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THE OPTIMAL MEASUREMENTS THEORY AS A NEW PARADIGM IN THE METROLOGY

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The article is an overview and contains a brief history of the theory of optimal dynamic measurements as one of the paradigms in Metrology. The introduction contains the main provisions of the paradigmatic concept of T. Kuhn and its criticism by P. Feyerabend from anarchist point of view. The conclusion about the coexistence of conflicting paradigms within the same science is made. In the first part, a mathematical model of measuring transducer is described and the conditions for the existence of a unique precise optimal dynamic measurement are given. In the second part, various approximate optimal measurements are proposed and the conditions for convergence of the sequence of approximate dynamic measurements to the precise optimal measurement are specified. The third part contains an approach to the study of a stochastic mathematical model of a measuring transducer based on the Nelson – Gliklikh derivative of the stochastic process. In the conclusion, the ways of further possible research are outlined. The list of publications contains all available sources related to the issue.

Keywords: deterministic mathematical model of measurement transducer; stochastic mathematical model of measurement transducer; precise optimal dynamic measurement; approximate optimal measurement; degenerate flow; stochastic optimal measurement; Nelson – Gliklikh derivative; Wiener process; "white noise".

Acronyms and Abbreviations

CSOM – computer simulation of optimal measurements
MBOM – mathematical basis of optimal measurements
MM – mathematical model
MT – measuring transducer
NMOM – numerical methods of optimal measurements
OM – optimal measurement
OMP – optimal measurements paradigm
SMM – stochastic mathematical model

Introduction

The term "paradigm" was introduced into scientific usage by T. Kuhn. In his fundamental treatise [26] the paradigm is understood as a set of theoretical and methodological prerequisites that determine scientific research at the historical stage. The paradigm is the basis for choosing problems, as well as a model for solving research problems. The paradigm allows us to solve problems that arise in research work, to fix changes in the data structure that occur as a result of the *scientific revolution.* For example, the Ptolemy geocentric model of the Universe that existed for more than one and a half thousand years was eventually replaced by the Copernicus heliocentric model of the Universe. The scientific revolution that took place changed not only the methods of calculating the movement of planets in their orbits, but also posed a number of new problems, the study of which led to a radical break in the scientific worldview.

Thus, the development of science according to T. Kun occurs as a uniform progressive movement only within one of the paradigms. Paradigms change each other during scientific revolutions, as a result of which the entire building of science is rebuilt from the foundation to the spire on the roof. P. Feyerabend [12] made a reasonable criticism of this concept. He admits simultaneous existence of several, perhaps even mutually exclusive, paradigms in science. For example, Newton's mechanics was not cancelled after the emergence of general theory of relativity or quantum mechanics. It just turns out that the theory of relativity is more accurate at describing the movement of objects at high speeds than Newtonian mechanics, and quantum mechanics is more accurate at describing the interaction of very small objects. All three theories are based on mutually exclusive principles. Therefore, P. Feyerabend requires the introduction of the *principle of incommensurability* into scientific usage, according to which none of the paradigms can be criticized from the positions of another paradigm.

Briefly recalling the current understanding of paradigms in science and their interaction, we will proceed to the presentation of *the paradigm of optimal measurements.* Within this paradigm, we study the processes of restoration of the input signal that is distorted by both flaws of the measuring transducer (for example, its inertia, resonances in circuits or degradation as a result of operation) and external influences (for example, incoming white noise). OMP is represented by three mutually exclusive parts. In the first part, which is called MBOM, MM of MT is constructed, the problem of finding OM is set, and the conditions for unique solvability of this problem are given. This solution is called *a precise solution.* In the second part, which is called NMOM, algorithms for construction of approximate solutions to the problem of finding OM are constructed and conditions for convergence of approximate solutions to a precise solution are formulated. Finally, in the third part of OMP, which is called CSOM, the codes are created based on the algorithms of the second part of OMP, checking procedures are performed to debug these codes, and finally a numerical experiment is set to restore the distorted measurement obtained during natural experiments.

