

# ENGINEERING MATHEMATICS

MSC 65L06, 80A32

DOI: 10.14529/jcem180301

## TRANSIENT COMBUSTION ANALYSIS OF IRON MICRO-PARTICLES IN A GASEOUS OXIDIZING MEDIUM USING A NEW ITERATIVE METHOD

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This paper presents analytical solution to the transient combustion analysis for iron micro-particles in a gaseous oxidizing medium using Daftardar – Gejji and Jeffari Method (DJM). Also, parametric studies are carried out to properly understand the chemistry of the process and the associated burning time. Combusting particle thermal radiation effect and its linear density variation with temperature are applied. The generated analytical solution obtained by DJM are verified with an efficient numerical (fourth order Runge – Kutta, RK4) scheme. Results show that DJM is an efficient scheme for the problem. Also, the parametric study performed in this work shows that as the heat realized parameter and the surrounding temperature are increased, the combusting particle temperature increased rapidly until an asymptotic behaviour is attained. This work will be useful in solving to a great extent one of the challenges facing industries on combustion of metallic particles such as iron particles as well as in the determination of different particles burning time.

*Keywords: iron particle combustion; thermal radiation; temperature history; Daftardar – Gejji and Jeffari Method.*

### Introduction

The combustion of metallic particles and accurate prediction of the burning period of the combustibles have been the major challenges facing the metal processing industries. Combustible dusts which approximates gaseous oxidizing media require an accurate knowledge of their explosion hazards in order to make their importance to different industrial application applaudable. As a result, researchers have ventured into better understanding and modeling the particle and dust combustion. In a recent study, Sun et al. [1] investigated the behaviour of iron particles when simultaneously combusted and suspended in air. In their work, they considered the combustion zone and behavior of each iron particles by employing a powerful high-speed photomicrograph with which they were able to determine which iron particle combust at the combustion zone without gas phase flame. They finally established a relationship between the burning time and diameter of iron particles for moderate particle diameter. Haghiri and Bidbadi [2] applied the principle of flame propagation to study the dynamic behavior of a two-phase mixture which consists of micro-iron particles and air by considering the effect of thermal radiation. They obtained results which show that the considered thermal radiation plays a significant role in the improvement of vaporization process and burning velocity of organic dust mixture, compared with the case where this effect is correspondingly neglected. Hertzberg

et al. [3] examined combustible metal dust explosion limits as well as the influence of external agents such as temperature and pressure. They argued that when this combusting particles are properly conditioned and monitored, their merit will be greatly felt especially in industrial applications. Different analytical approaches have been widely employing to obtain close form solution to nonlinear engineering problems such as the tremendous work done by Hatami et al. [4]. In their work, three weighted residual methods were applied to properly predict the transient combustion of iron micro-particles. They concluded that least square method gives the best result when verified with numerical method. He [5] – [9] applied Homotopy perturbation method to different nonlinear engineering problems. He also presented different approaches of improving the scheme in order to handle some strongly nonlinear engineering and mathematical problems. Saedodin and Shahababaei [10] applied homotopy perturbation method (HPM) to study and analyze heat transfer in longitudinal porous fins while Darvishi et al. [11] and Moradi et al. [12] and Ha et al. [13] utilized differential transformation method (DTM) and homotopy analysis method (HAM) to obtain close form solution to the natural convection and radiation in a porous and porous moving fins with temperature dependent property and internal heat generation. Sobamowo et al. [14] applied homotopy perturbation method to analyze convective-radiative porous fin with temperature-dependent, internal heat generation and magnetic field. They presented interesting results and the validation of their work proves the efficiency of the scheme. Also, Yaseen et al. [15] developed an exact analytical solutions of Laplace equation by using DJ method. They employed the iterative method in the treatment of the Laplace equation considering both Dirichlet and Neumann conditions with their results compared with some existing iterative methods.

Motivated by the previous works as mentioned above, this paper introduces an analytical method whose close form solutions will be used for predicting and realizing the temperature history of iron particle during and after burning, so DJM is applied. The method agrees excellently with numerical Forth order Runge – Kutta method with minimal error as compared to those of previous works; hence the verification of the schemes.

