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HEAT TRANSFER AND FLOW ANALYSIS OF MAGNETOHYDRODYNAMIC DISSIPATIVE CARREAU NANOFLUID OVER A STRETCHING SHEET WITH INTERNAL HEAT GENERATION

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The unsteady two-dimensional flow and heat transfer analysis of Carreau nanofluid over a stretching sheet subjected to magnetic field, temperature dependent heat source/sink and viscous dissipation is presented in this paper. Similarity transformations are used to reduce the systems of the developed governing partial differential equations to nonlinear third and second orders ordinary differential equation which are solved using differential transform method. Using kerosene as the base fluid embedded with the silver (Ag) and copper (Cu) nanoparticles, the effects of pertinent parameters on reduced Nusselt number, flow and heat transfer characteristics of the nanofluid are investigated and discussed. From the results, it is established temperature field and the thermal boundary layers of Ag-Kerosene nanofluid are highly effective when compared with the Cu-Kerosene nanofluid. Heat transfer rate is enhanced by increasing in power-law index and unsteadiness parameter. Skin friction coefficient and local Nusselt number can be reduced by magnetic field parameter and they can be enhanced by increasing the aligned angle. Friction factor is depreciated and the rate of heat transfer increases by increasing the Weissenberg number. Also, for the purpose of verification, the results of the analytical of the approximate analytical solutions are compared with the results of numerical solution using Runge-Kutta coupled with Newton method. A very good agreement is established between the results. This analysis can help in expanding the understanding of the thermo-fluidic behaviour of the Carreau nanofluid over a stretching sheet.

Keywords: MHD; Nanofluid; non-uniform heat source/sink; Carreau fluid; thermal radiation and free convection.

Introduction

Free convective heat transfer in thin film flow of nanofluids are often encountered in various industrial and engineering applications such as wire and fiber coating, heat exchangers, extrusion process, polymer processing and chemical processing equipment, etc. Also, the analysis of stretched flows with heat transfer is very significant in controlling the quality of the end product in the afore-mentioned areas of applications. Such processes have great dependence on the stretching and cooling rates [1]. Consequently, in the past few years, research efforts have been directed towards the analysis of this very important phenomenon of wide areas of applications. In an early study of MHD fluid flow over a stretching carried out by Anderson et al. [2, 3] effects of a power-law fluid caused by thin liquid film and magnetohydrodynamic on an unsteady stretching surface were investigated.

In such study, it was established that the power-law index is more effective on temperature field than velocity field. Few years later, Chen [4] investigated the power-law fluid film flow of unsteady heat transfer stretching sheet while Dandapat et al. [5, 6] analyzed the effect of variable viscosity and thermo- capillarity on the heat transfer of liquid film flow over a stretching sheet. Meanwhile, Wang [7] developed an analytical solution for the momentum and heat transfer of liquid film flow over a stretching surface. Also, Chen [8] and Sajid et al. [9] investigated the flow characteristics of a non-Newtonian thin film over an unsteady stretching surface considering viscous dissipation using homotopy analysis and homotopy perturbation methods. After a year, Dandapat et al. [10] presented the analysis of two-dimensional liquid film flow over an unsteady stretching sheet while in the same year, effect of power-law index on unsteady stretching sheet was studied by Abbasbandy [11] while Santra and Dandapat [12] numerically studied the flow of the liquid film over an unsteady horizontal stretching sheet. A numerical approach was also used by Sajid et al. [13] to analyze the micropolar film flow over an inclined plate, moving belt and vertical cylinder. A year later, Noor and Hashim [14] investigated the effect of magnetic field and thermocapillarity on an unsteady flow of a liquid film over a stretching sheet while Dandapat and Chakraborty [15]) and Dandapat and Singh [16] presented the thin film flow analysis over a non-linear stretching surface with the effect of transverse magnetic field. Heat transfer characteristics of the thin film flows considering the different channels have also been analyzed by Abdel-Rahman [17], Khan et al. [18], Liu et al. [19] and Vajaravelu et al. [20] Meanwhile, Liu and Megahad [21] used homotopy perturbation method to analyze thin film flow and heat transfer over an unsteady stretching sheet with internal heating and variable heat flux. Effect of thermal radiation and thermocapillarity on the heat transfer thin film flow over a stretching surface was examined by Aziz et al. [22]. In their study on the numerical simulation of Eyring Powell flow and unsteady heat transfer of a laminar liquid film over a stretching sheet using finite difference method, Khader and Megahed [23] established that increasing the Prandtl number reduces the temperature field across the thin film.

The promising significance of the MHD fluid behavior in various engineering and industrial applications (such as in the design of cooling system with liquid metals, accelerators, MHD generators, nuclear reactor, pumps and flow meters and blood flow) still provokes the continuous studies and interests of researchers. Indisputably, numerous studies have been presented in literature on the behavior of magnetohydrodynamic flow in different flow con?gurations. Also, the effects of MHD on the non-Newtonian nanofluid have been of research interests in recent times. In a recent study, Lin et al. [24] examined the effect of MHD pseudo-plastic nanofluid flow and heat transfer film flow over a stretching sheet with internal heat generation. Numerically, Raju and Sandeep [25] studied heat and mass transfer in MHD non-Newtonian flow while Tawade et al. [26] presented the unsteady flow and heat transfer of thin film over a stretching surface in the presence of thermal radiation, internal heating in the presence of magnetic field. Heat and mass transfer of MHD flows through different channels have been analyzed [27, 28, 29, 30, 31, 32]. Makinde and Animasaun [33] investigated the effect of cross diffusion on MHD bioconvection flow over a horizontal surface. In another study, Makinde and Animasaun [34] presented the MHD nanofluid on bioconvection flow of a paraboloid revolution with nonlinear thermal radiation and chemical reaction while Sandeep [35], Ramana Reddy et al. [36] and Ali et al. [37] studied the heat transfer behaviour of MHD flows.

