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STOCHASTIC INCLUSIONS WITH CURRENT VELOCITIES HAVING DECOMPOSABLE RIGHT-HAND SIDES

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An existence of solution theorem is obtained for stochastic differential inclusions given in terms of the so-called current velocities (symmetric mean derivatives, a direct analogs of ordinary velocity of deterministic systems) and quadratic mean derivatives (giving information on the diffusion coefficient) on the flat n-dimensional torus. Right-hand sides in both the current velocity part and the quadratic part are set-valued, lower semi-continuous but not necessarily have convex images. Instead we suppose that they are decomposable.

Keywords: mean derivatives; current velocities; decomposable set-valued mappings; differential inclusions.

Introduction

The notion of mean derivatives was introduced by Edward Nelson [1–3] for the needs of stochastic mechanics (a version of quantum mechanics). The equation of motion in this theory (called the Newton – Nelson equation) was the first example of equations in mean derivatives. Later it turned out that the equations in mean derivatives arose also in many other branches of science (mechanics, hydrodynamics, Navier – Stokes vortices, gauge fields, economics, etc.).

Nelson introduced forward and backward mean derivatives while only their half-sum, symmetric mean derivative called current velocity, is a direct analog of ordinary velocity for deterministic processes. In [4] another mean derivative called quadratic, is introduced. It gives information on the diffusion coefficient of the process and using Nelson's and quadratic mean derivatives together, one can in principle recover the process from its mean derivatives.

Since the current velocities are natural analogs of ordinary velocities of deterministic processes, investigation of equations and especially inclusions with current velocities is very much important for applications since there are a lot of models of various physical, economical etc. processes based on such equations and inclusions.

Here we investigate inclusions with current velocities who's right hand sides are lower semi-continuous. Unlike previous publications we do not suppose that the images of points are convex sets. Instead we suppose that they are decomposable. This property yields serious modification of all proofs and constructions. We obtain an existence of solution theorem for such inclusions.

Some words about notation. By $S(n)$ we denote the space of symmetric $n \times n$ matrices, by $S_+(n)$ the subset of positive defined symmetric matrices, and by $\bar{S}_+(n)$ its closure, the set of positive semi-definite symmetric matrices.

1. Preliminaries on the Mean Derivatives

Let $\xi(t)$ be a stochastic process in $\mathcal{T}^\backslash$, $t \in [0, T]$, that is given on a certain probability space $(\Omega, \mathcal{F}, \mathsf{P})$ so that $\xi(t)$ is L_1-random variable for all t. The minimal complete σ-algebra that contains the preimages of all Borel sets in $\mathcal{T}^n$ with respect to $\xi(t) : \Omega \to \mathcal{T}^n$ is called "the present" $\mathcal{N}_t^\xi$. We denote by E_t^ξ the conditional expectation with respect to $\mathcal{N}_t^\xi$.

Strictly speaking, almost surely (a.s.) the sample paths of $\xi(t)$ are not differentiable for almost all t. Thus its "classical" derivatives exist only in the sense of generalized functions. To avoid using the generalized functions, following Nelson (see, e.g., [1–3]) we give

Definition 1. [1,4] *(i) Forward mean derivative $D\xi(t)$ of $\xi(t)$ at time t is an L_1-random variable of the form*

$$D\xi(t) = \lim_{\Delta t \to +0} E_t^\xi \left(\frac{\xi(t + \Delta t) - \xi(t)}{\Delta t} \right), \tag{1}$$

where the limit is supposed to exists in $L_1(\Omega, \mathcal{F}, \mathsf{P})$ and $\Delta t \to +0$ means that Δt tends to 0 and $\Delta t > 0$.

(ii) Backward mean derivative $D_*\xi(t)$ of $\xi(t)$ at t is an L_1-random variable

$$D_*\xi(t) = \lim_{\Delta t \to +0} E_t^\xi \left(\frac{\xi(t) - \xi(t - \Delta t)}{\Delta t} \right), \tag{2}$$

where the conditions and the notation are the same as in (i).

