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COMPUTATION OF THE KAUFFMAN BRACKET SKELETON

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In this paper, we present an invariant called the Kauffman bracket skeleton, which is a simplification of the generalized Kauffman bracket polynomial in two variables. The idea is to take into account only the order and values of coefficients and disregard the degrees of one of the variables. However, the proposed simplification is more compact, and at the same time is not weaker than the original generalized Kauffman bracket polynomial in the sense of, for example, tabulation of prime knots and links in the thickened torus up to complexity 4 inclusively. In order to confirm this fact, we present tables of the proposed invariant for the tabulated prime knots and links in the thickened torus. Also, we construct an algorithm to compute the proposed simplification. The algorithm does not require to work with degrees of the disregarded variable and uses a symmetry of the Kauffman bracket presented by rows of the alternating Pascal's triangle. Finally, we give a remark on an interpretation of the proposed invariant and algorithm in the case of classical knots and links.

Keywords: knot; link; thickened torus; Kauffman polynomial; Kauffman bracket skeleton; Pascal's triangle.

Introduction

One of the main problems of the knot theory is to distinguish the objects under study. This approach involves the problem to construct and compute knot invariants and see if some of them are helpful in the considered particular situation. For example, the generalized Kauffman bracket polynomial turned out to be enough to distinguish all prime knots [1] and links [2] in the thickened torus having diagrams with at most 4 crossings.

In this paper, we present an invariant called the Kauffman bracket skeleton, which is a simplification of the generalized Kauffman bracket polynomial in two (a and x) variables. The idea is to take into account only the order and values of coefficients and disregard the degrees of the variable a. However, the proposed simplification is more compact, and at the same time is not weaker than the original generalized Kauffman bracket polynomial in the sense of, for example, tabulation of prime knots and links in the thickened torus up to complexity 4 inclusively. Also, we construct an algorithm to compute the proposed simplification in an easier way than the original generalized Kauffman bracket polynomial. Indeed, the algorithm does not require to work with degrees of the variable a and uses a symmetry of the Kauffman bracket presented by rows of the alternating Pascal's triangle.

The paper is organized as follows. Section 1 gives the necessary definitions and proves that the Kauffman bracket skeleton is invariant. The proposed algorithm to compute the Kauffman bracket skeleton is constructed in Section 2. A computational example is presented in Section 3, where we also give a remark on an interpretation of the proposed invariant and algorithm in the case of classical knots and links. In addition, in order to show that the proposed simplification is not weaker than the original generalized Kauffman bracket polynomial in the sense of tabulation of prime knots and links in the thickened torus up to complexity 4 inclusively, we present tables of the proposed invariant for the tabulated prime knots and links in the thickened torus.

1. Definitions

1.1. Knot and Link in the Thickened Torus

Consider a two-dimensional torus $T = S^1 \times S^1$ and an interval $I = [0, 1]$. By *a thickened torus* we mean a 3-dimensional manifold homeomorphic to the direct product $T \times I$.

A smooth embedding of m circles in the $Int(T \times I)$ is called a *link in* $T \times I$ having m components and denoted by $L \subset T \times I$. An one-component link in the thickened torus is called a *knot in* $T \times I$ and denoted by $K \subset T \times I$.

As in the classical case, knots and links in the thickened torus can be given by their diagrams. *A diagram* $D \subset T$ *of a knot* $K \subset T \times I$ *or link* $L \subset T \times I$ is defined by analogy with the diagram of the classical knot except that the knot (or link) is projected into the torus T instead of the plane. We represent the torus T as a square with identified opposite sides. An example of a knot diagram on torus is given in Fig. 5 on the left.

1.2. Generalized Kauffman Bracket Polynomial

Let us recall the definition of the generalized Kauffman bracket polynomial [1]. In contrast to the usual Kauffman bracket polynomial of classical knots [4] (see also [3] for the original version called the Jones polynomial), the generalized version [1] takes into account types of curves on the torus (trivial, i.e. bounded a 2-disk, and nontrivial). The papers [5] and [6] give other generalizations of the Kauffman bracket polynomial.