## 1. Problem Description and Governing Equation

Consider an iron spherical particle, Fig. 1 which is combusted in the gaseous oxidizing medium as a result of high reaction with oxygen which acts as an oxidizer. The assumptions used includes:

- (a) particle is considered to have constant temperature (Isothermal);
- (b) the Biot number is small ( $Bi \ll 0.1$ );
- (c) lumped system assumption is applied;
- (d) the spherical particle combusts in an ambient medium;
- (e) interactions with other particles is neglected;
- (f) forced convection effect are neglected;
- (g) constant thermo-physical properties for the particle and ambient gaseous oxidizer;

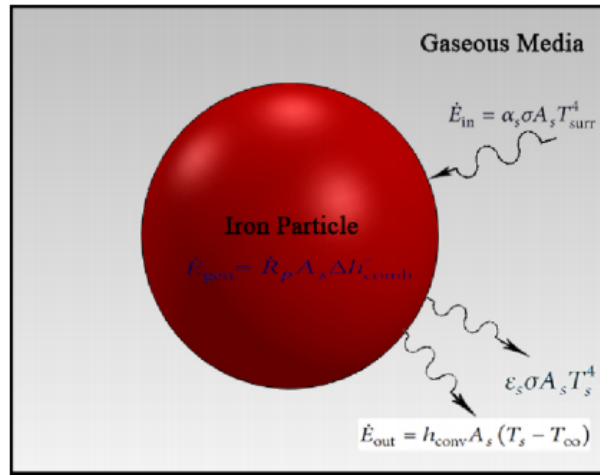
- (h) particle surface is assumed to be gray;
- (i) the surface reaction rate is treated as temperature independent with a constant convection heat coefficient;
- (j) Kirchhoff' slaw is invoked, hence the surface absorptivity ( $\alpha$ ) and the emissivity ( $\varepsilon$ ) at a given temperature and wavelength are equal;
- (k) particle density varies linearly with temperature as:

$$\rho_p = \rho_p^{(T)} = \rho_{p,a} [1 + \beta (T - T_a)];$$

- (l) ignition temperature is used as the initial condition ( $T^{(t=0)} = T_i$ ).

By incorporating the above assumptions (a – i) and performing energy balance:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \frac{dE_p}{dt} = \frac{dQ_p}{dt}. \quad (1)$$



**Fig. 1.** Schematic of combusted iron particle in gaseous media (Hatami et al 2014)

Representing the energy term [4], we have as shown below:

$$\dot{E}_{in} = \alpha \sigma A^s T_{sr}^4 \quad (2)$$

$$\dot{E}_{out} = h^{cv} A^s (T_s - T_a) + \varepsilon \sigma A^s T_s^4, \quad (3)$$

$$\dot{E}_{gen} = \dot{Q}_{cb} = \dot{R} A^s \delta h^{cb}, \quad (4)$$

$$\frac{dQ_p}{dt} = \rho^p c^p v^p \left( \frac{dT}{dt} \right)_s. \quad (5)$$

put Eqs. (2), (3), (4) and (5) into Eq. (1), we obtain

$$\alpha \sigma A^s T_{sr}^4 - h^{cv} A^s (T_s - T_a) - \varepsilon \sigma A^s T_s^4 + \dot{R} A^s (\delta h^{cb}) = \rho^p c^p v^p \left( \frac{dT}{dt} \right)_s. \quad (6)$$

In order to improve Eq. (6), the remaining assumptions are used. Applying the above assumptions ( $j - l$ ), Eq. (6) becomes,

$$\rho_{p,a} [1 + \beta (T - T_a)] V^p c^p \left( \frac{dT}{dt} \right)_s + h^{cv} A^s (T_s - T_a) + \varepsilon \sigma A^s (T_s^4 - T_{sr}^4) - \dot{R} A^s (\delta h^{cb}). \quad (7)$$

Introducing the following dimensionless parameters,

$$\begin{cases} \theta = \frac{T}{T_i}, & \theta_\infty = \frac{T_a}{T_i}, & \theta_{surr} = \frac{T_{sr}}{T_i}, & \varepsilon_1 = \beta T_i, \\ \tau = \frac{t}{(\rho_{p,a} v^p c^p / h^{cv} A^s)}, & \psi = \frac{\dot{Q}_{cb}}{h^{cv} A^s T_i}, & \varepsilon_2 = \frac{\varepsilon \sigma T_i^3}{h^{cv}}. \end{cases} \quad (8)$$