The above studies have been the consequent of the various industrial and engineering applications of non-Newtonian fluids. Among the classes of non-Newtonian fluids, Carreau fluid which its rheological expressions was first introduced by Carreau [38], is one of the non-Newtonian fluids that its model is substantial for gooey, high and low shear rates [39]. On account of this headway, it has profited in numerous innovative and assembling streams [39]. Owing to these applications, different studies have been carried out to explore the characteristics of Carreau liquid in flow under different conditions. Hayat et al. [40] studied the in?uence of induced magnetic field and heat transfer on peristaltic transport of a Carreau fluid. Olajuwon [41] presented a study on MHD flow of Carreau liquid over vertical porous plate with thermal radiation. Hayat et al. [42] investigated the convectively heated flow of Carreau fluid while in the same year, Akbar et al. [43] analyzed the stagnation point flow of Carreau fluid. Also, Akbar [44] presented blood flow of Carreau fluid in a tapered artery with mixed convection. A year later, Mekheimer [45] investigated the unsteady flow of a carreau fluid through inclined catheterized arteries haveing a balloon with time-variant overlapping stenosis. Elmaboud et al. [46] developed series solution of a natural convection flow for a Carreau fluid in a vertical channel with peristalsis. Using a revised model, flow of Carreau nanoliquid in the presence of zero mass flux condition at the stretching sheet has been examined by Hashim and Khan [47]. Raju and Sandeep [29] explored heat and mass transfer in Falkner-Skan flow of Carreau liquid past a wedge. The MHD flow of Carreau fluid with thermal radiation and cross diffusion effects was investigated by Machireddy and Naramgari [48]. Sulochana et al. [49] provided an analysis of magnetohydrodynamic stagnation-point flow of a Carreau nanofluid. In recent time, Raju and Sandeep [27] put forward the problem of homogeneous-heterogeneous reactions in non-linear stretched flow of Casson-Carreau fluid. Hayat et al. [1] presented radiative flow of Carreau liquid in presence of Newtonian heating and chemical reaction. Kumar and Kumar [39] applied Runge-Kutta and Newton's method to analyze the flow and heat transfer of electrically conducting liquid film flow of Carreau nanofluid over a stretching sheet by considering the aligned magnetic field in the presence of space and temperature dependent heat source/sink, viscous dissipation and thermal radiation.

The relatively new approximate analytical method, differential transformation method has proven proved to be more effective than most of the other approximate analytical solutions as it does not require many computations as carried out in Adomian decomposition method (ADM), homotopy analysis method (HAM), homotopy perturbation method (HPM), and variational iteration method ((VIM). Also, the differential transformation method as introduced by Zhou [50] has fast gained ground as it appeared in many engineering and scientific research papers because of its comparative advantages over the other approximate analytical methods. It is a method that could solve differential equations, difference equation, differential-difference equations, fractional differential equation, pantograph equation and integro-differential equation. It solves nonlinear integral and differential equations without linearization, discretization, restrictive assumptions, perturbation and discretization or round-off error. It reduces complexity of expansion of derivatives and the computational difficulties of the other traditional methods. Using DTM, a closed form series solution or approximate solution can be obtained as it provides excellent approximations to the solution of non-linear equation with high accuracy. It is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate. It is a more convenient method for engineering calculations compared with other approximate analytical or numerical methods. It appears more appealing than the numerical solution as it helps to reduce the computation costs, simulations and task in the analysis of nonlinear problems. Moreover, the need for small perturbation parameter as required in traditional PMs, the rigour of the determination of Adomian polynomials as carried out in ADM, the restrictions of HPM to weakly nonlinear problems as established in literatures, the lack of rigorous theories or proper guidance for choosing initial approximation, auxiliary linear operators, auxiliary functions, auxiliary parameters, and the requirements of conformity of the solution to the rule of coeficient ergodicity as done in HAM, the search Langrange multiplier as carried in VIM, and the challenges associated with proper construction of the approximating functions for arbitrary domains or geometry of interest as in Galerkin weighted residual method (GWRM), least square method (LSM) and collocation method (CM) are some of the difficulties that DTM overcomes. Therefore, in this study, differential transformation method is used to analysis the flow and heat transfer of an electrically conducting liquid film flow of Carreau nanofluid over a stretching sheet subjected to magnetic field, temperature dependent heat source/sink and viscous dissipation. Using kerosene as the base fluid embedded with the silver (Ag) and copper (Cu) nanoparticles, the effects of pertinent parameters on reduced Nusselt number, flow and heat transfer characteristics of the nanofluids are investigated and discussed.

1. Problem Formulation

Consider an unsteady, two-dimensional boundary layer flow of an electrically conducting and heat generating Carreau nanofluid over a stretching sheet bounded by a thin liquid film of uniform thickness $h(t)$ over a horizontal elastic sheet which emerges from a narrow slit at the origin of the cartesian coordinate system which is schematically represented in Fig.1. The sheet is stretched along the x-axis with stretching velocity $U(x,t)$ and y-axis is normal to it. An inclined magnetic field. The effects of non-uniform heat source/sink, thermal radiation, viscous is applied to the stretching sheet at angle dissipation and volume fraction are taken into consideration.

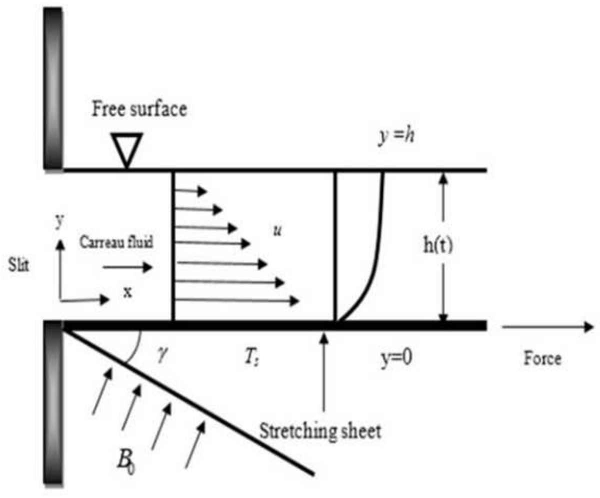

Fig. 1. Flow geometry of the problem (Kumar et al., 2017)

Following the assumptions, the equations for continuity and motion are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{nf}\left(1+\frac{3\left(n-1\right)\Gamma^{2}}{2}\left(\frac{\partial u}{\partial y}\right)^{2}\right)\frac{\partial^{2} u}{\partial y^{2}}-\sigma B_{o}^{2}u\cos^{2}\gamma, \tag{2}$$

$$\left(\rho C_{p}\right)_{nf}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\frac{\partial^{2} T}{\partial y^{2}}+\mu_{nf}\left(\frac{\partial u}{\partial y}\right)^{2}+q''', \tag{3}$$

where $\rho_{nf}=\rho_{f}\left(1-\varphi\right)+\rho_{s}\varphi$, $\left(\rho C_{p}\right)_{nf}=\left(\rho C_{p}\right)_{f}\left(1-\varphi\right)+\left(\rho C_{p}\right)\varphi$, $\mu_{nf}=\frac{\mu_{f}}{(1-\varphi)^{2.5}}$, $k_{nf}=k_{f}\left[\frac{k_{s}+2k_{f}-2\varphi\left(k_{f}-k_{s}\right)}{k_{s}+2k_{f}+\varphi\left(k_{f}-k_{s}\right)}\right]$.