From the properties of conditional expectation (see [5]) it follows that $D\xi(t)$ and $D_*\xi(t)$ can be represented as compositions of $\xi(t)$ and Borel measurable vector fields (regressions)

$$a(t, x) = \lim_{\Delta t \to +0} E \left(\frac{\xi(t + \Delta t) - \xi(t)}{\Delta t} \middle| \xi(t) = x \right),$$

$$a_*(t, x) = \lim_{\Delta t \to +0} E \left(\frac{\xi(t) - \xi(t - \Delta t)}{\Delta t} \middle| \xi(t) = x \right) \tag{3}$$

on $\mathbb{R}^n$. This means that $D\xi(t) = a(t, \xi(t))$ and $D_*\xi(t) = a_*(t, \xi(t))$.

Definition 2. [1,4] *The derivative $D_S = \frac{1}{2}(D + D_*)$ is called symmetric mean derivative. The derivative $D_A = \frac{1}{2}(D - D_*)$ is called anti-symmetric mean derivative.*

Consider the vector fields

$$v^\xi(t, x) = \frac{1}{2}(Y^0(t, x) + Y_*^0(t, x))$$

and

$$u^\xi(t, x) = \frac{1}{2}(Y^0(t, x) - Y_*^0(t, x)).$$

Definition 3. *$v^\xi(t) = v^\xi(t, \xi(t)) = D_S\xi(t)$ is called current velocity of $\xi(t)$;*
$u^\xi(t) = u^\xi(t, \xi(t)) = D_A\xi(t)$ is called osmotic velocity of $\xi(t)$.

Following [4, 6] we introduce the differential operator D_2 that differentiates an L_1 random process $\xi(t)$, $t \in [0, T]$ according to the rule

$$D_2\xi(t) = \lim_{\triangle t \to +0} E_t^\xi \left(\frac{(\xi(t + \triangle t) - \xi(t))(\xi(t + \triangle t) - \xi(t))^*}{\triangle t} \right), \tag{4}$$

where $(\xi(t+\triangle t)-\xi(t))$ is considered as a column vector (vector in $\mathbb{R}^n$), $(\xi(t+\triangle t)-\xi(t))^*$ is a row vector (transposed, or conjugate vector) and the limit is supposed to exists in $L_1(\Omega,\mathcal{F},\mathsf{P})$. We emphasize that the matrix product of a column on the left and a row on the right is a matrix so that $D_2\xi(t)$ is a symmetric positive semi-definite matrix function on $[0,T]\times\mathbb{R}^n$. We call D_2 the quadratic mean derivative.

Let $v(t,m)$ be a vector field and $\alpha(t,m)$ be a symmetric positive semi-definite $(2,0)$-tensor field on $\mathcal{T}^n$. The system

$$\left\{\begin{array}{lcl} D_S\xi(t) & = & v(t,\xi(t)),\\ D_2\xi(t) & = & \alpha(t,\xi(t)) \end{array}\right. \tag{5}$$

is called *the first order differential equation with current velocities.*

Definition 4. *We say that (5) on $\mathcal{T}^n$ has a solution on $[0,T]$ with initial condition $\xi(0)=\xi_0$ if there exists a probability space $(\Omega,\mathcal{F},\mathsf{P})$ and a process $\xi(t)$ given on $(\Omega,\mathcal{F},\mathsf{P})$ and taking values in $\mathcal{T}^n$ such that $\xi(0)=\xi_0$ and for almost all $t\in[0,T]$ equation (5) is satisfied P-a.s. by $\xi(t)$.*

Theorem 1. [7] *Let $v:[0,T]\times\mathbb{R}^n\to\mathbb{R}^n$ be smooth and $\alpha:[0,T]\times\mathbb{R}^n\to \mathrm{S}_+(n)$ be smooth. Let them also satisfy the estimates*

$$\|v(t,x)\|<K(1+\|x\|), \tag{6}$$

$$tr\ \alpha(t,x)<K(1+\|x\|^2) \tag{7}$$

and for all indices ij let the elements of matrix $\alpha(x)$ satisfy the inequality

$$\left|\frac{\partial a^{ij}}{\partial x^j}(x)\right|<K(1+\|x\|) \tag{8}$$

for some $K>0$. Let ξ_0 be a random element with values in $\mathbb{R}^n$ whose probability density ρ_0 is smooth and nowhere equal to zero. Then for the initial condition $\xi(0)=\xi_0$ equation (5) has a solution that is well posed on the entire interval $t\in[0,T]$ and unique as a diffusion process.