Let D be a diagram of a knot or link in the thickened torus. Endow each angle of each crossing of D with a marker A or B according to the rule given in the center of Fig. 1. Each *state* s of the diagram D is defined by a combination of ways to smooth each crossing of D such as to join together either two angles endowed with a marker A, or two angles endowed with a marker B, see Fig. 1 on the left and right, respectively. Obviously, if the diagram D has n crossings, then there exist exactly 2^n states of D.

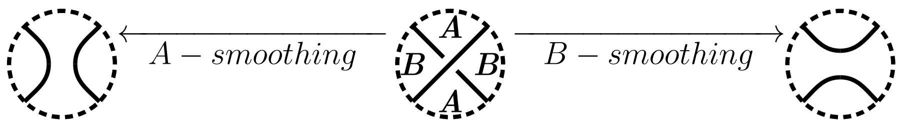

Fig. 1. A- and B-smoothings of a crossing

By the writhe of an oriented knot diagram D with n crossings we mean the sum over all crossings of D

$$w(D) = \sum_{i=1}^{n} \varepsilon(i),$$

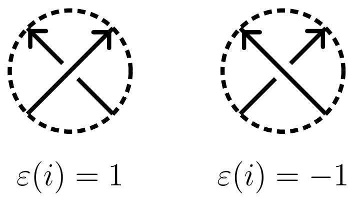

Fig. 2. Rules to define the sign $\varepsilon(i)$ of the i-th crossing

where $\varepsilon(i)$ is a sign of the i-th crossing of D defined by the rules given in Fig. 2. Note that the writhe of an oriented link diagram is the sum of signs of only those crossings of D that are self-intersections of the components.

The exact formula of the generalized Kauffman bracket polynomial [1] is as follows:

$$\mathcal{X}\,(a,x)_D = (-a)^{-3w(D)}\,\langle a,x\rangle_D\,, \tag{1}$$

where

$$\langle a,x\rangle_D = \sum_s a^{\alpha(s)-\beta(s)}(-a^2-a^{-2})^{\gamma(s)}x^{\delta(s)} \tag{2}$$

is the generalized Kauffman bracket [1]. Here $\alpha(s)$ and $\beta(s)$ are the numbers of markers A and B in the given state s, while $\gamma(s)$, $\delta(s)$ are the numbers of trivial and nontrivial curves in the torus obtained by smoothing of all crossings according to the state s, and $w(D)$ is the writhe of D. The sum is taken over all 2^n states of D.

Recall that the generalized Kauffman bracket polynomial $\mathcal{X}\,(a,x)_D$ is invariant under

1) isotopy (i.e., under all three Reidemeister moves $\Omega_1 - \Omega_3$ given in Fig. 3),
2) orientation preserving automorphisms of the torus,
3) orientation reversing automorphism of the torus (up to change of variables $a^n \to a^{-n}$),
4) simultaneous switching of all crossings (up to change of variables $a^n \to a^{-n}$).

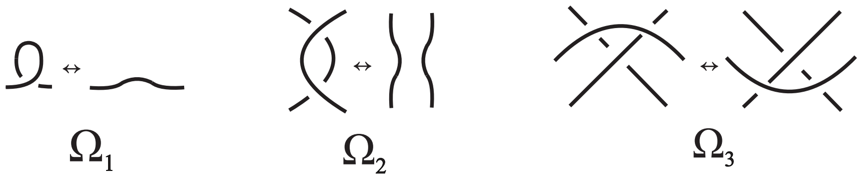

Fig. 3. Reidemeister moves $\Omega_1 - \Omega_3$

In most applications (for example, in tabulation), the orientation of knots and links is not of interest. Therefore, above we define the generalized Kauffman bracket polynomial (1) to be an invariant of nonoriented knots and links, i.e. an orientation is used as an auxiliary tool only. Indeed, the generalized Kauffman bracket $\langle a,x\rangle_D$ is invariant under the same transformations (1) – (4) as the generalized Kauffman bracket polynomial $\mathcal{X}\,(a,x)_D$ with the exclusion of the first Reidemeister move Ω_1. The invariance under Ω_1 is achieved by the factor $(-a)^{-3w(D)}$ taking into account the writhe $w(D)$ determined with the use of the orientation.