Assuming no slip condition, the appropriate boundary conditions are given as

$$u=U_{w},\quad v=0,\quad T=T_{s}\quad \text{at}\quad y=0,$$

$$\frac{\partial u}{\partial y}=0,\quad \frac{\partial T}{\partial y}=0\quad \text{at}\quad y=h,$$

$$v=\frac{dh}{dt}=-\frac{\alpha\beta}{2}\left(\frac{v_{f}}{b\left(1-\alpha t\right)}\right)^{\frac{1}{2}},\quad y=h(t)=\beta\left(\frac{v_{f}\left(1-\alpha t\right)}{b}\right)^{\frac{1}{2}}.$$

The non-uniform heat generation/absorption q''' is taken as

$$q'''=\frac{k_{f}U_{w}}{x\nu_{f}}\left[A^{*}\left(T_{s}-T_{o}\right)f'+B^{*}\left(T_{s}-T_{o}\right)\right],$$

where the surface temperature T_{s} of the stretching sheet varies with respect to distance x-from the slit as $T_{s}=T_{o}-T_{ref}\left(\dfrac{bx^{2}}{2v_{f}\left(1-at\right)^{\frac{3}{2}}}\right)$. And the stretching velocity varies with respect to x as $U=\dfrac{bx}{(1-at)}$.

On introducing the following stream functions

$$u=\frac{\partial\psi}{\partial y},\quad v=\frac{\partial\psi}{\partial x}. \tag{4}$$

And the similarity variables

$$\begin{aligned} &u=\tfrac{bx}{(1-at)}f'\left(\eta,t\right),\quad v=-\left(b\nu_{f}\right)^{-\frac{1}{2}}\left(1-at\right)^{-\frac{1}{2}}f\left(\eta,t\right),\\ &\eta=\left(b/\nu_{f}\right)^{\frac{1}{2}}\left(1-at\right)^{-\frac{1}{2}}y,\quad T=T_{o}-T_{ref}\left(bx^{2}/2v_{f}\right)\left(1-at\right)^{-\frac{3}{2}}\theta\left(\eta\right). \end{aligned} \tag{5}$$

Substituting Eq. (4) and (5) into Eq. (1), (2) and (3), we have a partially coupled third and second orders ordinary differential equation

$$f'''\left\{1+\frac{3\left(n-1\right)We\left(f''\right)^{2}}{2}\right\}+B_{1}\left\{B_{2}\left(S\left(f'+\frac{\eta}{2}f''\right)+ff''-\left(f'\right)^{2}\right)\right\}-Ha^{2}f'\cos^{2}\gamma=0, \tag{6a}$$

$$B_{3}\theta''+\frac{EcPr}{B_{1}}\left(f''\right)^{2}+\left\{A^{*}f'+B^{*}\theta\right\}-B_{4}Pr\left\{\frac{S}{2}\left(\left(\eta\theta'+3\theta\right)+2f'\theta-f\theta'\right)\right\}=0,\ y=h, \tag{6b}$$

where

$$We^2=\frac{b^3x^2\Gamma^2}{v_f\left(1-at\right)^3},\ Pr=\frac{\mu c_p}{k_f},\ Ha^2=\frac{\sigma B_o^2}{\rho_f b},\ Ec=\frac{U_w^2}{c_p\left(T_s-T_0\right)},\ S=\frac{\alpha}{b},\ R=\frac{4\sigma^* T_0^3}{k^* k_f},$$

$$B_1=\left(1-\varphi\right)^{2.5},\quad B_2=1-\varphi+\varphi\frac{\rho_s}{\rho_f},\quad B_3=\frac{k_{nf}}{k_f},\quad B_4=1-\varphi+\varphi\frac{\left(\rho c_p\right)_s}{\left(\rho c_p\right)_f}.$$

And the following boundary conditions becomes

$$\eta=0,\quad f=0,\quad f'=1,\quad \theta=0,\quad \eta=\beta,\quad f=\frac{S\beta}{2},\quad f''=0,\quad \theta'=0. \tag{7}$$

2. Method of Solution: Differential Transform Method

The above nonlinear Eqs. (6) is solved using differential transformation method as introduced by Zhou [50]. The basic definitions and the operational properties of the method are as follows.

If $u(t)$ is analytic in the domain T, then the function $u(t)$ will be differentiated continuously with respect to time t.

$$\frac{d^p u(t)}{dt^p}=\phi(t,p)\qquad \text{for all}\ \ t\in T \tag{8}$$

for $t=t_i$, then $\phi(t,p)=\phi(t_i,p)$, where p belongs to the set of non-negative integers, denoted as the p-domain. We can therefore write Eq. (8) as

$$U(p)=\phi(t_i,p)=\left[\frac{d^p u(t)}{dt^p}\right]_{t=t_i}, \tag{9}$$

where U_p is called the spectrum of $u(t)$ at $t=t_i$.

Expressing $u(t)$ in Taylor's series as

$$u(t)=\sum_{p}^{\infty}\left[\frac{\left(t-t_i\right)^p}{p!}\right]\ U(p), \tag{10}$$

where Eq. (8) is the inverse of $U(k)$ us symbol 'D' denoting the differential transformation process and combining (9) and (10), we have

$$u(t)=\sum_{p=0}^{\infty}\left[\frac{\left(t-t_i\right)^p}{p!}\right]\ U(p)=D^{-1}U(p).$$

Using the operational properties of the differential transformation method (Table 1), the differential transformation of the governing differential Eq. (6a) is given as

$$\begin{gathered}(p+1)\,(p+2)\,(p+3)F[p+3]+\frac{3\,(n-1)\,We}{2}\sum_{l=0}^{p}\sum_{m}^{l}\Big((l-m+1)\,(l-m+2)\,(p-l-m+1)\cdot\\ \cdot(p-l-m+2)F[l-m+2]F[p-l-m+2]\,(m+1)\,(m+2)\,(m+3)F[m+3]\Big)+\\ +B_1\Bigg(B_2\Bigg[\Big((k+1)F[k+1]+\frac{k\,(k+1)}{2}F[k+1]\Big)S+\sum_{l=0}^{p}(p-l+1)\,(p-l+2)F[l]F[p-l+2]-\\ -\sum_{l=0}^{p}(l+1)\,(p-l+1)F[l+1]F[p-l+1]\Bigg]-(M\cos\gamma)^2(p+1)F[p+1]\Bigg)=0.\end{gathered} \tag{11}$$