MBOM appeared relatively recently, in [36, 37] the problem of restoration of measurement distorted by inertia of MT was first set and studied. The Leontief-type system that appeared in the remote control theory was taken as MM of MT. Almost simultaneously with the emergence of MBOM, NMOM [18, 38] appeared, based on numerical methods for solving optimal control problems for Leontief-type systems [48].

Soon MM of MT was reconstructed [21,22] in order to restore measurements distorted not only by inertia of MT, but also by resonances in its circuits. Finally, MM of MT was once again upgraded to include interference caused by degradation of MT [19,39]. Note the first reviews of the history and development of MBOM and NMOM [31,40,41], as well as their solid mathematical foundation [20,42].

In parallel with the deterministic theory of optimal measurements, the stochastic theory of optimal measurements emerged [43] and developed [44] in the framework of OMP. It is based on the concept of Nelson – Gliklikh derivative [15]. To date, the Nelson-Gliclich derivative has been sufficiently well studied in various aspects [8–11, 47], and therefore naturally fits into OMP. It is based on the concept of "white noise" which is understood as the Nelson-Gliklikh derivative of the Wiener process. Stochastic MM of MT provides "white noise" not only as an external interference, but also possibly as occurring inside MT [45]. We also note recently appeared CSOM [46,49]. In conclusion of a brief overview of the history of OMP indicate currently existing paradigms in dynamic measurements [13,17,25,29,30,35].

1. Precise Optimal Measurement

Let L and M be square matrices of order n, $f(t) = \mathrm{col}(f_1(t), f_2(t), \dots, f_n(t))$ be some vector function. Consider a linear inhomogeneous equation of the form

$$L\dot{x}(t) = Mx(t) + f(t),$$

and assume the possibility of $\det L = 0$. Note that V. Leontief [27] was the first to study such equations. Therefore, we will call these equations *Leontief type equations*, considering the terms "differential-algebraic equations" [4], "algebra-differential systems" [28], "descriptor systems" [1] as synonyms.

By MM of MT we mean a Leontief type system of the form

$$L\dot{x}(t) = a(t)Mx(t) + Du(t), \quad y(t) = b(t)Nx(t) + Fu(t) \tag{1}$$

where D, N, F are square matrices of order n, $x(t) = \mathrm{col}(x_1(t), x_2(t), \dots, x_n(t))$, $y(t) = \mathrm{col}(y_1(t), y_2(t), \dots, y_n(t))$ and $u(t) = \mathrm{col}(u_1(t), u_2(t), \dots, U_n(t))$ are vector-functions, $a(t)$ and $b(t)$ are functions. Here the matrices L, M, D, N and F describe the construction of MT, the vector-function $x = x(t)$ describes the state of MT, the functions $a = a(t)$ and $b = b(t)$ describe the degradation of MT in long-term operation (for example, when operating in near-earth space), the vector function $u = u(t)$ corresponds to the input signal (*measurement*), the vector function $y = y(t)$ corresponds to the output signal (*observation*). The measurement and the observation in MM (1) have the same dimensions, but in practice the dimension of observation may be smaller.

The matrix M is called *regular with respect to the matrix* L (briefly, L-*regular*), if there exists $\alpha \in \mathbb{C}$ such that $\det(\alpha L - M) \neq 0$. It is clear that such a number $\alpha \in \mathbb{C}$ exists if $\det L \neq 0$. However, a careful analysis of real MT [23, 23] shows that the case $\det L = 0$ is quite common. So let the matrix M be L-regular, then [40, ch.12] there are such non-degenerate matrices A and B of order n that

$$BLA = \mathrm{diag}\{\overset{0}{\mathfrak{J}}_{p_1}, \overset{0}{\mathfrak{J}}_{p_2}, \dots, \overset{0}{\mathfrak{J}}_{p_l}, \mathbb{I}_{n-m}\}, \quad BMA = \mathrm{diag}\{\mathbb{I}_m, S\},$$

where $\overset{0}{\mathfrak{J}}_{p_k}$ is Jordan cell of order p_k with zeros on the main diagonal, $\sum\limits_{k=1}^{l} p_k = m$, $\mathbb{I}_k$ is a unit matrix of order k, S is a square matrix of order $n-m$. Take the number $p = \max\{p_1, p_2, \ldots, p_l\}$ and call the L-regular matrix M *(L,p)-regular.*

