# SHORT NOTES

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## THE ALGORITHMS FOR SOLVING VECTOR ENTROPY CONTROL PROBLEM, COMPARATIVE ANALYSIS

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The algorithms for solving the vector entropy control problem for Gaussian stochastic systems are considered in the article. To solve a nonlinear optimization problem for a conditional extremum, the method of penalty functions with unconditional optimization methods of various-orders is considered. A set of problem-oriented programs has been developed that implements the proposed algorithms. A comparative analysis of the computational efficiency of the proposed algorithms is performed based on Monte Carlo statistical simulation methods and simulation modeling.

*Keywords: differential entropy; Gaussian stochastic system; vector entropy control; nonlinear optimization.*

## Introduction

Let's take a complex stochastic system S as a multidimensional continuous random variable $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_m)$. Each $Y_i$ element of the vector $\mathbf{Y}$ is a one-dimensional random variable which is characterizing the functioning of the particular element of the system. Those elements can be either interdependent or independent of each other.

In [1] the differential entropy of random vector $\mathbf{Y}$ with $p_{\mathbf{Y}}(x_1, \ldots, x_m)$ joint probability density was introduced:

$$H(\mathbf{Y}) = -\int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} p_{\mathbf{Y}}(x_1, \ldots, x_m) \ln p_{\mathbf{Y}}(x_1, \ldots, x_m)\, dx_1 \ldots dx_m. \tag{1}$$

In some cases, we must consider the differential entropy as a vector $h(\mathbf{Y}) = (h_V; h_R)$, where $h_V$ is a randomness entropy and $h_R$ is a self-organization entropy [2]. In [3] the problem of entropy control in vector form for Gaussian stochastic systems is described. The control consists in transforming the system entropy vector from the state of $(h_v^0;\ h_R^0)$ with covariance matrix $\mathbf{\Sigma}_0$ to the state of $(h_v^*;\ h_R^*)$ with a minimum change in the covariance matrix.

$$\begin{cases} f(\mathbf{\Sigma}) = \sum\limits_{i=1}^{m} \sum\limits_{j=i}^{m} \left(\sigma_{ij}^0 - \sigma_{ij}\right)^2 \to \min_{\sigma_{ij}}, \\ \prod\limits_{i=1}^{m} \sigma_{ii} - \dfrac{e^{2h_v^* - m}}{(2\pi)^m} = 0, \\ |\mathbf{\Sigma}| - e^{2h_R^*} \prod\limits_{i=1}^{m} \sigma_{ii} = 0, \\ \sigma_{ij}^2 < \sigma_{ii}\sigma_{jj},\ \sigma_{ij} = \sigma_{ji}, \sigma_{ii} > 0, \quad \forall 1 \le i, j \le m, \\ \mathbf{\Sigma} > 0. \end{cases} \tag{2}$$

## Computational Experiments and Comparative Analysis

We solve constrained nonlinear optimization problem (1) using the method of penalty functions [4]. To solve unconstrained optimization problem, we use various-order methods. From zero-order methods the Nelder – Mead method was chosen, from first-order methods the conjugate gradient method was chosen, and from second-order methods the Newton's method was chosen.

To investigate the effectiveness of the proposed algorithms for vector entropy control, a set of problem-oriented programs in the R [5] language in the RStudio IDE [6] was developed to carry out computational experiments. For the above-mentioned unconstrained optimization methods, ready-made functions, present in the R language and its packages, were used. The "nmk" function from package "dfoptim" [7] was used for the Nelder – Mead method, the "Rcgminu" function from package "Rcgmin"[8] was applied for the conjugate gradient method, and the function "nlm" from standard package of R was selected for Newton's method. A detailed description of these functions can be found in the R language documentation and in the documentation of the respective packages.

For computational experiments five hundred $\Sigma^0$ covariance matrices and corresponding to these matrices effective $(h_V^*, h_R^*)$ end-points of entropy were generated. Such matrices and entropy end-points were generated for two-dimensional, three-dimensional, four-dimensional and five-dimensional systems.

