processes by the symbol $\mathbf{C}_K \equiv \mathbf{C}_K(\mathfrak{I} \times \Omega; \mathfrak{U})$. Isolate in $\mathbf{C}_K$ the subspace $\mathbf{C}_K\mathbf{L_2}$ of random processes, whose random variables belong to $\mathbf{L_2}(\Omega;\mathfrak{U}) = \left\{\xi : \int_\Omega ||\xi(\omega)||^2 d\mathbf{P}(\omega) < +\infty\right\}$, i.e. $\eta \in \mathbf{C}_K\mathbf{L_2}$, if $\eta(t,\cdot) \in \mathbf{L_2}(\Omega;\mathfrak{U})$ for each $t \in \mathfrak{I}$. Note that the space $\mathbf{C}_K\mathbf{L_2}$ contains, in particular, those K-random processes for which almost surely all trajectories are continuous, and all (independent) random variables are Gaussian.

We now introduce the Nelson – Gliklikh derivatives of K-random process

$$\overset{o}{\Theta}{}_K^{(l)}(t) = \sum_{j=1}^{\infty} \sqrt{\nu_j}\, \overset{o}{\eta}{}_j^{(l)}(t)\varphi_j, \tag{18}$$

provided that the derivatives up to the degree of l in the right side of (18) exist and the series uniformly converges on any compact from $\mathfrak{I}$.

Similarly, introduce the space $\mathbf{C}^{\mathbf{l}}_{\mathbf{K}}\mathbf{L}_2$ of K-random processes with a.s. continuous Nelson – Gliklikh derivatives up to order $l \in \mathbb{N}$, whose random variables belong to $\mathbf{L_2}(\Omega;\mathfrak{U})$.

As an example, consider the K-Wiener process

$$W_K(t) = \sum_{j=1}^{\infty} \sqrt{\nu_j}\beta_j(t)\varphi_j, \tag{19}$$

that exists on $\overline{\mathbb{R}}_+$.

Corollary 4. $\overset{o}{W}{}_K^{(l)}(t) = (-1)^{l+1}(2t)^{-l}W_K(t)$ *for all* $t \in \mathbb{R}_+$, $l \in \mathbb{N}$ *and nuclear operator* $K \in \mathcal{L}(\mathfrak{U})$.

Moreover, the K-Wiener process (19) satisfies the conditions

(WW1) a.s. $W_K(0) = 0$, a.s. all its trajectories $\beta(t)$ are continuous, and for all $t \in \overline{\mathbb{R}}_+ (= \{0\} \cup \mathbb{R})$ the random variable $W_K(t,\cdot)$ is Gaussian;

(WW2) the mathematical expectation $\mathbf{E}\left(W_K\left(t\right)\right) = 0$ and autocorrelation function $\mathbf{E}\left(\left(\beta\left(t\right) - \beta\left(s\right)\right)^2\right) = K\left|t - s\right|$ for all $s, t \in \overline{\mathbb{R}}_+$ and the following theorem is true.

Theorem 4. *For any nuclear operator $K \in \mathcal{L}(\mathfrak{U})$, there exists a K-Wiener process, satisfying the conditions (WW1), (WW2) and it can be represented in the form (2.4).*

3. The Cauchy problem for a Sobolev type higher order equation with additive white noise

Consider the linear stochastic Sobolev type equation of higher order

$$L \overset{o}{\eta}{}^{(n)} = M\eta + Nw, \tag{20}$$

where the absolute term will be specified later. Supplement the equation (20) with *the weakened* (in the sense of S.G. Krein) initial Showalter – Sidorov condition

$$\lim_{t\to 0+} \left[R^L_\alpha(M)\right]^{p+1} \left(\overset{o}{\eta}{}^{(m)}(t) - \xi_m \right) = 0,\ m = 0, ..., n-1. \tag{21}$$

which is the generalization of the condition [3]

$$\lim_{t\to 0+} L \overset{o}{\eta}{}^{(m)}(t) = L\xi_m,\ m = 0, ..., n-1,$$

and has advantages over the Cauchy condition

$$\lim_{t\to 0+} \overset{o}{\eta}{}^{(m)}(t) = \xi_m,\ m = 0, ..., n-1 \tag{22}$$

in the case of Sobolev type equations.

