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# CLASSIFICATION OF PRIME VIRTUAL LINKS OF GENUS 1 WITH AT MOST 4 CLASSICAL CROSSINGS

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One of the main problems of the knot theory is to classify studied objects, i.e. to construct a table of all inequivalent objects taking into account parameters represented some properties, as well as a list of invariants of the tabulated objects. The goal of this paper is to classify all genus 1 prime virtual links having virtual link diagrams with at most 4 classical crossings. The problem of classification is difficult, because there is no universal method to decide if two given objects are equivalent or not. We generalise Kauffman bracket of virtual link diagrams in order to obtain an invariant, which is enough to prove that constructed table contains only inequivalent objects. To this end, we propose an algorithm to compute the numbers of trivial and nontrivial curves. The results of the paper can be introduced into research on the proteins by means of a method to represent proteins as virtual links.

*Keywords: virtual links; genus one; table; Kauffman bracket; proteins.*

## Introduction

One of the main problems of the knot theory is to find an algorithm to recognize a knot (or link), i.e., to provide the studied object with a catalog number. This approach involves the problem to construct complete tables of knots and links arranged taking into account some properties of the object, and list of various invariants of tabulated objects. Many researchers worked in this aria during last 150 years. Most of the constructed tables consider knots and links in 3-dimensional sphere $S^3$. Recently, increasing interest to the theory of knots in arbitrary 3-dimensional manifolds leads to tabulation of knots and links in manifolds different from $S^3$. There are tables of links in the projective space $\mathbb{R}P^3$ [1], knots in the solid torus [2], prime knots in the lens spaces [3], as well as knots and links in the thickened surfaces, see [4] for table of knots in the thickened Klein bottle and [5]– [8] for tables of knots and links in the thickened torus.

Virtual knot theory was proposed by L. Kauffman [9]. At present, virtual knot theory is well developed, although there are just two tables of virtual knots, [10] and [11]. However, there are no tables of virtual links and their invariants. Therefore, the problem of tabulation of virtual links is actual, since classification of objects is one of the main problems of the knot theory. At present, there is no instrument to determine a genus of the giving virtual link, therefore we suggest to classify virtual links by two parameters (the number of classical crossings and the genus of virtual link). To this end, we propose tabulate links in the thickened surfaces of known genus and then convert constructed links into virtual link diagrams. Indeed, according to the Kuperberg theorem [12], every stable equivalence class of links in thickened surfaces has a unique irreducible representative. Therefore every virtual link diagram has a unique representative as a link in the thickened orientable surface having minimal genus among all other thickened orientable surfaces containing the link. Hence, the theory of virtual links can be considered as the theory of links in the thickened surfaces, which admit no destabilization.

The goal of this paper is to classify all genus 1 prime virtual links having virtual link diagrams with at most 4 classical crossings, see required definitions in Section 1, i.e. to construct a table of all inequivalent objects taking into account two parameters (the number of classical crossings and the genus of virtual link), as well as a list of invariants of the tabulated objects. To this end we extend the results of tabulation of prime links in the thickened torus to the case of genus 1 prime virtual links in terms of virtual link diagrams.

In Section 1 we define a prime genus 1 virtual link in terms of virtual link diagrams. In Section 2 we convert diagrams of links in the thickened torus tabulated in [8] into virtual link diagrams to show that there exist no more then 27 pairwise inequivalent genus 1 prime virtual link diagrams having at most 4 classical crossings. In order to provide the obtained table with a list of invariants computed by virtual link diagrams, in Section 3 we extend generalized Kauffman bracket polynomial for the case of virtual link diagrams having genus 1. To this end, in Section 4 we propose an algorithm to compute the numbers of trivial and nontrivial curves. Section 5 gives an example of computation of the generalized Kauffman bracket polynomial by a virtual link diagram. In Section 6 we discuss such interesting application of virtual links having genus 1 as a research on the proteins.