Using Eq. (8) in Eq. (7), the dimensionless form of Eq. (7) becomes,

$$(1 + \varepsilon_1 \theta - \varepsilon_1 \theta_\infty) \frac{d\theta}{d\tau} + \varepsilon_1 (\theta^4 - \theta_{surr}^4) + \theta - \psi - \theta_\infty = 0 \quad (9)$$

with initial condition

$$\theta(0) = 1. \quad (10)$$

## 2. Methods of Solution

Due to the nonlinear terms in Eq. (9), it is very difficult to develop a closed form or an exact analytical solution to the nonlinear equation. Therefore, the common practice is to make recourse to numerical method. However, in recent time, several semi- or approximate analytical methods have been developed to solve nonlinear equations. In this present study, the nonlinear equation in Eq. (9) is solved analytically using Daftardar – Gejji and Jeffari Method (DJM).

### 2.1. Basic Principle: Daftardar – Gejji and Jeffari Method (DJM)

As pointed previously, the Daftardar – Gejji and Jeffari Method (DJM) is an approximate analytical method for solving differential equations. However, a closed form series solution or approximate solution can be obtained for non-linear differential equations with the use of DJM. The basic definitions of the method is as follows.

Consider an equation with a functional of the form

$$A(t) = g(t) + N(A(t)), \quad (11)$$

with  $g(t)$  being a known function and  $N(A(t))$  representing the non-linear component of the general form  $A(t)$ . It is desired to obtain a series solution of the form

$$A(t) = \sum_{j=0}^{\infty} A_j(t). \quad (12)$$

The nonlinear operator  $N$  of Eq. (11) may be written in a decomposed form as

$$N \left( \sum_{j=0}^{\infty} A_j \right) = N(A_0) + \sum_{j=1}^{\infty} \left\{ N \left( \sum_{k=0}^j A_k \right) - N \left( \sum_{k=0}^{j-1} A_k \right) \right\}. \quad (13)$$

The general form as shown in Eq. (11) may then be written as

$$\sum_{j=0}^{\infty} A_j = g + N(A_0) + \sum_{j=1}^{\infty} \left\{ N \left( \sum_{k=0}^l A_k \right) - N \left( \sum_{k=0}^{l-1} A_k \right) \right\}. \quad (14)$$

Hence,

$$\begin{aligned} A_1 &= N(A_0), \\ A_2 &= N(A_0 + A_1) - N(A_0), \\ A_3 &= N(A_0 + A_1 + A_2) - N(A_0 + A_1), \\ A_4 &= N(A_0 + A_1 + A_2 + A_3) - N(A_0 + A_1 + A_2), \\ A_5 &= N(A_0 + A_1 + A_2 + A_3 + A_4) - N(A_0 + A_1 + A_2 + A_3), \\ &\vdots \end{aligned} \quad (15)$$

with a general solution of the form

$$A(t) = \sum_{j=0}^{\infty} A_j. \quad (16)$$

## 2.2. Method of Solution: Daftardar – Gejji and Jeffari Method (DJM)

Recall that the nonlinear governing equation as shown in Eq. (9) may be expressed as

$$\varepsilon_1 \theta \frac{d\theta}{d\tau} + (1 - \varepsilon_1 \theta_{\infty}) \frac{d\theta}{d\tau} + \varepsilon_1 (\theta^4 - \theta_{surr}^4) + \theta - \psi - \theta_{\infty} = 0, \quad \theta(0) = 1.$$