1.3. Kauffman Bracket Skeleton

Let us define the Kauffman bracket skeleton as a simplification of the generalized Kauffman bracket polynomial $\mathcal{X}(a,x)_D$. Obviously, $\mathcal{X}(a,x)_D$ can be represented in the form

$$\mathcal{X}(a,x)_D = \sum_m P_m(a)x^m,$$

where $P_m(a) = \sum_j b_{jm}a^j$ is a Laurent polynomial.

Let t_m be a tuple composed of nonzero coefficients b_{jm} of the polynomial $P_m(a)$, which are ordered in increasing order of j. Note that transition from $P_m(a)$ to t_m erases information on degrees of the variable a and remains information on order and values of the coefficients b_{jm}. For example, Laurent polynomials

$$P_{m_1}(a) = a^{-10} + 5a^{10} \text{ and } P_{m_2}(a) = 1 + 5a^2$$

correspond to the same tuple $t_{m_1} = t_{m_2} = (1,5)$.

The formal sum

$$S_D = \sum_{t_m \neq \emptyset} t_m x^m \tag{3}$$

is called the *Kauffman bracket skeleton*, and tuples t_m are called *coefficients of the Kauffman bracket skeleton*.

We say that the Kauffman bracket skeletons S_{D_1} and S_{D_2} are inverted to each other, if the coefficients of S_{D_1} are the corresponding coefficients of S_{D_2}, where numbers of each t_m are taken in reverse order. This transformation of the Kauffman bracket skeletons is called *inversion*. For example, $S_{D_1} = (1,-2)x + (3,-4,2)x^3$ is inverted to $S_{D_2} = (-2,1)x + (2,-4,3)x^3$.

Lemma 1. *Let diagrams D_1 and D_2 be equivalent. Then*

1. *$S_{D_1} = S_{D_2}$, if a finite sequence, which transforms D_1 to D_2, contains*
   (a) *any number of Ω_2, Ω_3 and orientation preserving automorphisms of the torus,*
   (b) *even number of Ω_1,*
   (c) *even number of simultaneous switching of all crossings and orientation reversing automorphisms of the torus;*
2. *$S_{D_1} = -S_{D_2}$, if only condition (b) of item (1) is unfulfilled;*
3. *S_{D_1} and S_{D_2} are inverted to each other, if only condition (c) of item (1) is unfulfilled;*
4. *$S_{D_1} = -S_{D_2}$, where S_{D_1} and S_{D_2} are inverted to each other, if both conditions (b) and (c) of item (1) are unfulfilled.*

Proof. The Kauffman bracket skeleton is an invariant under Ω_2, Ω_3 and orientation preserving automorphisms of the torus. Indeed, the Kauffman bracket skeleton is completely defined by the Kauffman bracket, which is invariant under these transformations.

Each Ω_1 leads to multiplication of the Kauffman bracket by $(-a)^{\pm 3}$. Therefore, the Kauffman bracket skeleton is multiplied by -1.

Each simultaneous switching of all crossings leads to change of variables $a^n \to a^{-n}$ in the Kauffman bracket, while degrees of x remains the same. Therefore, the Kauffman bracket skeleton is replaced by the inverted one. The same is true for orientation reversing automorphism of the torus.

□

Corollary 1. *The Kauffman bracket skeleton considered up to inversion and multiplication by -1 is an invariant of knots and links in the thickened torus.*

1.4. Alternating Pascal's Triangle

Recall that Pascal's Triangle is a triangle array of integer numbers constructed as follows.

In the 0-th row (i.e., the topmost row), there is the unique nonzero entry, which is equal to 1 and is situated on the axis of symmetry of Pascal's triangle, while all others entries of the 0-th row are equal to 0.