First let's define the parameters to be set for the penalty function method. The following values were empirically chosen:

1. $r^0 = 1; C = 3;\ \varepsilon = 10^{-4}$ .
2. Maximum number of iterations – 100.
3. The search starting point is set randomly.
4. The solution is considered to be found if for the target entropy vector $(h_V^*, h_R^*)$ and for the entropy vector of the found solution $(\widehat{h_V},\ \widehat{h_R})$, the following conditions are true: $\left|h_V^* - \widehat{h_V}\right| < 0.05$ and $\left|h_R^* - \widehat{h_R}\right| < 0.05$.

The obtained comparative characteristics are given in Tables 1, 2.

**Table 1**

The percentage of successful attempts

| Dimension | The percentage of successful attempts | | | | |
|---|---|---|---|---|---|
| | Nelder – Mead method | Conjugate gradient method | Newton's method | Nelder – Mead and Newton's methods | Conjugate gradient and Newton's methods |
| 2 | 98.2 | 95.8 | 48.2 | 94 | 88.4 |
| 3 | 99.6 | 80.2 | 31.4 | 93 | 81.2 |
| 4 | 96.8 | 78.8 | 18 | 93.2 | 86.2 |
| 5 | 93.4 | 85.4 | 7.2 | 88.8 | 89.2 |

**Table 2**

Average calculation time in seconds

| Dimension | Average calculation time, seconds | | | | |
|---|---|---|---|---|---|
| | Nelder – Mead method | Conjugate gradient method | Newton's method | Nelder – Mead and Newton's methods | Conjugate gradient and Newton's methods |
| 2 | 0.732 | 0.657 | 0.08 | 0.449 | 0.428 |
| 3 | 1.382 | 1.687 | 0.273 | 0.641 | 0.728 |
| 4 | 1.622 | 0.956 | 0.253 | 0.92 | 0.727 |
| 5 | 3.97 | 1.008 | 0.362 | 1.605 | 0.604 |

By analyzing the data of the first three columns given in Table 1, we can conclude that the highest percentage of successful attempts shows the zero-order method of Nelder – Mead. The higher the order of the method, the lower the percentage of successful attempts. This is due to the fact that the methods of the first and second order guarantee the convergence of the sequence of solutions to the minimum point for strongly convex functions [4].

Otherwise, these methods start to work unstably. But, on the other hand, the methods of the first and second order, and especially the Newton method, work faster than the Nelder – Mead method.

In practice, the following approach is adopted. With the first few iterations of the method of penalty functions, to solve the problem of unconditional optimization, to use methods of zero or first order, and then continue using second-order methods. Another approach was studied as well. For the first 7 iterations the Nelder – Mead method or conjugate gradient method was used, afterwards- the Newton method. Data for this approach are given in $4^{th}$ and $5^{th}$ columns of Tables 1, 2. You can see that the Nelder – Mead and Newton joint method works much faster than the Nelder – Mead method, and is not that inferior in terms of the percentage of successful attempts.

We can conclude that if the method is used in real-time systems where rapid execution of the algorithm is necessary, it is worth to use the Nelder – Mead and Newton joint method, and if the execution time is not critical, then it is better to use the Nelder – Mead method.

## Summary

1. Methods and algorithms for solving the problem of vector entropy control of Gaussian stochastic systems are considered.

2. A comparative analysis of the proposed methods is presented.

3. It is shown that in real-time systems it is necessary to choose a Nelder – Mead and Newton joint method from the proposed methods, and if the execution time is not critical, then the Nelder – Mead method is the best choice.

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# АЛГОРИТМЫ РЕШЕНИЯ ЗАДАЧИ ВЕКТОРНОГО ЭНТРОПИЙНОГО УПРАВЛЕНИЯ, ИХ СРАВНИТЕЛЬНЫЙ АНАЛИЗ

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Рассмотрены алгоритмы решения задачи векторного энтропийного управления гауссовскими стохастическими системами. Для решения нелинейной оптимизационной задачи на условный экстремум рассматривается метод штрафных функций вместе с методами разных порядков для решения задачи безусловной оптимизации. Разработан комплекс проблемно-ориентированных программ, реализующий предложенные алгоритмы. Выполнен сравнительный анализ вычислительной эффективности предложенных алгоритмов на основе методов статистических испытаний Монте-Карло и имитационного моделирования.

*Ключевые слова: дифференциальная энтропия; гауссовская стохастическая система; векторное энтропийное управление; нелинейная оптимизация.*

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