## 1. Prime Virtual Link of Genus 1

Consider a two-dimensional surface $S$ and an interval $I = [0, 1]$. By *a thickened surface* we mean a 3-dimensional manifold homeomorphic to the direct product $S \times I$. A smooth embedding of the set of $m$ pairwise disjoint circles in $Int(S \times I)$ is called *$m$-component link in $S \times I$* and denoted by $L \subset S \times I$.

Recall that a link having single component is a knot. Therefore, we do not consider 1-component links, because the tables of virtual knots were constructed in [10] and [11].

Two links $L$ and $L'$ in $S \times I$ are equivalent if there exists a homeomorphism $h\colon S \times I \to S \times I$ such that $h(L) = L'$.

As in the classical case, links in the thickened torus can be given by their diagrams. *A diagram $D \subset S$ of a link $L \subset S \times I$* is defined by analogy with the diagram of the classical link except that the link is projected into the surface $S$ instead of the plane.

In order to perform *a stabilization* of the surface, it is enough to glue a handle to the surface such that an intersection of the handle and a diagram of link is empty. For example, Fig. 1 (a) shows a surface of genus 2 as a result of stabilization of the surface of genus 1 (i.e., the torus).

In order to perform the inverse operation, *destabilization* of the surface $S$, it is enough to choose a nontrivial (i.e., not bounded a 2-dimensional disk) curve $l$ on $S$ such that an intersection of $l$ and a diagram of link is empty, and then to cut $S$ by $l$ and glue each component of boundary by a 2-dimensional disk. For example, Fig. 1 (b) shows a surface of genus 1 (i.e., the torus) as a result of destabilization of the surface of genus 2.

*A virtual link* is an equivalence class of links in the thickened surfaces modulo homeomorphisms of the form $h\colon S\times I \to S\times I$ such that $h(L)=L'$, where $L$ and $L'$ are links in the thickened surface $S\times I$, and stabilizations (destabilizations).

*A genus of a virtual link* $L$ is the minimal genus of a surface $S$ such that $L$ is situated in the thickened surface $S\times I$. Here *genus of a surface* is the number of its handles. We consider virtual links of genus 1, i.e., links in the thickened torus such that the links admit no destabilizations. In order to draw diagrams on the torus, we represent the torus as a square with identified opposite sides.

*A virtual link diagram* is a planar quadrivalent graph provided with the following structure. Each vertex of the graph either is a classical crossing having overcrossing or undercrossing information (i.e., a small part of the edge in a neighborhood of the vertex is removed to show which strand is going over the other) or a virtual crossing (i.e., marked by a small circle around the vertex). Recall that every virtual link diagram corresponds to a link in the thickened surface. Therefore, classical crossings are obtained as well as in the case of diagram $D\subset S$, while virtual crossings appear, if two parts of diagram $D\subset S$ without common points on the surface $S$ can not be presented in the plain without intersections, see examples given in Fig. 1 (c).

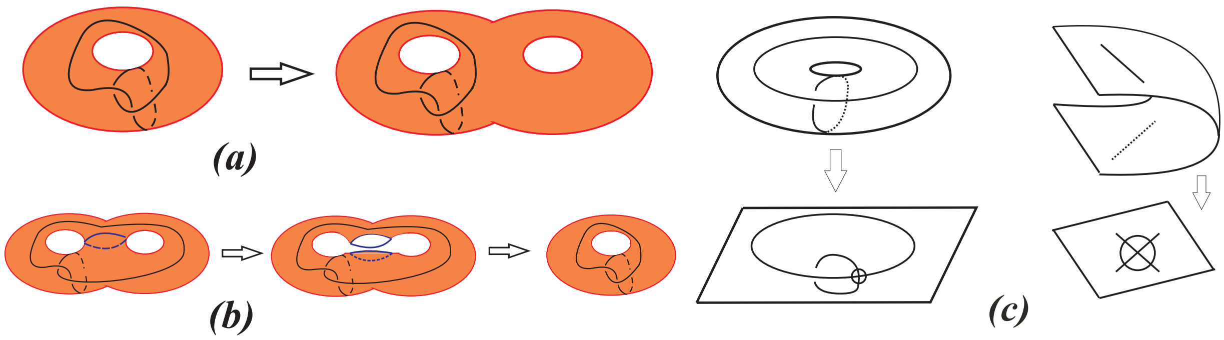


**Fig. 1**. (a) Stabilization of the surface of genus 1, (b) destabilization of the surface of genus 2, (c) virtual crossings appear