Corollary 1. *Let $v:[0,T]\times\mathcal{T}^n\to\mathbb{R}^n$ be smooth and $\alpha:[0,T]\times\mathcal{T}^n\to\mathrm{S}_+(n)$ be smooth. Let ξ_0 be a random element with values in $\mathcal{T}^\setminus$ whose probability density ρ_0 on $\mathcal{T}^n$, is smooth and nowhere equal to zero. Then for the initial condition $\xi(0)=\xi_0$ equation (5) has a solution that is well posed on the entire interval $t\in[0,T]$ and unique as a diffusion process.*

Indeed, the boundedness estimates (6), (7) and (8) follow from the fact that v, α and $|\frac{\partial a^{ij}}{\partial x^j}(x)|$ are smooth ant $\mathcal{T}^n$ is compact.

Consider a sequence of equations

$$\left\{\begin{array}{lcl} D_S\xi(t) & = & v_k(t,\xi(t)),\\ D_2\xi(t) & = & \alpha_k(t,\xi(t)) \end{array}\right. \tag{9}$$

on $\mathcal{T}^n$ of (5) type such that they all have solutions with the same initial condition. Denote by μ_k the measure on $(C^0([0,T],\mathcal{T}^n),\mathcal{C})$ generated by the solution of the k-th equation from (9).

Below we will use the following result:

Lemma 1. [8] *Let all v_k and α_k be uniformly bounded on $\mathcal{T}^n$ by the same upper bound. Then the set of measures $\{\mu_k\}$ on $(C^0([0,T],\mathcal{T}^n),\mathcal{C})$ is weakly compact.*

2. The Main Result

If the values of a lower semi-continuous set-valued mapping (generally speaking) are not convex, it may not have continuous selectors. Then the following construction is often very much useful.

Definition 5. *Let E be a separable Banach space. A non-empty set $\mathcal{M} \subset L^1([0,l];E)$ is called decomposable if $f \cdot \chi_{\mathfrak{M}} + g \cdot \chi_{[0,l]\setminus\mathfrak{M}} \in \mathcal{M}$ for all $f, g \in \mathcal{M}$ and for every measurable subset $\mathfrak{M}$ in $[0,l]$ where χ is the characteristic function of the corresponding set.*

The reader can find more details about decomposable sets in [9] and [10].

Theorem 2. (Bressan-Colombo Theorem) *Let (Ω, d) be a separable metric space, X be a Banach space and $(J,\mathcal{A},\mu)$ be a measurable space with a σ-algebra $\mathcal{A}$ and a non-atomic measure μ such that $\mu(J) = 1$. Consider the space $Y = L^1_X(J,\mathcal{A},\mu)$ of integrable mappings from $(J,\mathcal{A},\mu)$ into X. If a set-valued mapping $F : \Omega \multimap Y$ is lower semicontinuous and has close decomposable values, F has a continuous selector.*

The assertion of Theorem 2 is proved, e.g., as Lemma 9.2 in [9].

Recall some facts and notions involved in further considerations. Specify $l > 0$. In what follows we denote by λ the normalized Lebesgue measure on $[0,l]$, i.e., such that $\lambda([0,l]) = 1$.

Lemma 2. *Let (Ξ, d) be a separable metric space, X be a Banach space. Consider the space $Y = L^1(([0,l],\mathcal{B},\lambda), X))$ of integrable maps from $[0,l]$ into X. If a set-valued map $G : \Xi \to Y$ is lower semicontinuous and has closed decomposable images, it has a continuous selector.*

This is a particular case of Bressan-Colombo Theorem 2.