Table 1

Operational properties of differential transformation method

S/N	Function	Differential transform
1	$u(t) \pm v(t)$	$U(p) \pm V(p)$
2	$\alpha u(t)$	$\alpha U(p)$
3	$\dfrac{du(t)}{dt}$	$(p+1)\,U(p+1)$
4	$u(t)v(t)$	$\sum\limits_{r=0}^{p} V(r)U(p-r)$
5	$u^m(t)$	$\sum\limits_{r=0}^{p} U^{m-1}(r)U(p-r)$
6	$\dfrac{d^n u\,(t)}{dx^n}$	$(p+1)\,(p+2)\cdots(p+n)\,U\,(p+n)$
7	$\sin\left(\omega t+\alpha\right)$	$\dfrac{\omega^p}{p!}\sin\left(\frac{\pi p}{2!}+\alpha\right)$
8	$\cos\left(\omega t+\alpha\right)$	$Z\,(p)=\dfrac{\omega^p}{p!}\cos\left(\frac{\pi p}{2!}+\alpha\right)$

Equivalently, we can write the recursive relation for Eq. (6a) in DTM domain as

$$(p+1)\,(p+2)\,(p+3)F[p+3]+\frac{3\,(n-1)\,We}{2}\sum_{l=0}^{p}\sum_{m}^{l}\Big((l-m+1)\,(l-m+2)\,(p-l-m+1)\cdot$$
$$\cdot\,(p-l-m+2)F[l-m+2]F[p-l-m+2]\,(m+1)\,(m+2)\,(m+3)F[m+3]\Big)+$$
$$+B_1\Bigg(B_2\Bigg[\Bigg((k+1)F[k+1]+\frac{1}{2}\sum_{l=0}^{p}\delta\,(l-1)\,(k-l+1)\,(k-l+2)F[k-l+2]\Bigg)S+$$
$$+\sum_{l=0}^{p}(p-l+1)\,(p-l+2)F[l]F[p-l+2]-$$
$$-\sum_{l=0}^{p}(l+1)\,(p-l+1)F[l+1]F[p-l+1]\Bigg]-(M\cos\gamma)^2(p+1)F[p+1]\Bigg)=0. \quad (12)$$

$$F[p+3]=\frac{3\,(1-n)\,We}{2\,(p+1)\,(p+2)\,(p+3)}\sum_{l=0}^{p}\sum_{m}^{l}\Big((l-m+1)\,(l-m+2)\,(p-l-m+1)\cdot$$
$$\cdot\,(p-l-m+2)F[l-m+2]F[p-l-m+2]\,(m+1)\,(m+2)\,(m+3)F[m+3]\Big)-$$
$$-\frac{B_1}{(p+1)\,(p+2)\,(p+3)}\Bigg(B_2\Bigg[\Bigg((k+1)F[k+1]+$$
$$+\frac{1}{2}\sum_{l=0}^{p}\delta\,(l-1)\,(k-l+1)\,(k-l+2)F[k-l+2]\Bigg)S+\sum_{l=0}^{p}(p-l+1)\,(p-l+2)F[l]F[p-l+2]-$$
$$-\sum_{l=0}^{p}(l+1)\,(p-l+1)F[l+1]F[p-l+1]\Bigg]-(M\cos\gamma)^2(p+1)F[p+1]\Bigg), \quad (13)$$

where $\delta\,(l)=\begin{cases}1, & l=0,\\ 0, & l\neq 0.\end{cases}$

For the Eq. (6b), we have the recursive relation in DTM domain as

$$B_3\,(p+1)\,(p+2)\,\theta\,[p+2]+\frac{EcPr}{B_1}\sum_{l=0}^{p}(l+1)\,(l+2)\,(p-l+1)\,(p-l+2)F[l+2]\cdot$$
$$\cdot F[p-l+2]+A^*\,(p+1)F[p+1]+B^*\theta\,[p]-B_4Pr\left(\frac{S}{2}\,(p\theta[p]+3\theta[p])+\right.$$
$$\left.+2\sum_{l=0}^{p}(l+1)F[l+1]\,\theta\,[p-l]-\sum_{l=0}^{p}(l+1)\,\theta\,[l+1]F[p-l]\right)=0. \quad (14)$$

$$\theta\,[p+2]=\frac{1}{B_3\,(p+1)\,(p+2)}\left(\frac{-EcPr}{B_1}\sum_{l=0}^{p}(l+1)\,(l+2)\,(p-l+1)\,(p-l+2)F[l+2]\cdot\right.$$
$$\cdot F[p-l+2]-A^*\,(p+1)F[p+1]-B^*\theta[p]+B_4Pr\left(\frac{S}{2}\,(p\theta[p]+3\theta[p])+\right.$$
$$\left.\left.+2\sum_{l=0}^{p}(l+1)F[l+1]\,\theta\,[p-l]-\sum_{l=0}^{p}(l+1)\,\theta\,[l+1]F[p-l]\right)\right) \quad (15)$$

Also, recursive relation for the boundary conditions in Eq.(7) are

$$\begin{array}{lll} F\,[p]=0\Rightarrow F\,[0]=0,\ (p+1)\,F\,[p+1]=1 & \Rightarrow & F[1]=1,\ \theta\,[p]=0\ \Rightarrow\theta\,[0]=0,\\ \sum\limits_{l=0}^{p}(p+1)\,(p+2)\,F\,[p+2]=0 & \Rightarrow & F\,[2]=a_1,\ \ \sum\limits_{l=0}^{p}F\,[p]=\dfrac{S\beta}{2},\\ \sum\limits_{l=0}^{p}(p+1)\,\theta\,[p+1]=0 & \Rightarrow & \theta\,[1]=a_2. \end{array} \quad (16)$$

Therefore, we have the following boundary conditions in DTM domain

$$F\,[0]=0,\quad F[1]=1,\quad \theta\,[0]=0,\quad F\,[2]=a_1,\quad \theta\,[1]=a_2. \quad (17)$$

Using $p=0,1,2,3\ldots$ in the above recursive relations in Eq. (13), we arrived at

$$F\,[3]=-\frac{B_1\left(-B_2+B_2S-M^2\cos^2\gamma\right)}{6\left(1-6a_1^2W_e+6a_1^2nW_e\right)}, \quad (18)$$

$$F\,[4]=\frac{1}{24}\left(-B_1\left(B_2\left(-2a_1^2+3a_1S\right)-2a_1M^2\cos^2\gamma\right)-\right.$$
$$\left.-\frac{6a_1B_1^2\,(-1+n)\left(-B_2+B_2S-M^2\cos^2\gamma\right)^2W_e}{\left(1-6a_1^2W_e+6a_1^2nW_e\right)^2}\right), \quad (19)$$