*A virtual link* is an equivalence class of virtual link diagrams modulo generalized Reidemeister moves. These moves are classical Reidemeister moves $\Omega_1,\Omega_2,\Omega_3$, their virtual versions $\Omega_1',\Omega_2',\Omega_3'$, and semi-virtual version $\Omega_3''$ of $\Omega_3$, see Fig. 2.

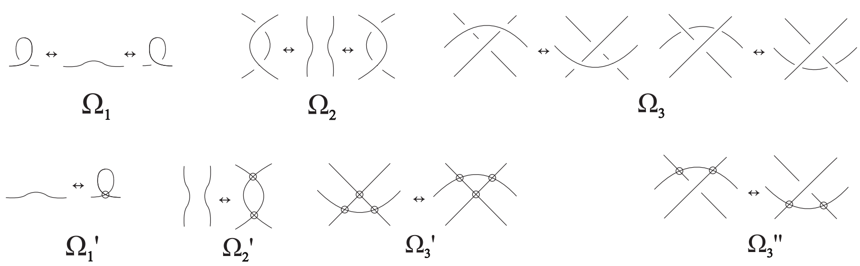


**Fig. 2**. Generalized Reidemeister moves

In the knot theory, recent tables includes only the so-called prime objects, which can not be obtained by some known operations from already tabulated objects. In order to define prime objects in our case, consider the following types of virtual link of genus 1 in terms of virtual link diagrams.

A virtual link $L$ of genus 1 is called *composite*, if $L$ admits a virtual link diagram $D$ such that at least one of the following conditions holds (exactly one of the virtual link diagrams, $D_1$ or $D_2$, can be a knot diagram).

1. $D$ is a connected sum of virtual link diagram $D_1$ of genus 1 and classical link diagram $D_2$. In this case, the connected sum is defined by analogy with the classical connected sum of two classical link diagrams.

2. $D$ is a circular connected sum of two non-trivial genus 1 virtual link diagrams $D_i$ of geometrical degree 1, $i = \overline{1,2}$. Here by a genus 1 virtual link diagram $D_i$ of geometrical degree 1 we mean a genus 1 virtual link diagram $D_i$ having an edge $e_i$ such that the endpoints of $e_i$ are classical crossings (perhaps, the same), $e_i$ passes only through virtual crossings and meet them all. Note that an existence of such edge $e_i$ is a property of virtual link diagram, whereas the same virtual link admits virtual link diagrams without such edge $e_i$. However, the circular connected sum is defined by analogy with the classical connected sum of two classical link diagrams with the exception that the break point $p_i$ is exactly on the edge $e_i$, $i = \overline{1,2}$. For two given genus 1 virtual link diagrams of geometrical degree 1, this operation is well defined up to choice of the break points in the case, when there are two such edges $e_i$ (intersected by a single virtual crossing) in at least one of virtual link diagrams.

A virtual link $L$ of genus 1 is called *split*, if $L$ admits a virtual link diagram $D$ such that either $D$ is not connected, or there exists a component $c$ of $D$ such that if $c$ passes through a classical crossing, then an overcrossing (simultaneous replacement to the term "undercrossing" is allowed) of this classical crossing is a strand of $c$.

A virtual link $L$ of genus 1 is called *essential*, if $L$ admits no diagram without virtual crossings.