Let $i \in \{1, 2, ...\}$ be a number of a row, $j \in \{0, 1, 2, ...\}$ be an order number (from left to right) of a nonzero entry in a row, and $e_{i,j}$ be a nonzero entry having the j-th order number in the i-th row. Each entry $e_{i,j}$ is computed as a sum of two entries of the $(i-1)$-th row, which are situated on the left and right of the desired entry $e_{i,j}$:

$$e_{i,j} = e_{i-1,j-1} + e_{i-1,j+1},$$

under the assumption that empty entries are equal to 0. In particular, the first and the last number of each row is equal to 1, which is obtained as a sum of 0 and 1. As an example, the first 5 rows of Pascal's triangle are given in Fig. 4(a).

One of possible interpretations of Pascal's triangle states that Pascal's triangle determines binomial coefficients which arise in binomial expansion. Indeed, if a binomial of the form $(x + y)$ is raised to a positive integer power i, then the binomial coefficients $e_{i,j}$ of the expansion

$$(x + y)^i = \sum_{j=0}^{i} e_{i,j} \cdot x^{i-j} \cdot y^j$$

coincide with the entries $e_{i,j}$ of the i-th row of Pascal's triangle taking in the same order.

In this paper, we use an alternating version of Pascal's triangle that corresponds to the case when both x and y are negative. Namely, according to (2), we consider

$$(x + y)^i = (-a^2 - a^{-2})^{\gamma(s)}.$$

In this case, in contrast to Pascal's triangle, each entire of the i-th row is endowed with a sign obtained by the rule $(-1)^i$, where $i \in \{0, 1, 2, ...\}$ is the number of the row. As an example, the first 5 rows of the alternating Pascal's triangle are given in Fig. 4(b).

	(a)									(b)								
$i=0$					1									1				
$i=1$				1		1							-1		-1			
$i=2$			1		2		1					1		2		1		
$i=3$		1		3		3		1			-1		-3		-3		-1	
$i=4$	1		4		6		4		1	1		4		6		4		1

Fig. 4. The first 5 rows of Pascal's triangle (a) and the alternating Pascal's triangle (b)

2. Algorithm to Compute the Kauffman Bracket Skeleton

Let D be a diagram of a knot or link in the thickened torus having n crossings. In order to compute the Kauffman bracket skeleton S_D without computation of the generalized Kauffman bracket polynomial $\mathcal{X}\left(a,x\right)_D$, we propose the following algorithm. Note that an algorithm to compute the generalized Kauffman bracket polynomial $\mathcal{X}\left(a,x\right)_D$ can be presented by STEPS 1–4 and additional steps on computation of the writhe and sum (1).

STEP 1. Enumerate crossings of D.

STEP 2. Endow each angle of each crossing of D with a marker A or B according to the rule given in the center of Fig. 1.

STEP 3. Determine all 2^n possible states of the diagram D, where each state is defined by a combination of ways to smooth each crossing of D such as to join together either two angles endowed with a marker A, or two angles endowed with a marker B, see Fig. 1 on the left and right, respectively.

STEP 4. For each state s, determine the numbers $\alpha(s)$, $\gamma(s)$ and $\delta(s)$ of markers A in the code of the state s, trivial and nontrivial curves in the torus obtained by smoothing of all crossings of the diagram D according to the state s, respectively.

STEP 5. Let $\{F_\ell(x)\}$ be an infinite sequence of the polynomials $F_\ell(x)$, where ℓ is an ordered number of the polynomial in the sequence $\{F_\ell(x)\}$ and x is a variable. At present, consider all the polynomials $F_\ell(x)$ to be zero. Further, for each state s, set $k=\alpha(s)$, i.e. $k\in\{0,1,2,...,n\}$, and redetermine the polynomials $F_\ell(x)$ as follows.

1. If $\gamma(s)=0$, then add the term $x^{\delta(s)}$ to the polynomial $F_k(x)$.

2. If $\gamma(s)=1$, then add the term $-x^{\delta(s)}$ to the polynomials $F_{k-1}(x)$ and $F_{k+1}(x)$.

3. If $\gamma(s)=2$, then add the term $x^{\delta(s)}$ to the polynomials $F_{k-2}(x)$ and $F_{k+2}(x)$, and add the term $2\cdot x^{\delta(s)}$ to the polynomial $F_k(x)$.