Grouping the coefficients, the above equation becomes:

$$\frac{d\theta}{d\tau} = - \left( \frac{\varepsilon_1}{1 - \varepsilon_1 \theta_{\infty}} \right) \theta \frac{d\theta}{d\tau} - \left( \frac{\varepsilon_2}{1 - \varepsilon_1 \theta_{\infty}} \right) \theta^4 - \left( \frac{1}{1 - \varepsilon_1 \theta_{\infty}} \right) \theta + \left( \frac{\psi + \theta_{\infty} + \varepsilon_2 \theta_{surr}^4}{1 - \varepsilon_1 \theta_{\infty}} \right), \quad (17)$$

re-representing the coefficients, we have

$$\frac{d\theta}{d\tau} = \beta_1 \theta \frac{d\theta}{d\tau} + \beta_2 \theta^4 + \beta_3 \theta + \beta_4, \quad (18)$$

where

$$\beta_1 = - \left( \frac{\varepsilon_1}{1 - \varepsilon_1 \theta_{\infty}} \right), \quad \beta_2 = - \left( \frac{\varepsilon_2}{1 - \varepsilon_1 \theta_{\infty}} \right), \quad \beta_3 = - \left( \frac{1}{1 - \varepsilon_1 \theta_{\infty}} \right), \quad \beta_4 = \left( \frac{\psi + \theta_{\infty} + \varepsilon_2 \theta_{surr}^4}{1 - \varepsilon_1 \theta_{\infty}} \right). \quad (19)$$

The leading term is obtained from the initial dimensionless ignition temperature as

$$\theta_0 = 1. \quad (20)$$

The remaining term of the series solution will be obtained by applying the DJM principle in Eq. (15) on the nonlinear Eq. (9) as shown below.

Since the combustion model is:

$$\frac{d\theta}{d\tau} = \beta_1 \theta \frac{d\theta}{d\tau} + \beta_2 \theta^4 + \beta_3 \theta + \beta_4.$$

Integrating both sides to obtain the dependent variable for the L.H.S,

$$\theta = \theta_0 + \int_0^\tau \left( \beta_1 \theta \frac{d\theta}{d\tau} + \beta_2 \theta^4 + \beta_3 \theta + \beta_4 \right) d\tau. \quad (21)$$

Using the initial condition,

$$\theta = 1 + \int_0^\tau \left( \beta_1 \theta \frac{d\theta}{d\tau} + \beta_2 \theta^4 + \beta_3 \theta + \beta_4 \right) d\tau. \quad (22)$$

Now, as described in Eq. (15),

$$\theta_1 = N(\theta_0) = \int_0^\tau \left( \beta_1 \theta_0 \frac{d\theta_0}{d\tau} + \beta_2 \theta_0^4 + \beta_3 \theta_0 + \beta_4 \right) d\tau.$$

Using the initial condition,

$$\theta_1 = (\beta_2 + \beta_3 + \beta_4) \tau.$$

Making necessary substitution, we have

$$\theta_1 = \left( -\frac{1}{1 - \epsilon_1 \theta_\infty} - \frac{\epsilon_2}{1 - \epsilon_1 \theta_\infty} + \frac{\psi + \epsilon_2 \theta_{surr}^4 + \theta_\infty}{1 - \epsilon_1 \theta_\infty} \right) \tau. \quad (23)$$

Similarly,

$$\theta_2 = \left\{ \begin{array}{l} \left( \frac{\epsilon_2 (-1 + \psi - \epsilon_2 + \epsilon_2 \theta_{surr}^4 + \theta_\infty) (-1 + \psi + \epsilon_2 (-1 + \theta_{surr}^4) + \theta_\infty)^3}{5(-1 + \epsilon_1 \theta_\infty)^5} \right) \tau^5 - \\ \left( \frac{\epsilon_2 (-1 + \psi - \epsilon_2 + \epsilon_2 \theta_{surr}^4 + \theta_\infty) (-1 + \psi + \epsilon_2 (-1 + \theta_{surr}^4) + \theta_\infty)^2}{(-1 + \epsilon_1 \theta_\infty)^4} \right) \tau^4 + \\ \left( \frac{2\epsilon_2 (-1 + \psi - \epsilon_2 + \epsilon_2 \theta_{surr}^4 + \theta_\infty) (-1 + \psi + \epsilon_2 (-1 + \theta_{surr}^4) + \theta_\infty)}{(-1 + \epsilon_1 \theta_\infty)^3} \right) \tau^3 + \\ \left( \frac{(-1 + \psi - \epsilon_2 + \epsilon_2 \theta_{surr}^4 + \theta_\infty) \left( \frac{1 + (-1 + \psi) \epsilon_1 + \epsilon_2 (4 - \epsilon_1 + \epsilon_1 \theta_{surr}^4 - 4\epsilon_1 \theta_\infty)}{2(-1 + \epsilon_1 \theta_\infty)^3} \right)}{2(-1 + \epsilon_1 \theta_\infty)^3} \right) \tau^2 - \\ \left( \frac{\epsilon_1 (-1 + \psi - \epsilon_2 + \epsilon_2 \theta_{surr}^4 + \theta_\infty)}{(-1 + \epsilon_1 \theta_\infty)^2} \right) \tau. \end{array} \right. \quad (24)$$