The expressions for $F[5],F[6],F[7],\ldots$ are too large to be included in this paper. Also, using $p=0,1,2,3\ldots$ in the above recursive relations in Eq. (15), we arrived at

$$\theta\,[2]=\frac{-AB_1-4a_1^2EcPr}{2B_1B_3}, \quad (20)$$

$$\theta\,[3]=\frac{1}{6B_3}\left(-2Aa_1-a_2B+B_4Pr\,(a_2+2a_2S)+\frac{4a_1EcPr\left(-B_2+B_2S-M^2\cos^2\gamma\right)}{1-6a_1^2W_e+6a_1^2nW_e}\right), \quad (21)$$

$$\theta\,[4]=\frac{1}{48B_3}\left[\frac{2B\left(AB_1+4a_1^2EcPr\right)}{B_1B_3}+\frac{B_4Pr\left(12a_1a_2B_1B_3-5AB_1S-20a_1^2EcPrS\right)}{B_1B_3}+\right.$$

$$+\frac{2AB_1\left(B_2\left(-1+S\right)-M^2\cos^2\gamma\right)}{1+6a_1^2\left(-1+n\right)W_e}-4EcPr\left\{\frac{B_1\left(B_2\left(-1+S\right)-M^2\cos^2\gamma\right)^2}{\left(1+6a_1^2\left(-1+n\right)W_e\right)^2}+\right.$$
$$\left.\left.+2a_1^2\left(B_2\left(2a_1-3S\right)+2M^2\cos^2\gamma-\frac{6B_1\left(-1+n\right)\left(B_2\left(-1+S\right)-M^2\cos^2\gamma\right)^2W_e}{\left(1+6a_1^2\left(-1+n\right)W_e\right)^2}\right)\right\}\right]. \quad (22)$$

Also, the expressions for $\theta[5], \theta[6], \theta[7], \ldots$ are too large to be included in this paper. Using the definition in Eq. (10), the solution of Eq. (7) using DTM is given as

$$\begin{aligned} F\left(\eta\right) &= F[0]+\eta F[1]+\eta^2F[2]+\eta^3F[3]+\eta^4F[4]+\eta^5F[5]+... \\ \theta\left(\eta\right) &= \theta[0]+\eta\theta[1]+\eta^2\theta[2]+\eta^3\theta[3]+\eta^4\theta[4]+\eta^5\theta[5]+... \end{aligned} \quad (23)$$

where a_1 and a_2 are unknown constants which are be determined through the boundary conditions in Eq. (7).

3. Results and Discussion

Tables 2 and 3 show the comparison of the results of numerical methods (NM) and that of DTM. The obtained results using DTM are in very good agreements with the

Table 2

Physical parameter values of $f''(0)$ and $-\theta'(0)$ for Cu-Kerosene nanofluid

Ha	ϕ	We	S	n	A^*	E	γ	***NM*** $f''(0)$	***DTM*** $f''(0)$	***NM*** $-\theta'(0)$	***DTM*** $-\theta'(0)$
1								−0.800673	−0.800673	3.183502	3.183502
2								−0.951051	−0.951050	3.137925	3.137923
3								−1.077238	−1.077238	3.097322	3.097320
	0.1							−0.951051	−0.951050	3.137925	3.137923
	0.2							−0.926769	−0.926768	2.900338	2.900336
	0.3							−0.843920	−0.843921	2.683437	2.683437
		1						−0.865479	−0.865478	3.155764	3.155763
		3						−0.611938	−0.611938	3.218581	3.218581
		5						−0.484571	−0.484571	3.252867	3.252868
			0.2					−1.090240	−1.090238	3.094797	3.094797
			0.4					−1.002314	−1.002312	3.125135	3.125137
			0.6					−0.894041	−0.894040	3.149859	3.149861
				1				−0.995049	−0.995047	3.129665	3.129667
				5				−0.796797	−0.796798	3.171534	3.171534
				10				−0.700307	−0.700307	3.195477	3.195475
					1			−0.951051	−0.951050	3.002623	3.002622
					2			−0.951051	−0.951050	2.833496	2.833495
					3			−0.951051	−0.951050	2.664369	2.664368
						1		−0.951051	−0.951050	2.534955	2.534956
						2		−0.951051	−0.951050	1.864989	1.864989
						3		−0.951051	−0.951050	1.195023	1.195024
							$\pi/6$	−1.077238	−1.077237	3.097322	3.097321
							$\pi/4$	−0.951051	−0.951050	3.137925	3.137926
							$\pi/3$	−0.800673	−0.800673	3.183502	3.183501

results of the numerical method (NM) using Runge-Kutta coupled with Newton method as presented by Kumar and Kumar (2017). The high accuracy of DTM gives high confidence about validity of the method in providing solutions to the problem.

Table 3

Physical parameter values of $f''(0)$ and $-\theta'(0)$ for Ag-Kerosene nanofluid

Ha	ϕ	We	S	n	A^*	E	γ	***NM*** $f''(0)$	***DTM*** $f''(0)$	***NM*** $-\theta'(0)$	***DTM*** $-\theta'(0)$
1								–0.841593	–0.841595	–3.090642	–3.090644
2								–0.987394	–0.987396	–3.045404	–3.045403
3								–1.110328	–1.110327	–3.005010	–3.005011
	0.1							–0.987394	–0.987392	–3.045404	–3.045402
	0.2							–0.982125	–0.981227	–2.739717	–2.739718
	0.3							–0.907088	–0.907089	–2.473053	–2.473053
		1						–0.894212	–0.894211	–3.064925	–3.064924
		3						–0.627126	–0.627125	–3.132014	–3.132012
		5						–0.495591	–0.495590	–3.168031	–3.168030
			0.2					–1.133904	–1.133903	–2.982102	–2.982104
			0.4					–1.041797	–1.041795	–3.026448	–3.026447
			0.6					–0.926340	–0.926341	–3.063119	–3.063118
				1				–1.036497	–1.036498	–3.036220	–3.036221
				5				–0.820912	–0.820911	–3.081939	–3.081940
				10				–0.719234	–0.719235	–3.107549	–3.107547
					1			–0.987394	–0.987392	–2.906601	–2.906603
					2			–0.987394	–0.987392	–2.733098	–2.733096
					3			–0.987394	–0.987392	–2.559594	–2.559592
						1		–0.987395	–0.987394	–2.390294	–2.390293
						2		–0.987395	–0.987394	–1.662395	–1.662396
						3		–0.987395	–0.987394	–0.934497	–0.934497
							$\pi/6$	–1.110328	–1.110327	–3.005010	–3.005011
							$\pi/4$	–0.987394	–0.987395	–3.045404	–3.045406
							$\pi/3$	–0.841593	–0.841591	–3.090642	–3.090641

Also, the tables depict the effect of physical parameters on $f''(0)$ and $-\theta'(0)$ (for the Cu-kerosene and Ag-Kerosene nanofluids) which are the friction factor and local Nusselt number, respectively. As it can be seen from the tables that increasing values of the magnetic field parameter leads to decreasing values of the friction factor and heat transfer rate. An increase in the value of volume fraction of nanoparticles increases the friction factor and decreases the rate of heat transfer. Additionally, it is depicted as the value of aligned angle parameter increases, both friction factor and heat transfer rate increase. The Weissenberg and unsteadiness parameters have tendency to improve or enhance the rate of heat transfer.