4. If $\gamma(s)=3$, then add the term $-1\cdot x^{\delta(s)}$ to the polynomials $F_{k-3}(x)$ and $F_{k+3}(x)$, and add the term $-3\cdot x^{\delta(s)}$ to the polynomials $F_{k-1}(x)$ and $F_{k+1}(x)$.

5. ...

6. If $\gamma(s)=i$, then add the terms of the form $e_{i,j}\cdot x^{\delta(s)}$ to the polynomials $F_\ell(x)$, where $e_{i,j}$ are the entries of the i-th row of the alternating Pascal's triangle and $\ell\in\{k-i,k-i+2,...,k+i-2,k-i\}$, i.e. ℓ takes all integer values in the interval $[k-i;k+i]$ such that $\ell \ mod \ 2=(k-i) \ mod \ 2$.

7. ...

STEP 6. Let p be the maximal degree of the polynomials in the sequence $\{F_\ell(x)\}$. For each $m \in \{0, 1, 2, ..., p\}$, determine the tuple t_m in formal sum (3) as the sequence of nonzero coefficients of x^m in the polynomials $F_\ell(x)$ taking in increasing order of numbers ℓ of nonzero polynomials $F_\ell(x)$.

STEP 7. Obtain the Kauffman bracket skeleton S_D by substituting the tuples t_m into formal sum (3).

3. Computational Example

We use the algorithm proposed in the previous section in order to compute the Kauffman bracket skeleton S_{3_2} for the knot diagram 3_2 given in Fig. 5 on the left. The results of STEPS 1 – 3 are given in Fig. 5, while the results of STEPS 4 – 5 are given in Tables 1 and 2.

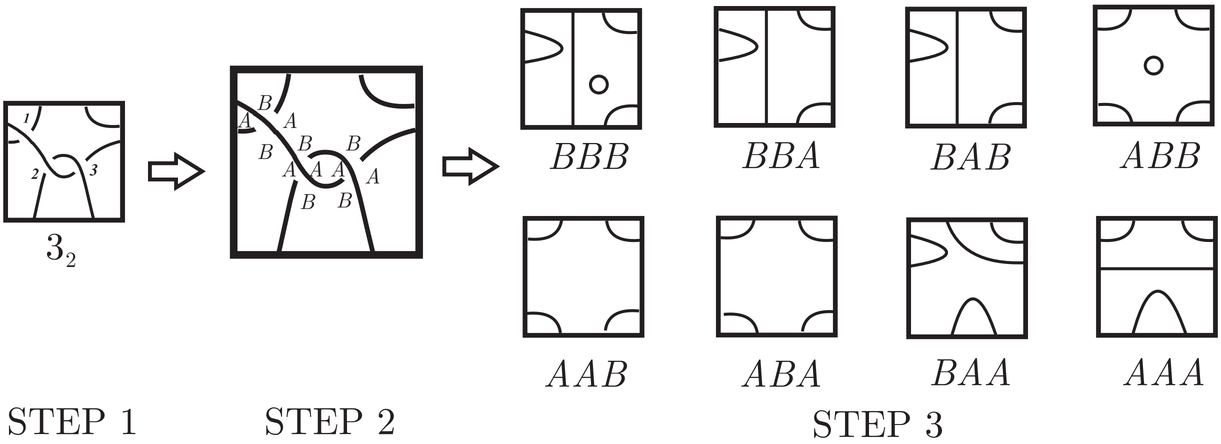

Fig. 5. Results of STEPS 1 – 3 of the algorithm for the knot diagram 3_2

Table 1

Results of STEPS 4 – 5 of the algorithm for the knot diagram 3_2

Code	**Numbers for state**			**The terms added to**				
	$k = \alpha(s)$	$i = \gamma(s)$	$\delta(s)$	$F_{-1}(x)$	$F_0(x)$	$F_1(x)$	$F_2(x)$	$F_3(x)$
BBB	0	1	2	$-x^2$	–	$-x^2$	–	–
BBA	1	0	2	–	–	x^2	–	–
BAB	1	0	2	–	–	x^2	–	–
ABB	1	2	0	1	–	2	–	1
AAB	2	1	0	–	–	-1	–	-1
ABA	2	1	0	–	–	-1	–	-1
BAA	2	1	0	–	–	-1	–	-1
AAA	3	0	2	–	–	–	–	x^2

Therefore, we obtain the values of nonzero polynomials in the sequence $\{F_\ell(x)\}$ given in Table 2.