Consider $\mathfrak{I} = (0, \tau)$. Let $K \in \mathcal{L}(\mathfrak{U})$ be a nuclear operator with eigenvalues $\{\nu_j\} \subset \mathbb{R}_+$. The K-random process $\eta \in \mathbf{C}^n_K\mathbf{L}_2$ is called (*a classical*) *solution of equation* (20), if a.s. all its trajectories satisfy equation (20) for some K-random process $w \in \mathbf{C}_K\mathbf{L}_2$, operator $N \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ and $t \in (0, \tau)$. The solution $\eta = \eta(t)$ of equation (20) is called (*the classical*) *solution* of problem (20), (21) if the condition (21) is also fulfilled.

Consider firstly the problem (22) for the homogeneous equation

$$L \overset{o}{\eta}{}^{(n)} = M\eta. \tag{23}$$

In this case (and only in this case) consider $\mathfrak{I} = \mathbb{R}_+$.

Definition 4. The set $\mathfrak{P} \subset \mathfrak{U}$ is called *the phase space* of equation (23) if

(i) a.s. every trajectory of the solution $\eta = \eta(t)$ lies in $\mathfrak{P}$ pointwise, i.e. $\eta(t) \in \mathfrak{P}$ for all $t \in \mathbb{R}_+$;

(ii) for all random variables $\xi_m \in L_2(\Omega; \mathfrak{P}),\ m = 0, 1, \dots, n-1$, there exists a unique solution $\eta \in \mathbf{C}^n_K\mathbf{L}_2$ of the problem (22), (23).

Theorem 5. *Let the operator M be (L, n, p)-sectorial, $p \in \{0\} \cup \mathbb{N}$ and (12), (13) be fulfilled. Then the subspace $\mathfrak{U}^1$ is the phase space of equation* (23).

In fact, due to corollaries 1, 2, equation (23) can be reduced to the equivalent system

$$H \overset{o}{\eta}{}^{0(n)} = \eta^0,\ \overset{o}{\eta}{}^{0(n)} = S\eta^1, \tag{24}$$

where $\eta^0 = (\mathbb{I} - P)\eta$, $\eta^1 = P\eta$. After applying Nelson – Gliklikh differentiation n times to the first equation in (24) and using operator H on it, we consecutively obtain

$$0 = H^{p+1} \overset{o}{\eta}{}^{0(n(p+1))} = ... = H^2 \overset{o}{\eta}{}^{0(2n)} = ... = H \overset{o}{\eta}{}^{0(n)} = \eta^0. \quad (25)$$

Thus, the condition (i) of definition 4 is true. To prove the fulfillment of the condition (ii), note that if $\xi_m \in \mathfrak{U}^1$, $m = 0, 1, \dots, n-1$, then there exists a unique solution of the problem (22), (24) and it is given by $\eta^1 = \eta^1(t) = \sum\limits_{m=0}^{n-1} V_m^t \xi_m$. Then the unique solution of the problem (22), (23) for $\xi_m \in \mathfrak{U}^1$, $m = 0, 1, \dots, n-1$, is given by $\eta(t) = \eta^0(t) + \eta^1(t) = \sum\limits_{m=0}^{n-1} V_m^t \xi_m$.

Corollary 5. *Under the conditions of theorem 5 the solution of the problem* (22), (23) *is the Gaussian K-random process if the random variables ξ_m, $m = 0, 1, \dots, n-1$, are Gaussian.*

We need to make a few remarks. Conditions (21) are respectively equivalent to the following conditions:

$$P(\overset{o}{\eta}{}^{(m)}(0) - \xi_m) = 0 \text{ and } \lim_{t \to 0+} P(\overset{o}{\eta}{}^{(m)}(t) - \xi_m) = 0. \quad (26)$$

Thus the following lemma is true.