A virtual link $L$ of genus 1 is called *prime*, if $L$ is essential, not oriented, not split, not composite and contains more than one component.

## 2. Table of Prime Virtual Link Diagrams of Genus 1 with at Most 4 Classical Crossings

According to the Kuperberg theorem [12], there is a natural bijection between genus 1 virtual links and links in the thickened torus admitting no destabilization. By comparing the conditions on links given in [8] and Section 1, we see that links constructed in [8] are prime virtual links of genus 1 defined in Section 1. Therefore, the following theorem is equivalent to Theorem 2 [8].

**Theorem 1.** *There exist no more than* 27 *pairwise inequivalent prime virtual links of genus 1 with at most 4 classical crossings. All tabulated virtual links are given in Fig. 4, where the components of links are colored in the same colors as in [8].*

Note that we say "no more than" instead of "exactly", because today we can examine all conditions in the definition of prime virtual link of genus 1 except that the given virtual link of genus 1 is not circular connected sum of two non-trivial genus 1 virtual link diagrams of geometrical degree 1. Therefore, some tabulated virtual links can be non-prime, if they are circular connected sums.

We can easy obtain the table of prime genus 1 virtual link diagrams in the plane (see Fig. 4) using the list of link diagrams in the torus constructed in Theorem 2 [8]. To this end we propose the following two steps for each diagram.

1. Close link diagram in the plane (see an example in Fig. 3) by analogy with the braid closure. It means we connect corresponding ends on the opposite square sides in pairs. The obtained arcs intersect by the virtual crossings.

2. Apply a sequence of virtual and semi-virtual versions of Reidemeister moves for removing some virtual crossings.

Fig. 3 shows an example.

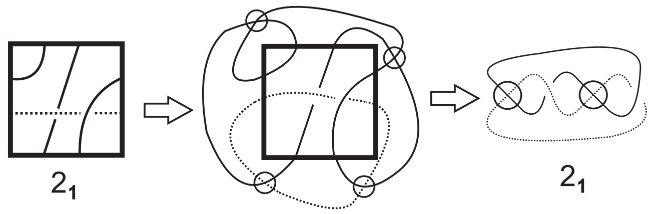


**Fig. 3**. The diagram $2_1$ on the torus is converted into a virtual link diagram

The obtained virtual link diagrams are prime and pairwise inequivalent, because they represent links in the thickened torus, which are prime and pairwise inequivalent according to [8]. Also, the list of invariants, i.e. Kauffman bracket polynomials, obtained in [8] remains the same.

## 3. Kauffman Bracket Polynomial for Genus 1 Virtual Link Diagram

The problem of tabulation is difficult, because there is no universal method to decide if two given objects are equivalent or not. We propose to take into account types of curves (trivial, i.e. bounded a 2-disk, and nontrivial) by analogy with the case of link diagrams on torus [8], where generalized Kauffman bracket polynomial of a virtual link $L$ having genus 1 is

$$X(D) = (-a)^{-3w(D)} \sum_{s} a^{\alpha(s)-\beta(s)}(-a^2 - a^{-2})^{\gamma(s)} x^{\delta(s)}.$$

Here $D$ is a diagram of genus 1 virtual link $L$, $\alpha(s)$ and $\beta(s)$ are the numbers of markers $A$ and $B$ in the given state $s$, and $\gamma(s)$, $\delta(s)$ are the numbers of trivial and nontrivial curves in the torus obtained by resolving all crossings according to the state $s$, and $w(D)$ is the sum of signs of the crossings of $D$ that are self-intersections of the components. The sum is taken over all $2^n$ states, where $n$ is the number of classical crossings in $D$.