The influence of pertinent parameters such as magnetic field parameter, unsteadiness parameter, heat source/sink parameter, Eckert number, volume fraction of nanoparticles etc. on the flow and heat transfer of the thin film flow are investigated. Figs. 2 and 3 show the effects of magnetic field (Ha) on the velocity and temperature fields, respectively.

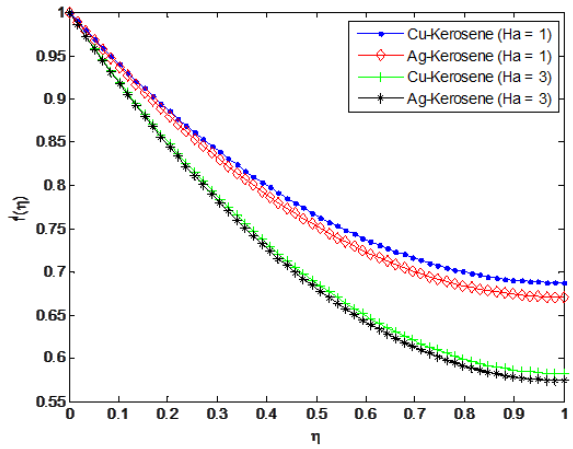

Fig. 2. Effect of Magnetic field parameter (Hartmann number) on the fluid velocity distribution

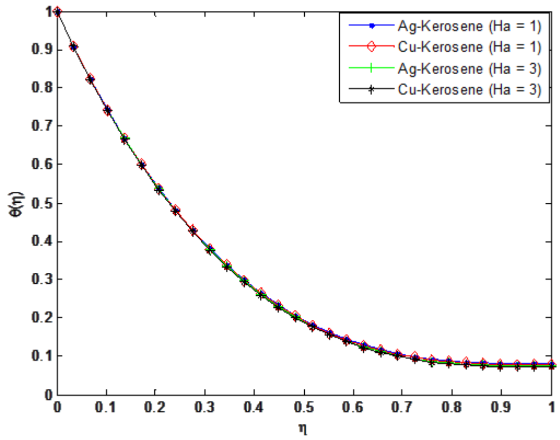

Fig. 3. Effect of Magnetic field parameter (Hartmann number) on the fluid temperature distribution

It is revealed that as the magnetic field increases, the velocity field decreases while the temperature field increases. The velocity distribution decreases with increase in magnetic field parameter due to the Lorentz force as a result of the presence of magnetic field which slows fluid motion at boundary layer and hence retards the velocity field.

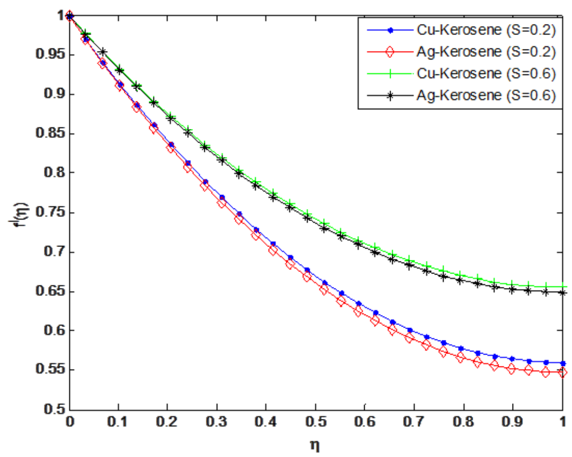

Fig. 4. Effect of unsteadiness parameter on the fluid velocity distribution

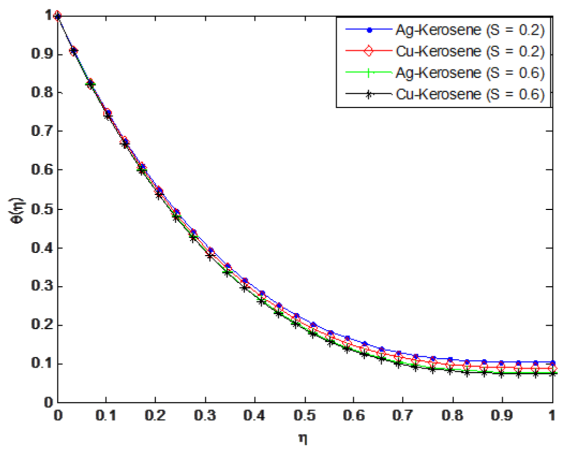

Fig. 5. Effect of unsteadiness parameter on the fluid temperature distribution

The effects of unsteadiness parameter on velocity and temperature profiles are shown in Figs. 4 and 5, respectively. It is observed that increasing values of S increases the velocity field while decreases the temperature field. This is because as the rate of heat loss by the thin film increases as the value of unsteadiness parameter increases. Figs. 6 and 7 depict the effects of Weissenberg number (We) on the velocity and temperature profiles. It is shown from the figures that the velocity increases for increasing values of We and opposite trend was observed in temperature field. The observed trends in the velocity and temperature fields are due to the fact that a higher value of *We* will reduce the viscosity forces of the Carreau fluid. The influence of aligned angle on velocity and temperature profiles is presented in Figs. 8 and 9. From the figures, it is shown that as the value of aligned parameter increases, the velocity field increases while temperature field decreases.