Lemma 4. *Let the operator M be (L, n, p)-sectorial, $p \in \{0\} \cup \mathbb{N}$ and (12), (13) be fulfilled. For all independent random variables $\xi_m \in \mathbf{L}_2$, $m = 0, 1, \dots, n-1$, there exists a.s. a unique solution $\eta \in \mathbf{C}_K^\infty \mathbf{L}_2$ of the problem* (21), (23)*, represented in the form $\eta(t) = \sum\limits_{m=0}^{n-1} V_m^t \xi_m$, $t \in \mathbb{R}$. If in addition ξ_m, $m = 0, 1, \dots, n-1$ take values only in $\mathfrak{U}^1$, then this solution is the unique solution of the problem* (22), (23).

Return to equation (20) and note that now $\mathfrak{I} = (0, \tau)$. Let the K-Wiener process $w = w(t), t \in [0, \tau)$ be such that

$$(\mathbb{I} - Q)Nw \in \mathbf{C}_K^{n(p+1)} \mathbf{L}_2 \text{ and } QNw \in \mathbf{C}_K \mathbf{L}_2, \quad (27)$$

then the K-random process

$$\eta(t) = -\sum_{q=0}^{p} H^q M_0^{-1} (\mathbb{I} - Q) N \overset{o}{w}{}^{(qn)}(t) + \int_0^t V_{n-1}^{t-s} L_1^{-1} QNw(s) ds \quad (28)$$

is a unique classical solution of the problem (20), (21) with $\xi_m \in \mathfrak{V}^0$, $m = 0, ..., n-1$.

Lemma 5. *Let the operator M be (L, p)-bounded, $p \in \{0\} \cup \mathbb{N}$. For any K-random process $w = w(t)$ satisfying* (27)*, and all independent random variables $\xi_m \in L_2(\Omega; \mathfrak{U}^0)$, $m = 0, 1, \dots, n-1$, independent with w, there exists a.s. a unique solution $\eta \in \mathbf{C}_K^n \mathbf{L}_2$ of the problem* (20), (21)*, given by* (28)*. If in addition*

$$\xi_m = -\sum_{q=0}^{p} H^q M_0^{-1} (\mathbb{I} - Q) N \overset{o}{w}{}^{(qn+m)}(0),$$

then this solution is a unique solution of the problem (20), (22).

Theorem 6. *Let the operator M be (L,n,p)-sectorial, $p \in \{0\} \cup \mathbb{N}$ and (12), (13) be fulfilled. For any $N \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ and K-random process $w = w(t)$ satisfying* (27), *and for all independent random variables $\xi_m \in L_2(\Omega;\mathfrak{U}^0)$, $m = 0, 1, \ldots, n-1$, independent with w, there exists a.s. a unique solution $\eta \in \mathbf{C}^n_K\mathbf{L}_2$ of the problem* (20), (21), *represented in the form*

$$\eta(t) = \sum_{m=0}^{n-1} V^t_m \xi_m - \sum_{q=0}^{p} H^q M_0^{-1}(\mathbb{I} - Q)N \overset{o}{w}{}^{(qn)}(t) + \int_0^t V^{t-s}_{n-1} L_1^{-1} QNw(s)ds. \tag{29}$$

If in addition ξ_m, $m = 0, 1, \ldots, n-1$, satisfy

$$(P - \mathbb{I})\xi_m = \sum_{q=0}^{p} H^q M_0^{-1}(\mathbb{I} - Q)N \overset{o}{w}{}^{(qn+m)}(0), \tag{30}$$

then the solution (29) *is the solution of the problem* (20), (22).