The same generalized Kauffman bracket polynomial can be constructed in the case of genus 1 virtual link diagrams. The problem is to distinguish types of curves in the plain obtained as a result of smoothing of genus 1 virtual link diagram according to the given

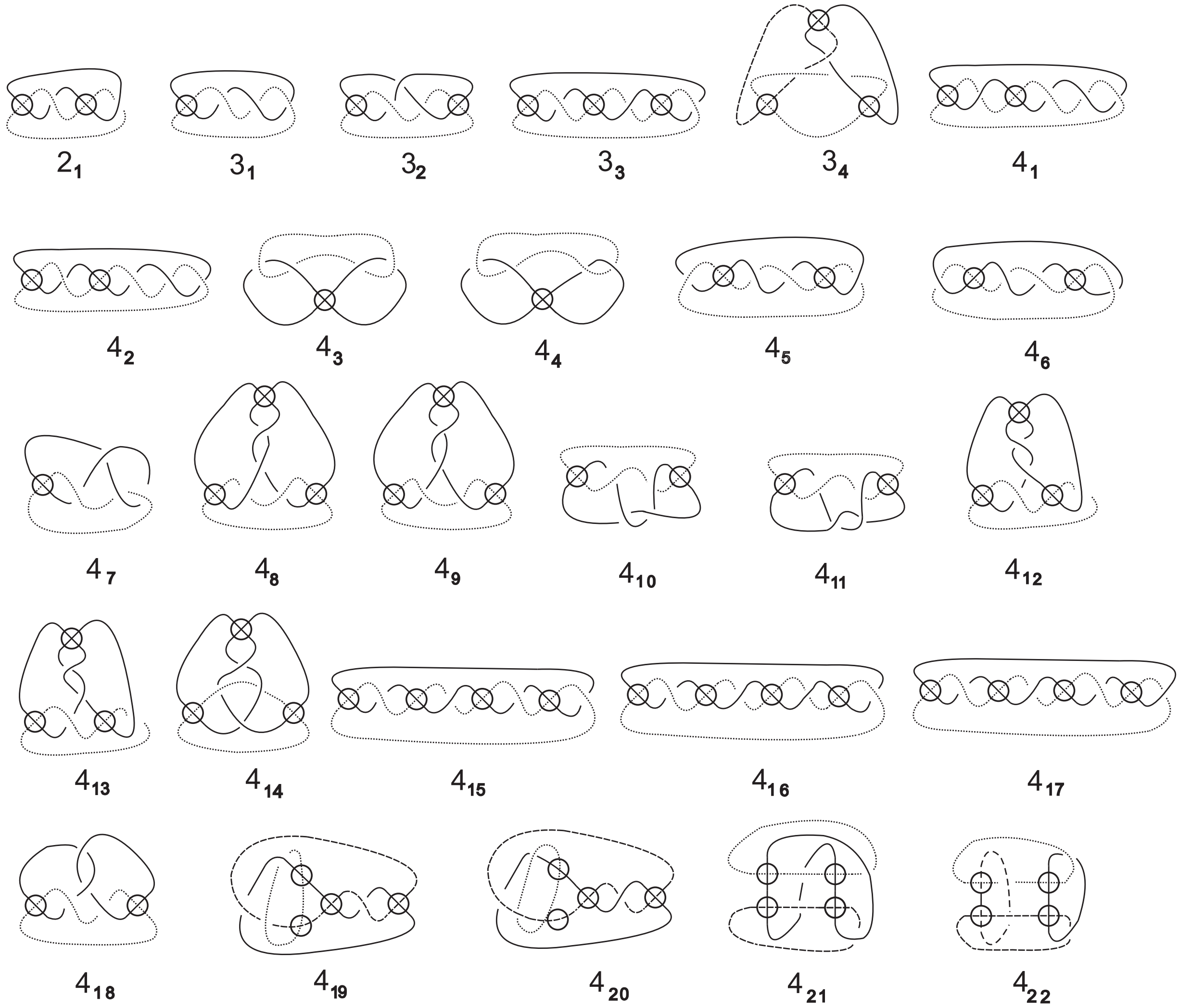


**Fig. 4**. Prime virtual link diagrams of genus 1 with at most 4 classical crossings

state $s$. We mean that each obtained curve in the plain determines trivial or nontrivial curve on the torus.