Table 2
Values of nonzero polynomials
in the sequence $\{F_\ell(x)\}$
for the knot diagram 3_2

x^m	$F_{-1}(x)$	$F_1(x)$	$F_3(x)$
1	1	-1	-2
x	–	–	–
x^2	-1	1	1

At STEP 6, according to Table 2, we have $p = 2$, therefore, it is necessary to determine the tuples t_0, t_1, and t_2 in formal sum (3). Rows of Table 2 immediately give that

$$t_0 = (1, -1, -2), \quad t_1 = 0, \quad t_2 = (-1, 1, 1).$$

At STEP 7, we obtain that the Kauffman bracket skeleton S_{3_2} for the knot diagram 3_2 is as follows:

$$(1, -1, -2) + (-1, 1, 1)x^2,$$

which is considered up to inversion and multiplication by -1 (see Corollary 1) and, therefore, is in agreement with the corresponding generalized Kauffman bracket polynomial [1]

$$\mathcal{X}\,(a, x)_{3_2} = 2a^{-6} + a^{-10} - a^{-14} + (-a^{-6} - a^{-10} + a^{-14})x^2.$$

It turned out that the proposed simplification is not weaker than the original generalized Kauffman bracket polynomial in the sense of tabulation of prime knots and links in the thickened torus up to complexity 4. Namely, the Kauffman bracket skeleton, as well as the original generalized Kauffman bracket polynomial, turned out to be enough to distinguish all 21 prime knots [1] and 26 prime links [2] in the thickened torus having diagrams with at most 4 crossings, see Tables 3 and 4. Note the pair of equivalent links (4_{10} and 4_{11}) given in the paper [2]. In order to transform 4_{10} to 4_{11}, it is enough to slide the horizontal component in the diagram of 4_{10} out the top of the square in the bottom, then rotate the square by π, and switch all the crossings, see Fig. 6. Obviously, the Kauffman bracket skeletons of these links are inverted (i.e., equivalent) to each other, see Table 4.

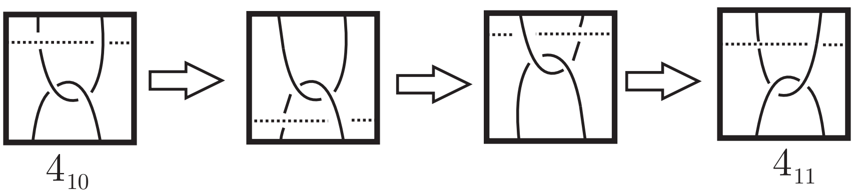

Fig. 6. Pair of equivalent links (4_{10} and 4_{11}) given in the paper [2]

Table 3

The Kauffman bracket skeletons of prime knots in the thickened torus given in [1]

2_1: $(1,1,-1)x$	4_7: $(-1,-1,1,1,1)x$
3_1: $(1,1,-1,-1,1)x$	4_8: $(2,-1,1,-1)x$
3_2: $(2,1,-1)+(-1,-1,1)x^2$	4_9: $(-1,-1,1,2,1,-1)x$
3_3: $(1,2-1)+(-2,1)x^2$	4_{10}: $(2,-1,1)+(-1,1,-1)x^2$
4_1: $(1,1,-1,1,-1)x$	4_{11}: $(1,1,-1,1)+(-2,2,-1)x^2$
4_2: $(1,-1,2,1,-1,-1)x$	4_{12}: $(1,-2,2)x+(1,-1)x^3$
4_3: $(1,1,-1,-1,1,1,-1)x$	4_{13}: $(-1,1,1)x+(1,-1)x^3$
4_4: $(-1,1,1,-2,-1)+(-2,2)x^2$	4_{14}: $(1,-1,-1,1,1)x+(1,-1)x^3$
4_5: $(1,-2-2,1)+(-1,1,1,-1)x^2$	4_{15}: $(-1,-1,1,-1,-1)x+x^3$
4_6: $(1,-2,-1,1,2)x$	4_{16}: $(-1,-1,1,-1,-2,1)x+x^3$
	4_{17}: $(-3,1,1,-1,-1)x+x^3$