However, "white noise" $w(t) = \overset{o}{W}_K(t) = (2t)^{-1}W_K(t)$ does not satisfy condition (27), therefore it cannot stand in the right side of (20). One approach to solving this problem is suggested in [24, 25] (incidentally, it also works for traditional white noise). To use this approach, transform the second term in the right side of (28) as follows:

$$\int_\varepsilon^t V^{t-s}_{n-1} L_1^{-1} QN \overset{o}{W}_K(s)ds = -V^{t-\varepsilon}_{n-1} L_1^{-1} QNW_K(\varepsilon) - \int_\varepsilon^t \frac{d}{dt} V^{t-s}_{n-1} L_1^{-1} NW_K(s)ds =$$

$$= -V^{t-\varepsilon}_{n-1} L_1^{-1} QNW_K(\varepsilon) - \int_\varepsilon^t V^{t-s}_{n-2} L_1^{-1} NW_K(s)ds. \tag{31}$$

This integration by parts makes sense for any $\varepsilon \in (0,t), t \in \mathbb{R}_+$ due to definition of the Nelson – Gliklikh derivative. Letting $\varepsilon \to 0$ in (31) we get

$$\int_0^t V^{t-s}_{n-1} L_1^{-1} QN \overset{o}{W}_K(s)ds = -\int_0^t V^{t-s}_{n-2} L_1^{-1} NW_K(s)ds. \tag{32}$$

Corollary 6. *Let the operator M be (L,n,p)-sectorial, $p \in \{0\} \cup \mathbb{N}$, and (12), (13) be fulfilled, the operator N satisfies*

$$QN = N. \tag{33}$$

Let $\mathfrak{I} \subset \overline{\mathbb{R}}_+$. For all independent random variables $\xi_m \in \mathbf{L}_2$, $m = 0, 1, \ldots, n-1$, independent with W_K, there exists a.s. a unique solution $\eta \in \mathbf{C}^n_K\mathbf{L}_2$ of the problem (21) *for the equation*

$$L \overset{o}{\eta}{}^{(n)} = M\eta + N \overset{o}{W}_K, \tag{34}$$

given by

$$\eta(t) = \sum_{m=0}^{n-1} V^t_m \xi_m - \int_0^t V^{t-s}_{n-2} L_1^{-1} NW_K(s)ds. \tag{35}$$

If in addition ξ_m, $m = 0, ..., n-1$ *take values only in* $\mathfrak{U}^1$, *then* (35) *is the unique solution of the problem* (22) *for equation* (34).

Obviously, (35) is obtained from (28) due to (33) and limit transition (31) $\to$ (32), whereas condition (33) plays the main role here.

Theorem 7. *Let the operator* M *be* (L, n, p)*-sectorial,* $p \in \{0\} \cup \mathbb{N}$*, and (12), (13) be fulfilled. For any operator* $N \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ *and for all independent* $\mathfrak{U}$*-valued random variables* $\xi_m \in \mathbf{L}_2$*, independent with* W_K*, there exists a.s. unique solution* $\eta = \eta(t)$ *of the problem* (26), (34) *given by*

$$\eta(t) = \sum_{m=0}^{n-1} V_m^t \xi_m - \int_0^t V_{n-2}^{t-s} L_1^{-1} Q N W_K(s) ds - \sum_{q=0}^{p} H^q M_0^{-1} (\mathbb{I} - Q) N \overset{o}{W}{}_K^{(qn+1)}(t).$$

4. The Cauchy problem for the stochastic Boussinesq – Lòve equation with additive "white noise"

Let $D \subset \mathbb{R}^d$ be a bounded domain with the boundary ∂D of class C^∞. Fix $l \in \{0\} \cup \mathbb{N}$ and set $\mathfrak{G} = W_2^l(D)$, $\mathfrak{V} = \{u \in W_2^{l+2}(D) : u(x) = 0, x \in \partial D\}$. Obviously, $\mathfrak{V}$ is a real separable Hilbert space densely and continuously embedded in $\mathfrak{G}$.