The resulting series solution is

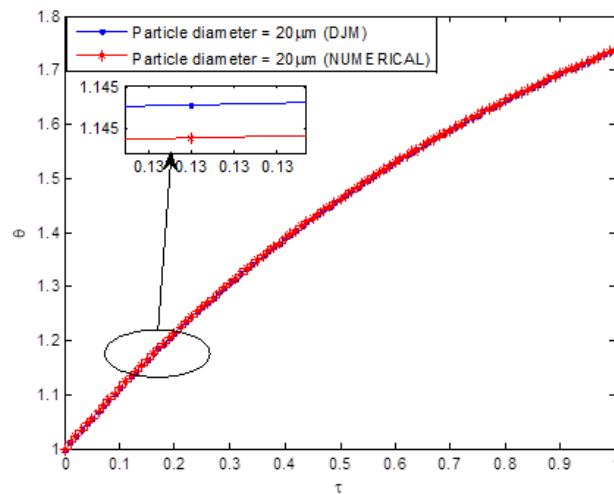
$$\theta(\tau) = \sum_{j=0}^{\infty} \theta_j, \quad (25)$$

which in expanded form becomes:

$$\begin{aligned}
 \theta(\tau) = & 1 + \left( -\frac{1}{1 - \epsilon_1\theta_\infty} - \frac{\epsilon_2}{1 - \epsilon_1\theta_\infty} + \frac{\psi + \epsilon_2\theta_{surr}^4 + \theta_\infty}{1 - \epsilon_1\theta_\infty} \right) \tau + \\
 & \left( \frac{\epsilon_2(-1 + \psi - \epsilon_2 + \epsilon_2\theta_{surr}^4 + \theta_\infty)(-1 + \psi + \epsilon_2(-1 + \theta_{surr}^4) + \theta_\infty)^3}{5(-1 + \epsilon_1\theta_\infty)^5} \right) \tau^5 - \\
 & \left( \frac{\epsilon_2(-1 + \psi - \epsilon_2 + \epsilon_2\theta_{surr}^4 + \theta_\infty) \left( \left( \frac{-1 + \psi + \epsilon_2(-1 + \theta_{surr}^4) + \theta_\infty}{\epsilon_2(-1 + \theta_{surr}^4) + \theta_\infty} \right) \right)^2}{(-1 + \epsilon_1\theta_\infty)^4} \right) \tau^4 + \\
 & \left( \frac{2\epsilon_2(-1 + \psi - \epsilon_2 + \epsilon_2\theta_{surr}^4 + \theta_\infty) \left( \left( \frac{-1 + \psi + \epsilon_2(-1 + \theta_{surr}^4) + \theta_\infty}{\epsilon_2(-1 + \theta_{surr}^4) + \theta_\infty} \right) \right)}{(-1 + \epsilon_1\theta_\infty)^3} \right) \tau^3 + \\
 & \left( \frac{(-1 + \psi - \epsilon_2 + \epsilon_2\theta_{surr}^4 + \theta_\infty)(1 + (-1 + \psi)\epsilon_1 + \epsilon_2(4 - \epsilon_1 + \epsilon_1\theta_{surr}^4 - 4\epsilon_1\theta_\infty))}{2(-1 + \epsilon_1\theta_\infty)^3} \right) \tau^2 - \\
 & \left( \frac{\epsilon_1(-1 + \psi - \epsilon_2 + \epsilon_2\theta_{surr}^4 + \theta_\infty)}{(-1 + \epsilon_1\theta_\infty)^2} \right) \tau + \dots
 \end{aligned} \tag{26}$$

### 3. Results and Discussion

Fig. 2 depicts the verification of the analytical scheme used with a numerical fourth order Runge – Kutta. The scheme, DJM ascertain a good agreement with the numerical method. In order to visualize the accuracy of the scheme, a super-imposed plot which shows the temperature profile of a 20 $\mu$ m combusting iron particle is inspected as shown in Fig. 2 together with Table. From the figure, it is evident that DJM shows a good agreement with the Numerical scheme and as such is efficient for the problem in concern.