Let $\mathbf{v}(t,m)$ be a set-valued vector field and $\boldsymbol{\alpha}(t,m)$ be a set-valued symmetric positive semi-definite $(2,0)$-tensor field on $\mathcal{T}^n$. The system of the form

$$\begin{cases} D_S\xi(t) \in \mathbf{v}(t,\xi(t)), \\ D_2\xi(t) \in \boldsymbol{\alpha}(t,\xi(t)) \end{cases} \tag{10}$$

is called a first order differential inclusion with current velocities.

The definition of solution of inclusion (10) is quite analogous to Definition 4.

Theorem 3. *Let the set-valued fields $\mathbf{v}$ and $\boldsymbol{\alpha}$ on $\mathcal{T}^n$ be lower semicontinuous, uniformly bounded and have closed decompisable images of points.*

Consider a random element ξ_0 with values in $\mathcal{T}^n$ that has the density ρ_0 smooth and nowhere equal to zero. Then for the initial condition $\xi(0) = \xi_0$ inclusion (10) has a solution well-posed on the entire interval $t \in [0,T]$.

Proof.

By Lemma 2 set-valued fields $\mathbf{v}$ and $\boldsymbol{\alpha}$ have continuous selectors v and α, respectively. Construct a sequence of theie smooth approximations v_k and α_k that converge to v and α with respect to the supremum norm. In addition, it is possible to construct α_k to be symmetric and positive definite since we deal with approximations in the space of symmetric matrices and any positive semi-definite matrix is a limit point of the space of positive definite matrices.

Consider the sequence of equations of (5) type:

$$\begin{cases} D_S\xi_k(t) &= v_k(\xi_k(t)), \\ D_2\xi_k(t) &= \alpha_k(\xi_k(t)). \end{cases}$$

For all those equations we consider the same initial condition ξ_0.

The approximations v_k and α_k are uniformly bounded by the same constant by the construction. Thus, all the above equations satisfy the conditions of Theorem 1, i.e., for every equation there exists a solution ξ_k.

Introduce on the Banach manifold $C^0([0,T],\mathcal{T}^n)$ of continuous curves in $\mathcal{T}^n$ the σ–algebra $\mathcal{C}$ generated by cylinder sets and denote by μ_k the measure on $(C^0([0,T],\mathcal{T}^n),\mathcal{C})$, generated by the solution $\xi_k(t)$. Introduce also the family of complete σ-sub-algebras $\mathcal{P}_t$, generated by cylinder sets with bases over $[0,t]$, $t\in[0,T]$, and the family of complete σ-sub-algebras $\mathcal{N}_t$, generated by primages of Borel sets in $\mathcal{T}^n$ under the mapping $x(\cdot)\mapsto x(t)$. It is clear that $\mathcal{N}_t$ is a σ-sub-algebra in $\mathcal{P}_t$ and that $\mathcal{P}_t$ is the "past" while $\mathcal{N}_t$ is the "present" for the coordinate process on $(C^0([0,T],\mathcal{T}^n),\mathcal{C},\mu_k)$.

By Theorem 1 the set $\{\mu_k\}$ of measures on $(C^0([0,T],\mathcal{T}^n),\mathcal{C})$ is weakly compact. Hence, we can select a subsequence that weakly converges to some measure μ. Without loss of generality we can suppose that the sequence μ_k weakly converges to μ. Consider the coordinate process $\xi(t)$ on the probability space $(C^0([0,T],\ \mathcal{T}^n),\ \mathcal{C},\mu)$, i.e., for any elementary ivent $x(\cdot)\in C^0([0,T],\mathcal{T}^n)$ by definition $\xi(t,x(\cdot))=m(t)$. Recall that $\mathcal{P}_t$ is the "past" for $\xi(t)$, and $\mathcal{N}_t$ is the "present" for this coordinate process.