## 4. Algorithm to Compute the Numbers of Trivial and Nontrivial Curves

In order to distinguish the curves in the plain, we propose an algorithm based on the following ideas.

1. Each trivial curve bounds a 2-dimensional disk, and, therefore, one of the sides of trivial curve can not be connected by an arc with any other curve (with the exclusion of the case of nested curves, but the algorithm recognize them starting with the inner curve).

2. All obtained nontrivial curves are isotopic in the torus. Therefore, each side of nontrivial curve can be connected by arc with some (perhaps, the same) nontrivial

curve. The algorithm recognize such arcs for the given curve as arcs connected two arcs formed each smoothed classical crossings.

3. Information given by smoothed classical crossings is enough, because both sides of each nontrivial curve pass through some (perhaps, the same) smoothed classical crossings, otherwise virtual link admits destabilization and, therefore, is not genus 1.

***The algorithm*** *to determine numbers of trivial and nontrivial curves obtained as a result of smoothing of genus 1 virtual link diagram according to the given state $s$.*

1. Smooth all classical crossings of virtual link diagram according to the state $s$ and denote the total number of obtained curves by $k$.

2. For $i = \overline{1, k}$ convert the thread of $i$-th curve into a $i$-th ribbon, which boundaries are colored with different colors $a_i$ and $b_i$. Note that $a_i \neq a_j$ and $b_i \neq b_j$ for $i \neq j$.

3. For each smoothed classical crossing $c_l$ write equality of the form $a_i = a_j$, $a_i = b_j$ or $b_i = b_j$ included two colors of two ribbon boundary arcs located in the central part of smoothed classical crossing $c_l$. Here $i \in \{1, k\},\ j \in \{1, k\}$ (perhaps, $i = j$), $l = \overline{1, n}$ and $n$ is the number of classical crossings.

4. If at least one of colors, $a_i$ or $b_i$, is not mentioned in obtained system of equalities, then $i$-th curve is trivial. Remove another color (if any) from the system of equalities. Repeat step (4) while there exists a color, which is not mentioned in the system of equalities. All remained colors belong to ribbons based on nontrivial curves.

## 5. Computational Example

Let us compute the generalized Kauffman bracket polynomial $X(2_1)$ for the virtual link diagram $2_1$ given in Fig. 4.

1. Consider virtual link diagram $2_1$ given in Fig. 5 (a).

2. Provide each classical crossing with sequence number in the form $c_l$, $l = 1, 2$, as well as with markers $A$ and $B$ (see Fig. 5 (b)) according to the rule given in Fig. 5 (c) (center).

3. In order to describe all states, we enumerate all 4 possible combinations to smooth classical crossings: $AA$, $AB$, $BA$, $BB$.

4. For each state, replace each classical crossing with corresponding smoothing of type $A$ or type $B$ (see Fig. 5 (d), where obtained curves are colored) according to the rule given in Fig. 5 (c) (left and right).

5. In order to determine numbers of trivial and nontrivial curves $\gamma(s)$ and $\delta(s)$, for each state we use an algorithm proposed above. The results are given in Table 1. For example, consider state $AA$, see Fig. 5 (e). Convert the single curve into a ribbon, which boundaries are colored with $a_1$ (firm line) and $b_1$ (dashed line). The system of equations given by classical crossings $c_1$ and $c_2$ contains no color $b_1$, since both ribbon boundary arcs located in the central part of smoothed classical crossing $c_1$ (similarly, $c_2$) are colored with $a_1$ (firm line). Therefore, the considered single curve is trivial.