So, let the matrix M be (L,p)-regular, $p \in \{0, 1, \ldots, n\}$, set the initial *Showalter – Sidorov condition* [41,45]

$$\lim_{t\to 0+} [R^L_\mu(M)]^p (x(t) - x_0) = 0, \tag{2}$$

where $R^L_\mu(M) = (\mu L - M)^{-1} L$ is *the right L-resolvent of the matrix M*, and $x_0 \in \mathbb{R}^n$ is some vector. Fix the number $\tau \in \mathbb{R}_+$ and consider *the space of measurements*

$$\mathfrak{U} = \{u \in L_2((0,\tau);\mathbb{R}^n) : u^{(p)} \in L_2((0,\tau);\mathbb{R}^n)\},$$

the space of observations $\mathfrak{Y} = L_2((0,\tau);\mathbb{R}^n)$ and *the state space* $\mathfrak{X} = \mathfrak{Y}$.

Theorem 1. [34] *Let the matrix M be (L,p)-regular, $p \in \{0,1,\ldots,N\}$. Then for any $x_0 \in \mathbb{R}^n$, $a \in C([0,\tau];\mathbb{R}_+) \cap C^p((0,\tau);\mathbb{R}_+)$, $b \in C([0,\tau];\mathbb{R}_+)$ and $u \in \mathfrak{U}$ there exists a unique solution $y \in \mathfrak{Y}$ of* (1), (2) *given by*

$$y(t) = b(t) N x(t) + F u(t), \tag{3}$$

where

$$x(t) = X(t,0)x_0 + \int_0^t X(t,s) L_1^{-1} Q D u(s) ds + \sum_{q=0}^{p} H^q M_0^{-1} (Q - \mathbb{I}_n) \left(\frac{1}{a(t)} \frac{d}{dt}\right)^q \frac{Du(t)}{a(t)}. \tag{4}$$

In (4) $X(t,s) = \lim\limits_{k\to\infty} \left(\left(L - \frac{1}{k} \int_0^t a(r) dr \, M \right)^{-1} L \right)^k$ is *a degenerate flow,* i.e. $X(t,r)X(r,s) = X(t,s)$ for all t, r, $s \in \mathbb{R}$ such that $t \geq r \geq s$, moreover $X(t,t) \neq \mathbb{I}_n$ for all $t \in \mathbb{R}$;

$$L_1^{-1} = \lim_{k\to\infty} \left(L - \frac{1}{k} M\right)^{-1} Q, \quad Q = \lim_{k\to\infty} \left(kL\left(L - kM\right)^{-1}\right)^{pk},$$

$$M_0^{-1} = \lim_{k\to\infty} \left(\frac{1}{k} L - M\right)^{-1} (\mathbb{I}_n - Q), \quad L_0 = L(\mathbb{I}_n - P),$$

$$P = \lim_{k\to\infty} \left(k\left(L - kM\right)^{-1} L\right)^{pk}, \quad H = M_0^{-1} L_0.$$

The main part of our MM of MT is the penalty function

$$J(u) = \varepsilon \int_0^\tau ||y(t) - \widetilde{y}(t)||^2 dt + (1-\varepsilon) \int_0^\tau \langle Cx(t), x(t) \rangle dt. \tag{5}$$