**Fig. 2.** Verification of DJM with Numerical

Table

Comparism of the two analytical scheme with a numerical method  
for a 20  $\mu\text{m}$  iron particle

$\theta(\tau)$ for a Particle diameter of ( $\mu\text{m}$ )		
	Numerical	DJM
0.0	1.0000000000000000	1.0000000000000000
0.1	1.113333969181095	1.113333969181095
0.2	1.215117348008129	1.215117348008133
0.3	1.306590963764452	1.306590963764455
0.4	1.388844932815013	1.388844932815017
0.5	1.462843666959360	1.462843666959367
0.6	1.529449065177065	1.529449065177069
0.7	1.589444597054137	1.589444597054139
0.8	1.643562984179010	1.643562984179017
0.9	1.692520185796611	1.692520185796617
1.0	1.737058395009000	1.737058395009900

### 3.1. Effect of Particle Diameter on the Temperature History

Fig. 3 and Fig. 4 depict the effect of the combusting particle diameter on temperature profile and burning rate using DJM. From the graphs, it can be easily seen that particle diameter have evident influence on the temperature profile. A particle with 60 $\mu\text{m}$  diameter was observed to possess a higher temperature profile which means that an increase in the combusting particle diameter causes a corresponding increase in the temperature profile as well as the burning time. As a result of this evident impact, the particle diameter may be used as a controlling agent in reducing the hazardous effects that normally propagate from iron particle combustion.

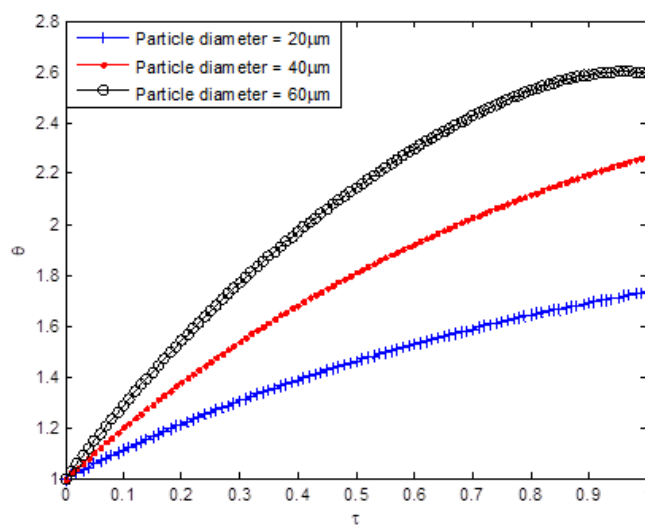


Fig. 3. Effect of particle diameter on the temperature



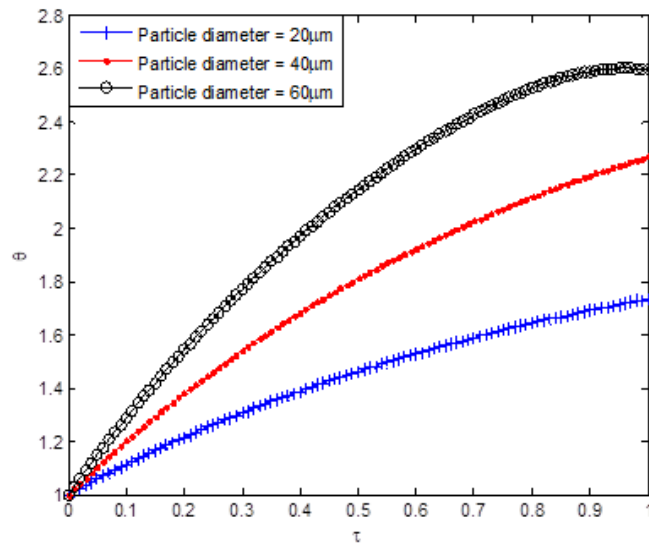


Fig. 4. Effect of particle diameter on the temperature Profile with DTM Profile with HPM