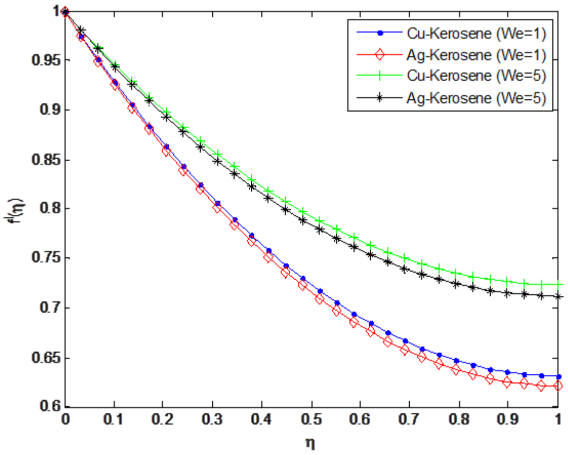

Fig. 6. Effect of Weissenberg number on the fluid velocity distribution

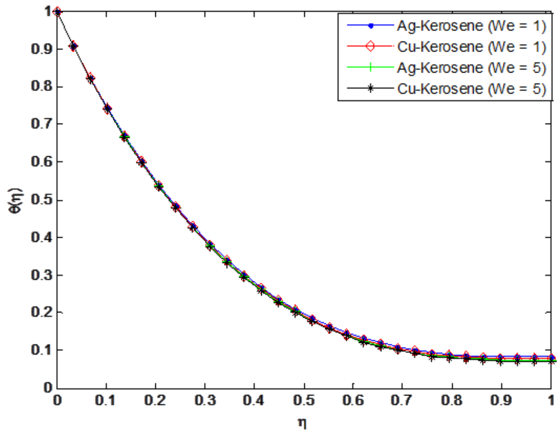

Fig. 7. Effect of Weissenberg number on the fluid temperature distribution

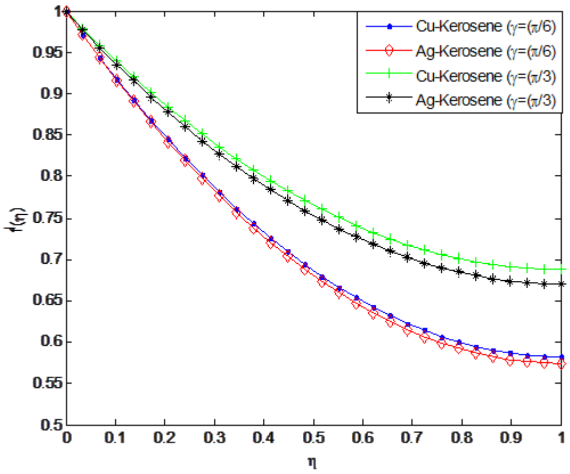

Fig. 8. Effect of aligned angle on the fluid velocity distribution

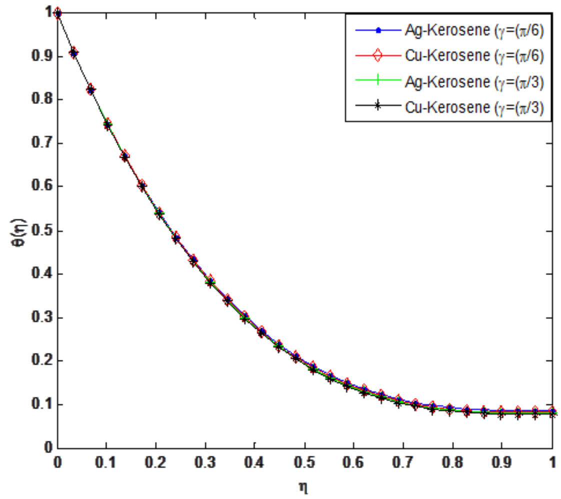

Fig. 9. Effect of aligned angle on the fluid temperature distribution

Figs. 10 and 11 demonstrated the effect of power law index on velocity and temperature fields. As the power index is increased, it was observed that the velocity profile increases while the temperature profile decreases. This is because, increasing value of the power law index, thickens the liquid film associated with an increase of the thermal boundary layer. The effects of nanoparticles volume fraction on the velocity and temperature profiles are depicted in Figs. 12 and 13, respectively. The result shows that as the solid volume fraction of the film increases both the velocity and temperature field increases. This is because as the nanoparticle volume increases, more collision occurs between nanoparticles and particles with the boundary surface of the plate and consequently the resulting friction enhances the thermal conductivity of the flow and gives rise to increase the temperature within the fluid near the boundary region.

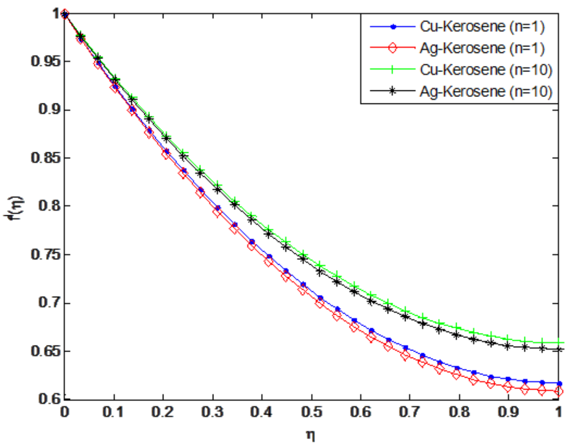

Fig. 10. Effect of power-law index on the fluid velocity distribution

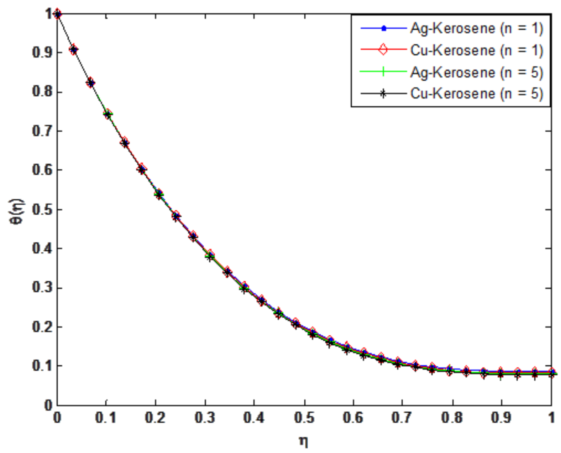

Fig. 11. Effect of power-law index on the fluid temperature distribution

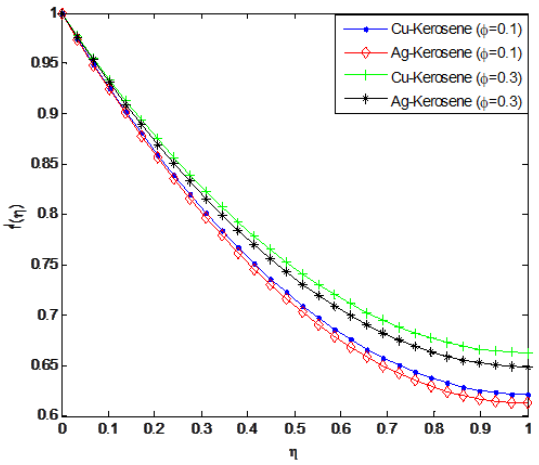

Fig. 12. Effect of nanoparticle volume fractions on the fluid velocity distribution

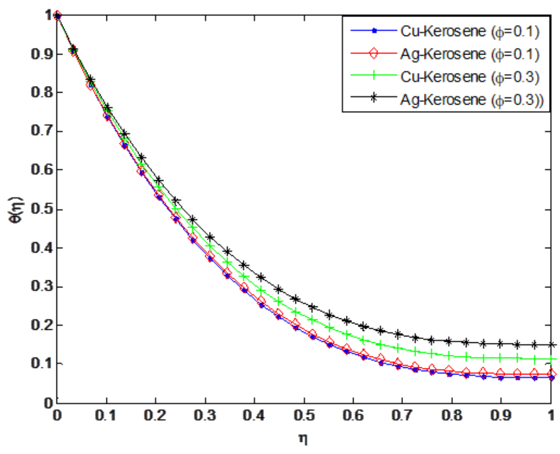

Fig. 13. Effect of nanoparticle volume fractions on the fluid temperature distribution

Fig. 14 and 15 depict the influence of non-uniform heat source/sink parameter on the temperature field. It is revealed that increasing the nonuniform heat source/sink parameter enhances the temperature fields. The effect of Eckert number on temperature profile is shown in Fig. 16. It was established that as the values of Eckert number increases, the values of the temperature distributions in the fluid increases. This is because as Ec increases, heat energy is saved in the liquid due to the frictional heating.