Here $||\cdot||$ and $\langle\cdot,\cdot\rangle$ are Euclidean norm and inner product in $\mathbb{R}^n$, $y(t)$ is calculated using (3), (4), so it depends linearly on $u(t)$. Snce $x(t)$ is calculated using (4) it also depends linearly on $u(t)$. Using a priori information construct a convex and closed subset of $\mathfrak{U}_\partial \subset \mathfrak{U}$, which is called *a set of admissible measurements*. By minimizing the first term of the functional (5), we reduce the impact of MT inertia on the measurement. And by minimizing the second term, we reduce the impact of resonances in the MT circuits. (Note that a square symmetric matrix C of order n characterizes the mutual influence of resonances in MT chains). The constant $\varepsilon \in (0,1)$ takes into account the researcher's preferences. Finally, $\widetilde{y}(t)$ is an observation obtained as a result of a computational or field experiment. So, the problem of searching for *optimal measurement* $v(t)$ is to find the minimum

$$J(v) = \min_{u \in \mathfrak{U}_\partial} J(u). \tag{6}$$

Theorem 2. *Let the matrix M be (L,p)-regular, $p \in \{0,1,\ldots,n\}$. Then for all $x_0 \in \mathbb{R}^n$, $a \in C([0,\tau];\mathbb{R}_+) \cap C^p((0,\tau);\mathbb{R}_+)$ и $b \in C([0,\tau];\mathbb{R}_+)$, there exists a unique $v \in \mathfrak{U}_\partial$ such that* (6) *holds.*

A vector-function $v = v(t)$ that exists by Theorem 2, is still called *precise optimal measurement*. Strictly speaking, after replacing $u(t) = v(t)$, the function (4) will no longer be a solution of the system of equations

$$L\dot{x}(t) = a(t)Mx(t) + Dv(t)$$

even in a generalized sense. However, when substituting (4) into (3) and replacing $u(t) = v(t)$, we get a vector function $y = y(t)$, which is called *a precise optimal observation*. Note that the vector functions $v = v(t)$ and $y = y(t)$ obtained by applying Theorem 1 and Theorem 2 are *virtual precise optimal measurement* and *virtual precise optimal observation*. The algorithms for construction of v and y will be proposed in Section 2.

However, before proceeding to the construction of algorithms, let's make a couple of comments that will simplify the solution of this problem. First, note that without loss of generality $\det M \neq 0$. Indeed, by replacing $x(t) = \exp\left(\alpha \int\limits_0^t a(\tau)d\tau\right) z(t)$ in (1) we get

$$L\dot{z} = a(t)(M - \alpha L)z(t) + Dv(t),$$

$$w(t) = b(t)Nz(t) + Fv(t),$$

where $v(t) = \exp\left(-\alpha \int\limits_0^t a(\tau)d\tau\right) u(t)$ and $w(t) = \exp\left(-\alpha \int\limits_0^t a(\tau)d\tau\right) y(t)$. Reassigning $M - \alpha L$ to M, we get the required.

Second, instead of solution (4), we will consider a particular solution

$$x(t) = \int\limits_0^t X(t,s)L_1^{-1}QDu(s)ds + \sum_{q=0}^{p} H^q M_0^{-1}(Q - \mathbb{I}_n)\left(\frac{1}{a(t)}\frac{d}{dt}\right)^q \frac{Du(t)}{a(t)}, \tag{7}$$

which is obtained if we take $x_0 \in \ker[R_\mu^L(M)]^p = \ker X(t,0)$ in (4). By substituting (7) for (4) in (3) and (5), we can find (6).

Theorem 3. *Let the matrix M be (L,p)-regular, $p \in \{0,1,\ldots,n\}$, and* $\det M \neq 0$. *Then for any $x_0 \in \ker[R_\mu^L(M)]^p$, $a \in C([0,\tau];\mathbb{R}_+) \cap C^p((0,\tau);\mathbb{R}_+)$ and $b \in C([0,\to];\mathbb{R}_+)$, there exists a unique $v \in \mathfrak{U}_\partial$ such that* (6) *holds.*

Later the optimal measurement $v \in \mathfrak{U}_\partial$ found by Theorem 3 will be called *precise partial optimal measurement*, and optimal observation $y = y(t)$, found by (7) and (3), will be called *precise partial optimal observation*.