### 3.2. Effect of $\epsilon_1$ and $\epsilon_2$ on the Temperature History

Fig. 5 and Fig. 6 depict the influence of  $\epsilon_1$  and  $\epsilon_2$  on the temperature profile. From the figures, it can be seen that increasing  $\epsilon_1$  and  $\epsilon_2$  decreases the combustion temperature with this effect more pronounced with  $\epsilon_2$ . The decrease in combustion temperature with a corresponding increase in  $\epsilon_1$  and  $\epsilon_2$  is as a result of an increase in the radiation heat transfer term in the combustion particle.

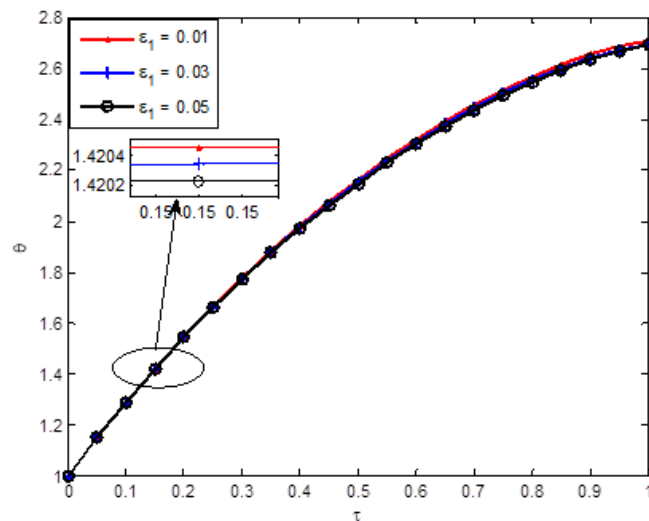


Fig. 5. Effect of  $\epsilon_1$  on the temperature profile

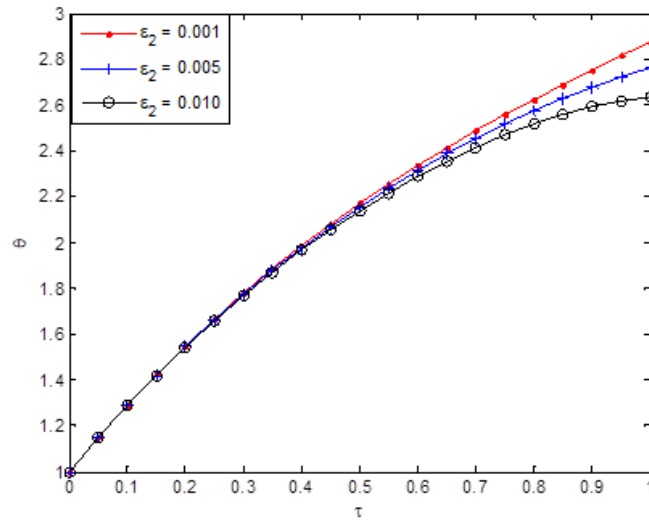


Fig. 6. Effect of  $\epsilon_2$  on the temperature profile

### 3.3. Effect of the Heat Realized Parameter and Surrounding Temperature on the Temperature History

Fig. 7 and Fig. 8 depict the influence of the heat realized parameter and the surrounding temperature on the combustion temperature. From the plots, we can conclude that increasing the heat realized parameter and the surrounding temperature increases the combustion temperature. This increase is significant for the heat realized parameter variation than that of the surrounding temperatures except for high values of surrounding temperature.