Table 4

The Kauffman bracket skeletons of prime links in the thickened torus given in [2]

2_1: $(-1,-2,-1)+2x^2$	4_9: $(-1,1,-3,-1)+(-1,3)x^2$
3_1: $(-1,1,-1,-1)x$	4_{10}: $(-1,-2,-1,2,1,-1)x$
3_2: $(1,-1,-2)x$	4_{11}: $(-1,1,2,-1,-2,-1)x$
3_3: $(1,1,-1,1)x-x^3$	4_{12}: $(-2,-1,2,1)+(1,2,-2)x^2$
3_4: $(-1,1,3,1)-3x^2$	4_{13}: $(1,1,-2,-1,1)+(-2,3)x^2$
4_1: $(-1,-2,-1)+(1,-1,2)x^2$	4_{14}: $(1,-1,-4,-1,1)+(-1,4,-1)x^2$
4_2: $(-1,-1,-2)+(1,-1,1,1)x^2$	4_{15}: $(1,-1)+(-1,-2)x^2+x^4$
4_3: $(-1,-1,2,-1,-1)x$	4_{16}: $(-1,-1,-1)x^2+x^4$
4_4: $(1,-2,-2,1,1,-1)x$	4_{17}: $(1,-2,1)+(-2,1,-2)x^2+x^4$
4_5: $(1,-2,1)+(-1,3,-1)x^2$	4_{18}: $(1,-1,2,2)x$
4_6: $(2,-2)+(1,-2,1,1)x^2$	4_{19}: $(1,-2,2,-1)x+x^3$
4_7: $(1,-2,-1,1,1,-1,-1)x$	4_{20}: $(-1,-1,2)x+x^3$
4_8: $(1,-2,-3)+(-1,1,2)x^2$	4_{21}: $(-1,1,4,1,-1)x$
	4_{22}: $(1,-2,-6,-2,1)+6x^2$

Remark. In this paper, we construct the Kauffman bracket skeleton in the case of knots and links in the thickened torus. The proposed invariant can be easily interpreted for the case of classical knots and links. It is enough to set

$$\gamma(s)=\gamma(s)-1 \text{ and } \delta(s)\equiv 0$$

in all the formulas and steps of the algorithm and apply obvious simplifications. In this case, a value of the invariant is an unique tuple t_0.

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ВЫЧИСЛЕНИЕ СКЕЛЕТА СКОБКИ КАУФМАНА

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В этой статье мы строим инвариант, называемый скелетом скобки Кауфмана, который является упрощением обобщенного полинома скобки Кауфмана от двух переменных. Идея состоит в том, чтобы учитывать только порядок и значения коэффициентов и игнорировать степени одной из переменных. Предлагаемое упрощение более компактно и, в то же время, не слабее, чем оригинальный обобщенный полином скобки Кауфмана в смысле, например, табулирования примарных узлов и зацеплений в утолщенном торе до сложности 4 включительно. Чтобы подтвердить этот факт, мы приводим таблицы предложенного инварианта для табличных примарных узлов и зацеплений в утолщенном торе. В статье построен алгоритм для вычисления предложенного упрощения. Алгоритм не требует работы со степенями игнорируемой переменной и использует симметрию скобки Кауфмана, представленную строками альтернированного треугольника Паскаля. Наконец, мы приводим замечание об интерпретации предложенного инварианта и алгоритма в случае классических узлов и зацеплений.

Ключевые слова: узел; зацепление; утолщенный тор; полином Кауфмана; скелет скобки Кауфмана; треугольник Паскаля

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