Be the construction of $\xi_k(t)$, its quadratic derivative equals $\alpha_k(\xi_k(t))$. This means that for any bounded continuous real function f on $C^0([0,T],\mathcal{T}^n)$, measurable with respect to $\mathcal{N}_t$, for all k the equality

$$\lim_{\Delta t\to 0}\int\limits_{C^0([0,T],\mathcal{T}^n)}\left[\frac{(m(t+\Delta t)-m(t))(m(t+\Delta t)-m(t))^*}{\Delta t}-\right.$$
$$\left.-\,\alpha_k(m(t))\right]f(m(\cdot))d\mu_k=0 \qquad (11)$$

holds. Since $\alpha_k(t,m)$ converge uniformly to $\alpha(t,m)$ as $k\to\infty$, we derive that $\alpha_k(t,m(t))$ tends to $\alpha(t,m(t))$ uniformly for all μ_k including μ.

The field $\alpha(m(t))$ is continuous on some set of complete measure in $C^0([0,T],\mathcal{T}^n)$ since $\alpha(m)$ is continuous on $\mathcal{T}^n$.

Since we have uniform convergence (see above) for all k, we derive from boundedness of $f(m(\cdot))$ that for k large enough we get

$$\left\|\int\limits_{C^0([0,T],\mathcal{T}^n)}[\alpha_k(m(t))-\alpha(m(t))]\,f(m(\cdot))d\mu_k\right\|<\delta.$$

Since $f(m(\cdot))$ is bounded, there exists a certain number $\Xi > 0$ such that $|f(m(\cdot))| < \Xi$ $m(\cdot)$. Recall that all $\alpha_k(m)$ and $\alpha(m)$ are uniformly bounded, i.e., their norms are not greater than a certain $Q > 0$. Then

$$\left\| \int\limits_{C^0([0,T],\mathcal{T}^n)} [\alpha_k(m(t)) - \alpha(m(t))]\, f(m(\cdot)) d\mu_k \right\| < 2\delta Q \Xi$$

for all k large enough. Since δ is an arbitrary positive number,

$$\lim_{k\to\infty} \int\limits_{C^0([0,T],\mathcal{T}^n)} [\alpha_k(m(t)) - \alpha(m(t))] f(m(\cdot)) d\mu_k = 0.$$

The function $\alpha(m(t))$ is μ a.s. continuous and bounded on $C^0\ ([0,T],\mathcal{T}^n)$ (as it is shown above). Since the measures μ_k weakly converge to μ, by Lemma [11, Section VI.1]

$$\lim_{k\to\infty} \int\limits_{C^0([0,T],\mathcal{T}^n)} \alpha(m(t)) f(m(\cdot)) d\mu_k = \int\limits_{C^0([0,T],\mathcal{T}^n)} \alpha(m(t)) f(m(\cdot)) d\mu.$$

Evidently $$\lim_{k\to\infty} \int\limits_{C^0([0,T],\mathcal{T}^n)} [(m(t+\Delta t) - m(t))(m(t+\Delta t) - m(t))^*] f(m(\cdot)) d\mu_k =$$

$$= \int\limits_{C^0([0,T],\mathcal{T}^n)} [(m(t+\Delta t) - m(t))(m(t+\Delta t) - m(t))^*] f(m(\cdot)) d\mu.$$

Thus, $$\lim_{\Delta t\to 0} \int\limits_{C^0([0,T],\mathcal{T}^n)} \left[\frac{(m(t+\Delta t) - m(t))(m(t+\Delta t) - m(t))^*}{\Delta t} - \right.$$

$$\left. - \alpha(m(t)) \right] f(m(\cdot)) d\mu = 0. \qquad (12)$$

Since $f(m(\cdot))$ is an arbitrary bounded continuous function that is measurable with respect to $\mathcal{N}_t$, this means that $D_2\xi(t) = \alpha(\xi(t))$. But by construction $\alpha(\xi(t)) \in \boldsymbol{\alpha}(\xi(t))$ μ-a.s.

Now let us turn to the current velocity of solution.