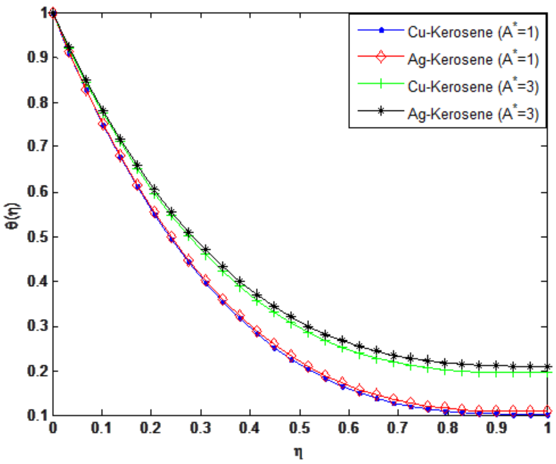

Fig. 14. Effect of non-uniform heat source/sink parameter (A^*) on the fluid temperature distribution

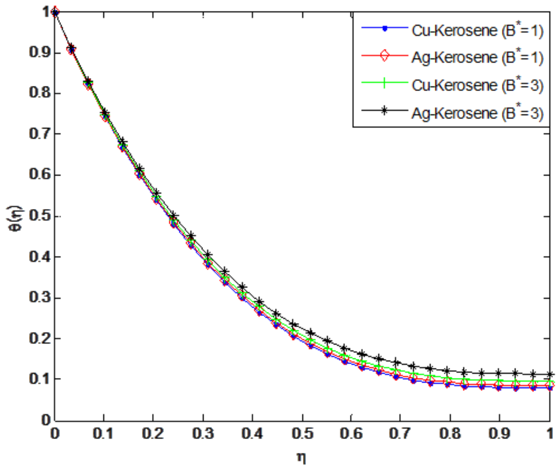

Fig. 15. Effect of non-uniform heat source/sink parameter (B^*) on the fluid temperature distribution

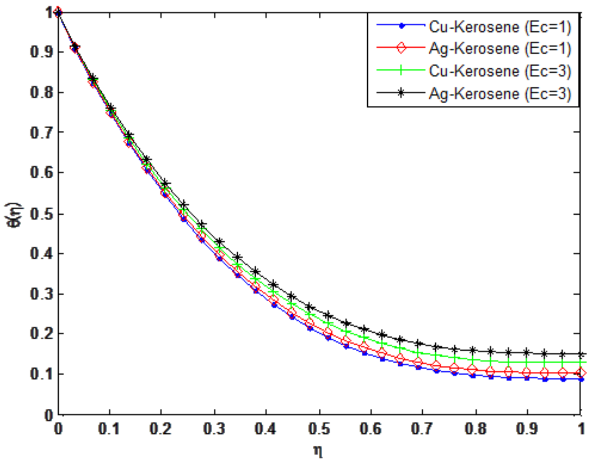

Fig. 16. Effect of Eckert number on the fluid temperature distribution

Conclusion

In this paper, the flow and heat transfer analysis of an electrically conducting liquid film flow of of Carreau nanofluid over a stretching sheet subjected to magnetic field, temperature dependent heat source/sink and viscous dissipation have been analyzed using differential transform method. The approximate analytical solutions were verified numerically using Runge-Kutta coupled with Newton method. Using kerosene as the base fluid embedded with the silver (Ag) and copper (Cu) nanoparticles, the effects of pertinent parameters on reduced Nusselt number, flow and heat transfer characteristics of the nanofluid were investigated and discussed. From the results, it was established temperature field and the thermal boundary layers of Ag-kerosene nanofluid are highly effective when compared with the Cu-kerosene nanofluid. Thermal and momentum boundary layers of

Cu-kerosene and Ag-kerosene nanofluids are not uniform. Heat transfer rate is enhanced by increasing in power-law index and unsteadiness parameter. Skin friction coefficient and local Nusselt number can be reduced by magnetic field parameter and they can be enhanced by increasing in aligned angle. Friction factor is depreciated and the rate of heat transfer increases by increasing the Weissenberg number. This analysis can help in expanding the understanding of the thermo-fluidic behaviour of the Carreau nanofluid over a stretching sheet.

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ТЕПЛОПЕРЕДАЧА И АНАЛИЗ ПОТОКА МАГНИТОГИДРОДИНАМИЧЕСКОЙ ДИССИПАТИВНОЙ НАНОЖИДКОСТИ КАРРО НА РАСТЯЖИМОМ ЛИСТЕ С ВНУТРЕННИМ ТЕПЛОВЫДЕЛЕНИЕМ

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В работе представлен нестационарный двумерный анализ потока и теплопередачи наножидкости Карро над растягивающимся листом, подвергнутым воздействию магнитного поля, температурно-зависимого источника / поглотителя тепла и вязкой диссипации. Для того, чтобы привести системы разработанных управляющих уравнений в частных производных к нелинейным обыкновенным дифференциальным уравнениям третьего и второго порядков, которые решаются методом дифференциального преобразования, применяются преобразования подобия. Используя керосин в качестве базовой жидкости, внедренной в наночастицы серебра (Ag) и меди (Cu), мы исследуем и обсуждаем влияния соответствующих параметров на уменьшенное число Нуссельта, характеристики потока и теплопередачи наножидкости. Согласно полученным результатам, температурное поле и тепловые пограничные слои наножидкости Ag-керосина являются высокоэффективными по сравнению с наножидкостью Cu-керосина. Скорость теплопередачи повышается за счет увеличения показателя степени и параметра неустойчивости. Коэффициент трения кожи и локальное число Нуссельта могут быть понижены параметром магнитного поля и повышены путем увеличения выровненного угла. С увеличением числа Вейссенберга коэффициент трения понижается, а скорость теплопередачи – повышается. Для того, чтобы проверить эти утверждения, результаты анализов приближенных аналитических решений сравниваются с результатами численного решения с использованием метода Рунге-Кутты в сочетании с методом Ньютона. Установлено, что полученные результаты хорошо согласуются. Проведенный анализ может помочь в расширении понимания терможидкостного поведения наножидкости Карро на растягивающемся листе.

Ключевые слова: MHD; наножидкость; неоднородный источник / поглотитель тепла; жидкость Карро; тепловое излучение и свободная конвекция.

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