Let $\{\nu_j\}$ be the sequence of eigenvalues of the Laplace operator (defined in D with homogenous Dirichlet boundary conditions), numbered in nondecreasing order according to multiplicity, and by $\{\varphi_j\}$ denote the set of corresponding eigenfunctions, orthonormal in the sense of $\mathfrak{V}$.

Introduce the $\mathfrak{V}$-valued K-random processes. Construct the operator $\Lambda = (-1)^{m-1} \Delta^m$ with domain dom$\Lambda = \{W_2^{l+2(m+1)}(D) : \Delta^k u(x) = 0, x \in \partial D, k \in 0, 1, ..., m-1\}, m \in \mathbb{N}$. Note that the operator Λ has the same eigenfunctions $\{\varphi_j\}$, as the Laplace operator, but its spectrum consists of eigenvalues $|\nu_j|^m$. Since their asymptotic $|\nu_j|^m \sim j^{\frac{2m}{d}} \to \infty, j \to \infty$, we consider that number $m \in \mathbb{N}$ is taken such that the series $\sum\limits_{j=1}^{\infty} |\nu_j|^{-m}$ converges for a fixed $d \in \mathbb{N}$. Then the operator Λ is continuously invertible on $\mathfrak{V}$, whereas the inverse operator (i.e. the Green operator) has the spectrum consisting of eigenvalues $\mu_j = |\nu_j|^{-m}$. That very operator we take as the nuclear operator K.

In the cylinder $D \times [0, T]$, $T \in \mathbb{R}_+$ consider the Cauchy – Dirichlet problem

$$\xi(x, 0) = \xi_0(x), \quad \xi_t(x, 0) = \xi_1(x), \quad x \in D, \tag{36}$$

$$\xi(x, t) = 0, \quad (x, t) \in \partial D \times [0, T] \tag{37}$$

for the equation

$$(\lambda - \Delta_x) \overset{o}{\xi}_{tt} = \alpha \Delta_x \xi + \overset{o}{W}_K, \tag{38}$$

where $W_K = W_K(t)$ is a $\mathfrak{V}$-valued K-Wiener process.

Set $A = \lambda - \Delta, B = \alpha \Delta$.

Lemma 6. *For arbitrary* $\lambda \in \mathbb{R}$, $\alpha \in \mathbb{R}_+$ *the operator* M *is* $(L, 2, 0)$*-sectorial.*

Proof.

The L-spectrum of the operator M has the form

$$\sigma^L(M) = \left\{ \mu_k = \frac{\alpha\nu_k}{\lambda - \nu_k},\ k \in \mathbb{N} \setminus \{l : \nu_l = \lambda\} \right\}. \qquad (39)$$

Since $\nu_k \sim -k^2/d$ for $k \to \infty$, then, firstly, there exists a sector, including $\sigma^L(M)$, and consequently the set

$$S^L_{\theta,2}(M) = \{\mu \in \mathbb{C} : |\arg(\mu^2)| < \theta, \quad \mu \neq 0\} \subset \rho^L_2(M).$$

Secondly, for sufficiently large $|\mu|$, lying outside of this set

$$\max\ \left\{ \|R^L_{\mu^2}(M)\|_{\mathcal{L}(\mathfrak{U})}, \|L^L_{\mu^2}(M)\|_{\mathcal{L}(\mathfrak{F})} \right\} \leq \text{const}\ |\mu|^{-2}$$

$$\forall \mu \in S^L_{\theta,2}(M).$$

This means that the operator M is $(L, 2, 0)$-sectorial.

□

Lemma 7. *For arbitrary $\lambda \in \mathbb{R}$, $\alpha \in \mathbb{R}_+$ the conditions (12), (13) are fulfilled.*

Proof.