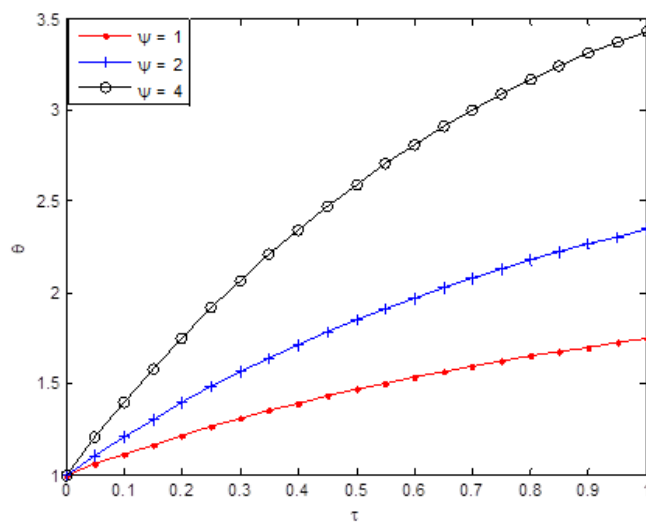
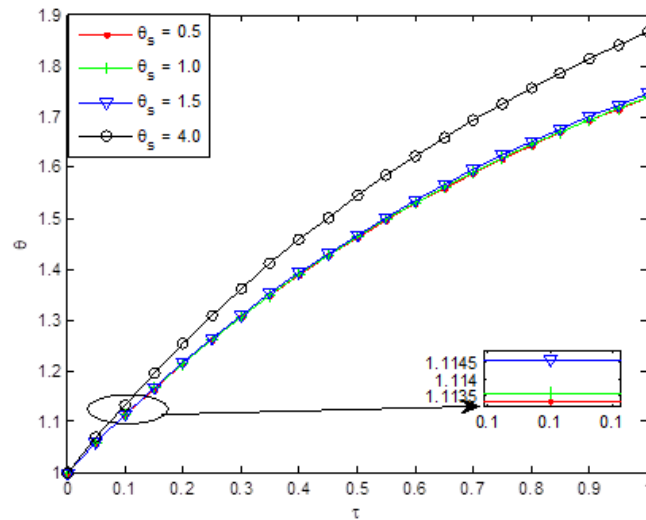


Fig. 7. Effect of heat realized term on the temperature profile



**Fig. 8.** Effect of surrounding temperature on the temperature profile

## Conclusion

In this work, a study has been carried out for the determination of the temperature history of iron particle during combustion process. The results of the DJM solution were verified numerically. It was established that DJM gives a good result and is efficient for the problem investigated. Also, parametric studies were performed to fully understand how the combusting particle diameter, density, radiative term, heat realized term and other parameters affect the burning time as well as the combustion temperature. The results revealed that by increasing the heat realized parameter, combustion temperature increased until a steady state was reached. It is hoped that the present study will enhance the understanding of the combustion of the particle and also obviate the challenges facing industries on combustion of metallic particles such as iron particles as well as in the determination of different particles burning time.

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*Received May 25, 2018*

## ПЕРЕХОДНЫЙ АНАЛИЗ ГОРЕНИЯ ЖЕЛЕЗНЫХ МИКРОЧАСТИЦ В ГАЗООБРАЗНОЙ СРЕДЕ ОКИСЛЕНИЯ С ИСПОЛЬЗОВАНИЕМ НОВОГО ИТЕРАЦИОННОГО МЕТОДА

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В настоящей работе представлено аналитическое решение для переходного анализа горения микрочастиц железа в газообразной окисляющей среде с использованием метода Дафтардара – Гейджи и Джеффри. Кроме того, для правильного понимания химии процесса и времени горения проводятся параметрические исследования. Применяются эффект теплового излучения частиц горения и изменение его линейной плотности в зависимости от температуры. Сгенерированное аналитическое решение, полученное с помощью метода Дафтардара – Гейджи и Джеффри, проверяется эффективной разностной схемой (четвертый порядок Рунге – Кутты). Результаты показывают, что метод Дафтардара – Гейджи и Джеффри эффективен для решения рассматриваемой задачи. Кроме того, параметрическое исследование, проведенное в этой работе, показывает, что с увеличением измеряемого теплового параметра и температуры окружающей среды температура горючей частицы быстро возрастает до достижения асимптотического поведения. Работа имеет большое значение для решения одной из задач, стоящих перед отраслью промышленности, производства которых связаны со сжиганием металлических частиц (железа), а также при определении времени горения различных частиц.

*Ключевые слова:* сгорание частицы железа; тепловая радиация; история температур; метод Дафтардара – Гейджи и Джеффри.

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*Поступила в редакцию 25 мая 2018 г.*