By construction $D_S\xi_k(t) = v_k(t, \xi_k(t))$ for all k. This means that for any real bounded continuous function f on $C^0([0,T],\mathcal{T}^n)$, measurable with respect $\mathcal{N}_t$, for any k the equality

$$\lim_{\Delta t\to 0} \int\limits_{C^0([0,T],\mathcal{T}^n)} \left[\frac{m(t+\Delta t) - m(t-\Delta t)}{\Delta t} - v_k(m(t)) \right] f(m(\cdot)) d\mu_k = 0.$$

holds.

Specify an arbitrary $\varepsilon > 0$. Since μ_k weakly converges to μ, there exists $K(\varepsilon)$ such that for $k > K(\varepsilon)$

$$\left\| \int\limits_{C^0([0,T],\mathcal{T}^n)} [m(t+\Delta t) - m(t-\Delta t)]\, f(m(\cdot)) d\mu_k - \right.$$

$$\left. - \int\limits_{C^0([0,T],\mathcal{T}^n)} [m(t+\Delta t) - m(t-\Delta t)]\, f(m(\cdot)) d\mu \right\| < \varepsilon$$

and $$\left\| \int\limits_{C^0([0,T],\mathcal{T}^n)} f(m(\cdot)) v(m(t)) d\mu_k - \int\limits_{C^0([0,T],\mathcal{T}^n)} f(m(\cdot)) v(m(t)) d\mu \right\| < \varepsilon.$$

By analogous arguments, as above, we can show that

$$\lim_{k\to\infty} \int\limits_{C^0([0,T],\mathcal{T}^n)} [v_k(m(t)) - v(m(t))] f(m(\cdot)) d\mu_k = 0.$$

and that v is continuous. Recall that v is bounded since it is a selector of the bounded set-valued mapping.

Then by Lemma [11, Section VI.1] we obtain that

$$\lim_{k\to\infty} \int\limits_{C^0([0,T],\mathcal{T}^n)} v(m(t)) f(m(\cdot)) d\mu_k = \int\limits_{C^0([0,T],\mathcal{T}^n)} v(m(t)) f(m(\cdot)) d\mu.$$

It is obvious that

$$\lim_{k\to\infty} \int\limits_{C^0([0,T],\mathcal{T}^n)} [(m(t+\Delta t) - m(t-\Delta t))] f(m(\cdot)) d\mu_k =$$

$$= \int\limits_{C^0([0,T],\mathcal{T}^n)} [(m(t+\Delta t) - m(t-\Delta t))] f(m(\cdot)) d\mu.$$

Thus, $$\lim_{\Delta t\to 0} \int\limits_{C^0([0,T],\mathcal{T}^n)} \left[\frac{(m(t+\Delta t) - m(t-\Delta t)}{\Delta t} - v(m(t)) \right] f(m(\cdot)) d\mu \ = 0.$$

Since $f(m(\cdot))$ is an arbitrary bounded function, measurable with respect to $\mathcal{N}_t$, this means that $D_S\xi(t) = v(\xi(t))$. But by construction $v(\xi(t)) \in \mathbf{v}(\xi(t))$ μ-a.s.

□

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References

1. Nelson E. Derivation of the Schrödinger Equation from Newtonian Mechanics. *Physical Review*, 1966, vol. 150, pp. 1079–1085. doi: 10.1103/PhysRev.150.1079.

2. Nelson E. *Dynamical Theory of Brownian Motion*. Princeton, NJ, Princeton University Press, 1967.

3. Nelson E. *Quantum Fluctuations*. Perinceton, NJ, Princeton University Press, 1985.

4. Azarina S.V., Gliklikh Yu.E. Differential Inclusions with Mean Derivatives. *Dynam. Systems Appl.*, 2007, vol. 16, no. 1, pp. 49–71.

5. Parthasarathy K.R. *Introduction to Probability and Measure*. New York, Springer-Verlag, 2005. doi: 10.1007/978-93-86279-27-9.

6. Gliklikh Yu.E. *Global and Stochastic Analysis with Applications to Mathematical Physics*. London, Springer-Verlag, 2011. doi: 10.1007/978-0-85729-163-9.