Find out if the conditions (12), (13) take place. Sine the spaces $\mathfrak{U}$ and $\mathfrak{F}$ are reflexive, then by Yagi – Fedorov theorem [4] and lemma 6 the condition (12) is fulfilled and

(i) $\mathfrak{U}^0 = \mathfrak{F}^0 = \{0\}$, $\mathfrak{U}^1 = \mathfrak{U}$, $\mathfrak{F}^1 = \mathfrak{F}$, if $\lambda \neq \nu_k$;

(ii) $\mathfrak{U}^0 = \mathfrak{F}^0 = \ker L = \text{span}\,\{\varphi_j, j : \lambda = \nu_j\}$,
$\mathfrak{U}^1 = \{u \in \mathfrak{U} : \langle u, \varphi_j\rangle = 0, j : \lambda = \nu_j\}$,
$\mathfrak{F}^1 = \{f \in \mathfrak{F} : \langle f, \varphi_j\rangle = 0\} = \text{im}\,L$, if $\lambda = \nu_j$;

The condition (13) also takes place and the operators

$$L_1^{-1} = \sum_k{}' \frac{\langle \cdot, \varphi_k\rangle\, \varphi_k}{\lambda - \nu_k},\ \ M_0^{-1} = \sum_{k:\lambda-\nu_k=0} \frac{\langle \cdot, \varphi_k\rangle\, \varphi_k}{\alpha\nu_k}.$$

A single quote by the sum means the lack of summands for which $\lambda - \nu_k = 0$.

□

Construct the propagators of the homogenous equation (38):

$$V_0^t = \sum_{\lambda>\lambda_k} (\cdot, \varphi_k)\varphi_k \operatorname{ch} \sqrt{\frac{\alpha\lambda_k}{\lambda - \lambda_k}} t + \sum_{\lambda<\lambda_k} (\cdot, \varphi_k)\varphi_k \cos \sqrt{\frac{\alpha\lambda_k}{\lambda_k - \lambda}} t,$$

$$V_1^t = \sum_{\lambda<\lambda_k} (\cdot, \varphi_k)\varphi_k \sqrt{\frac{\lambda - \lambda_k}{\alpha\lambda_k}} \operatorname{sh} \sqrt{\frac{\alpha\lambda_k}{\lambda - \lambda_k}} t + \sum_{\lambda>\lambda_k} (\cdot, \varphi_k)\varphi_k \sqrt{\frac{\lambda_k - \lambda}{\alpha\lambda_k}} \sin \sqrt{\frac{\alpha\lambda_k}{\lambda_k - \lambda}} t.$$

Moreover,

$$V_0^{t-s} A_1^{-1} = \sum_{\lambda>\lambda_k} \frac{(\cdot, \varphi_k)\varphi_k}{\lambda - \lambda_k} \operatorname{ch} \sqrt{\frac{\alpha\lambda_k}{\lambda - \lambda_k}} (t-s) +$$

$$+\sum_{\lambda<\lambda_k}\frac{(\cdot,\varphi_k)\varphi_k}{\lambda-\lambda_k}\cos\sqrt{\frac{\alpha\lambda_k}{\lambda_k-\lambda}}(t-s).$$

Thus, due to theorem 7, the following theorem is true.

Theorem 8. *For any $\alpha\in\mathbb{R}_+$, $\lambda\in\mathbb{R}$, $T\in\mathbb{R}_+$, for all independent $\xi_0,\xi_1\in L_2(\Omega;\mathfrak{V}^1)$, independent with W_K for every fixed t, there exists a.s. a unique solution of the problem* (36)–(38), *given by*

$$\xi(t)=V_0^t\xi_0+V_1^t\xi_1-\int\limits_0^t V_0^{t-s}A_1^{-1}QW_K(s)ds-M_0^{-1}(\mathbb{I}-Q)\overset{o}{W}_K(t). \tag{40}$$

Proof.

Due to lemmas 6, 7 all the conditions of theorem 7 are fulfilled.

□

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