Therefore,

$$\langle 2_1 \rangle = (\underbrace{a^2(-a^2-a^{-2})^1x^0}_{AA} + \underbrace{a^0(-a^2-a^{-2})^0x^2}_{AB} + \underbrace{a^0(-a^2-a^{-2})^0x^2}_{BA} + \underbrace{a^{-2}(-a^2-a^{-2})^1x^0}_{BB}),$$

i.e.

$$X(2_1) = (-a^{-4} - 2 - a^4) + 2x^2.$$

**Table 1**

Computation of $\gamma(s)$ and $\delta(s)$ for the diagram $2_1$ by the algorithm

| **Code of state** $s$ | **Computations** | | | **Results** | |
|---|---|---|---|---|---|
| | *Number of curves* | *Equation by* $c_1$ | *Equation by* $c_2$ | $\gamma(s)$ | $\delta(s)$ |
| $AA$ | $k = 1$ | $a_1 = a_1$ | $a_1 = a_1$ | 1 | 0 |
| $AB$ | $k = 2$ | $a_1 = a_2$ | $b_1 = b_2$ | 0 | 2 |
| $BA$ | $k = 2$ | $a_1 = a_2$ | $b_1 = b_2$ | 0 | 2 |
| $BB$ | $k = 1$ | $a_1 = a_1$ | $a_1 = a_1$ | 1 | 0 |

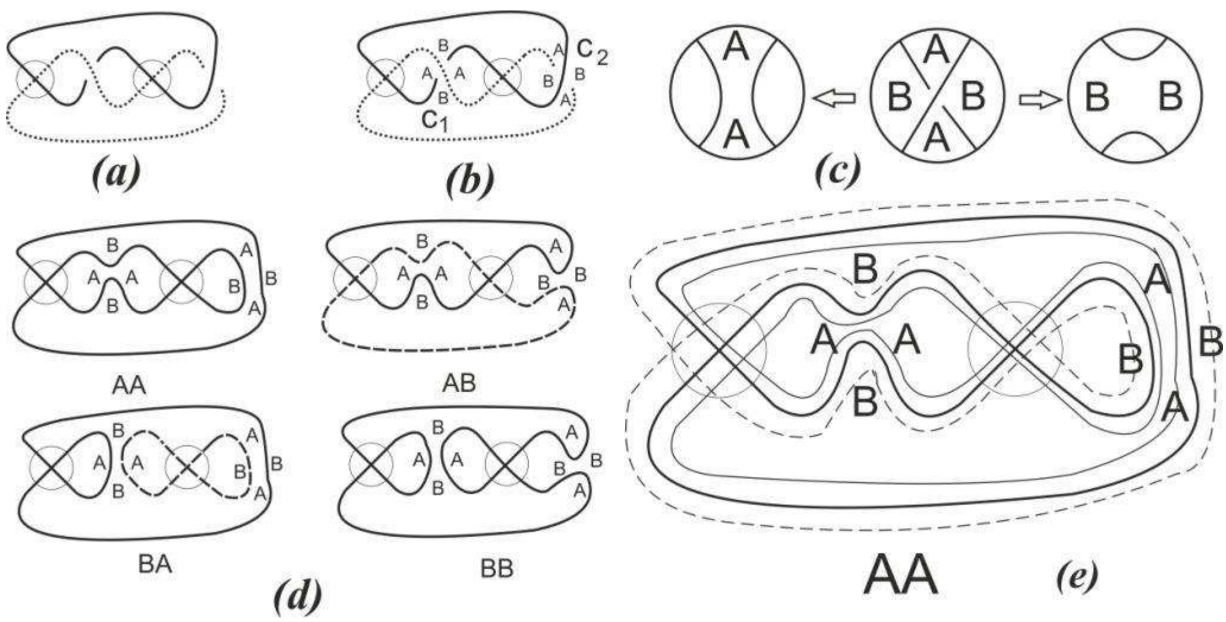


**Fig. 5**. Computation of the generalized Kauffman bracket polynomial $X(2_1)$

## 6. Practical Significance and Implementation Proposals

Many of the processes essential to life involve proteins, i.e. long molecules which fold into three-dimensional shapes allowing them to perform their biological role. A folded protein molecule consists of strings of amino acids. According to [14], open protein chains formed from a string of carbon and nitrogen atoms can be considered as long, knotted curves having distinct endpoints. Mathematically, open protein chains can be represented by knotoids proposed by V. Turaev in [15]. Here by knotoid we mean an open knot diagram which differs from the classical knot diagram in that the underlying curve is an interval rather than a curve.