7. Azarina S.V., Gliklikh Yu.E. On the Solvability of Nonautonomous Stochastic Differential Equations with Current Velocities. *Mathematical Notes*, 2016, vol. 100, issue 1–2, pp. 3–10. doi: 10.1134/S0001434616070014.

8. Gliklikh Yu.E., Makarova A.V. On Existence of Solutions to Stochastic Differential inclusions with Current Velocities II. *Journal of Computational and Engineering Mathematics*, 2016, vol. 3, no. 1, pp. 48–60. doi: 10.14529/jcem160106.

9. Deimling K. *Multivalued Differential Equations*. Berlin, New York, Walter de Gruyter, 1994. doi: 10.1002/zamm.19940740812.

10. Kamenskii M., Obukhovskii V., Zecca P. *Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Spaces*. Berlin, New York, Walter de Gruyter, 2001. doi: 10.1515/9783110870893.

11. Gihman I.I., Skorohod A.V. *The Theory of Stochastic Processes I*. Berlin, Springer-Verlag, 2004. doi: 10.1007/978-3-642-61943-4.

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СТОХАСТИЧЕСКИЕ ВКЛЮЧЕНИЯ С ТЕКУЩИМИ СКОРОСТЯМИ, ИМЕЮЩИЕ РАЗЛОЖИМЫЕ ПРАВЫЕ ЧАСТИ

Ю. Е. Гликлих, А. В. Макарова

Получена теорема существования для стохастических дифференциальных включений, заданных в терминах так называемых текущих скоростей (симметрических производных в среднем, прямых аналогов обычных скоростей детерминированных систем) и квадратичных производных в среднем (дающих информацию о коэффициенте диффузии) на плоском n-мерном торе. Правые части и с текущей скоростью, и с квадратичной производной многозначны, полунепрерывны снизу, но не обязательно имеют выпуклые образы. Вместо этого мы предполагаем, что они разложимы.

Ключевые слова: производные в среднем; текущие скорости; разложимые многозначные отображения; дифференциальные включения.

Литература

1. Nelson, E. Derivation of the Schrödinger Equation from Newtonian Mechanics / E. Nelson // Physical Review. – 1966. – V. 150. – P. 1079–1085.
2. Nelson, E. Dynamical Theory of Brownian Motion / E. Nelson. – Princeton: Princeton University Press, 1967.
3. Nelson, E. Quantum Fluctuations / E. Nelson. – Princeton: Princeton University Press, 1985.
4. Azarina, S.V. Differential Inclusions with Mean Derivatives / S.V. Azarina, Yu.E. Gliklikh // Dynam. Systems Appl. – 2007. – V. 16, № 1. – P. 49–71.
5. Партасарати, К.Р. Введение в теорию вероятностей и теорию меры / К.Р. Партасарати. – М.: Мир, 1988.
6. Gliklikh, Yu.E. Global and Stochastic Analysis with Applications to Mathematical Physics / Yu.E. Gliklikh. – London: Springer-Verlag, 2011.
7. Азарина, С.В. О разрешимости неавтономных стохастических дифференциальных уравнений с текущими скоростями / С.В. Азарина, Ю.Е. Гликлих // Математические заметки. – 2016. – Т. 100, № 1. – С. 3–12.
8. Gliklikh, Yu.E. On Existence of Solutions to Stochastic Differential inclusions with Current Velocities II / Yu.E. Gliklikh and A.V. Makarova // Journal of Computational and Engineering Mathematics. – 2016. – V. 3, № 1. – P. 48–60.
9. Deimling, K. Multivalued Differential Equations / K. Deimling. – Berlin; New York: Walter de Gruyter, 1994.
10. Kamenskii, M. Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Spaces / M. Kamenskii, V. Obukhovskii, P. Zecca. – Berlin; New York: Walter de Gruyter, 2001.

11. Гихман, И.И. Теория случайных процессов. Т. 1 / И.И. Гихман, А.В. Скороход. – М.: Физматлит, 1971.

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