Earlier research on knotted proteins attempted to close the protein into a curve using different ways of connection. A method to represent proteins as virtual knots was proposed in [14] and allows do not consider many different possibilities. The method corresponds to the virtual closure of the classical knotoid proposed in [15]. The virtual closure determines a well-defined map from knotoids in $S^2$ to virtual knots, which is non-injective and non-surjective, see [16]. Since all virtual crossings obtained as a result of virtual closure necessarily occur sequentially along the same arc, the genus of virtual knots obtained by virtual closure is at most one. Therefore, in order to describe the knotted structure of proteins that form knots and knotoids, a tables of classical knots and virtual knots of genus 1 are enough. In [14], these tables are used to analyze the database KnotProt 2.0 [17], which collects information about proteins that form knots and knotoids. A virtual knot table is essential for understanding these results fully.

However, knots are not enough to describe a structure of all proteins. The database LinkProt [18] collects information about protein chains and complexes that form links and provides an exhaustive list of open linked proteins and topologically linked proteins in the Protein Database [19]. Therefore, a table of virtual links of genus 1 is necessary for analogical analysis of the database LinkProt. Namely, for a set of linked open curves, a virtual link analysis can be performed in much the same way as the virtual knot analysis in [14].

## Conclusion

We begin resolution of one of the main problems of virtual knot theory, i.e. classification of virtual links. To this end, we extend the results of classification of prime links in the thickened torus to the case of genus 1 prime virtual links in terms of virtual link diagrams: define a prime genus 1 virtual link, show that there exist no more then 27 pairwise inequivalent genus 1 prime virtual link diagrams having at most 4 classical crossings, provide the obtained table with a list of invariants. In order to extend generalized Kauffman bracket polynomial for the case of virtual link diagrams, we propose an algorithm to compute the numbers of trivial and nontrivial curves. One of possible future applications of the constructed table is an analysis of the database LinkProt that collects information about protein chains and complexes that form links.

Note that long, flexible physical strands, from macroscopic string to long-chain molecules, are naturally knotted, that determines their configuration and properties. Results obtained in [14] emphasise that virtual knotting is a generic feature of certain geometrical classes of curves, arising from relatively weak geometric constraints even in the absence of the physical complexity of protein chains. Therefore, the results of the paper can be introduced into a research on the proteins. New mathematical techniques for the analysis and exploitation of knots and links from a wide range of complex physical structures need further study.

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# КЛАССИФИКАЦИЯ ПРИМАРНЫХ ВИРТУАЛЬНЫХ ЗАЦЕПЛЕНИЙ РОДА 1, ИМЕЮЩИХ НЕ БОЛЕЕ 4 КЛАССИЧЕСКИХ ПЕРЕКРЕСТКОВ

***А. А. Акимова, В. В. Таркаев***

Одной из основных проблем теории узлов является классификация изучаемых объектов, т. е. построение таблицы всех неэквивалентных объектов с учетом параметров, представляющих некоторые свойства, а также списка инвариантов табулированных объектов. Цель этой статьи – классифицировать все примарные виртуальные зацепления рода 1, имеющие виртуальные диаграммы с не более чем 4 классическими перекрестками. Для того, чтобы обобщить скобочный полином Кауффмана на случай виртуальных диаграмм, мы предлагаем алгоритм, позволяющий определить число тривиальных и нетривиальных кривых. Результаты работы могут быть использованы при исследовании белков с помощью метода представления белков в виде виртуальных зацеплений рода 1.

*Ключевые слова: виртуальные зацепления; род 1; таблица; скобка Кауффмана; белки.*

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