2. Approximate Particular Optimal Measurements

Let L, M, D, N and F be square matrices of order n, where the matrix M is (L,p)-regular, $p \in \{0, 1, \ldots, n\}$, and $\det M \neq 0$. Suppose that the vector $x_0 \in \ker[R^L_\mu(M)]^p$, and note that $\ker[R^L_\mu(M)]^p$ does not depend on $\mu \in \mathbb{C}$ such that $\det(\mu L - M) \neq 0$. Construct an algorithm to find numerically the particular optimal measurement.

2.1. The First Approximation

Represent the space $\mathfrak{U}$ as $\mathfrak{U} = \bigoplus\limits_{j=1}^{n} \mathfrak{U}_j$, where

$$\mathfrak{U}_j = \{u \in L_2((0,\tau);\mathbb{R}) : u^{(p)} \in L_2((0,\tau);\mathbb{R})\}.$$

By construction, the space $\mathfrak{U}_j$ is Hilbert and separable, $j = 1, 2, \ldots, n$. Denote by $\{\varphi_i\}$ an orthonormal sequence of basis functions. It is obvious that the sequence $\{\varphi_i\}$ can be taken to be equal in each $\mathfrak{U}_j$. Construct the finite-dimensional lineal

$$\mathfrak{U}^k_j = \operatorname{span}\{\varphi_i : i = 1, 2, \ldots, k\} \quad \text{and the subset} \quad \mathfrak{U}^k = \bigoplus_{j=1}^{n} \mathfrak{U}^k_j.$$

Find a subset $\mathfrak{U}^k_\partial = \mathfrak{U}^k \cap \mathfrak{U}_\partial$. The subset $\mathfrak{U}^k_\partial \subset \mathfrak{U}_\partial$ can be empty. However, in any case, the subset $\mathfrak{U}^k_\partial \subset \mathfrak{U}_\partial$ is closed and convex. Obviously, some terms of the sequence $\{\mathfrak{U}^k_\partial\}$ are nonempty sets, since the sequence is monotonic and

$$\lim_{k\to\infty} \mathfrak{U}^k_\partial = \mathfrak{U}_\partial.$$

Let $\mathfrak{U}^k_\partial \neq \emptyset$. Consider the vector $u_k \in \mathfrak{U}^k$ and construct the vectors

$$x_k(t) = \int_0^t X(t,s) L_1^{-1} Q u_k(s)\,ds + \sum_{q=0}^{p} H^q M^{-1}(Q - \mathbb{I}_n) \left(\frac{1}{a(t)}\frac{d}{dt}\right)^q \frac{Du(t)}{a(t)}, \tag{8}$$

$$y_k(t) = b(t) N x_k(t) + F u_k(t). \tag{9}$$

Substitute x_k and y_k into the penalty functional J and find the minimum

$$J(v_k) = \min_{u_k \in \mathfrak{U}^k_\partial} J(u_k). \tag{10}$$

If $\mathfrak{U}^k_\partial \neq \emptyset$, then such a vector v_k exists and is unique by virtue of Theorem 3. If $\mathfrak{U}^k_\partial = \emptyset$, then we increase the number k in order to obtain $\mathfrak{U}^k_\partial \neq \emptyset$ (see the reasoning given above). The vector $v_k \in \mathfrak{U}^k_\partial$ is called the *first approximate particular optimal measurement*.

Lemma 1. *Let the matrix M be (L,p)-regular, $p \in \{0, 1, \ldots, n\}$, and such that* $\det M \neq 0$. *Let the functions $a \in C([0,\tau];\mathbb{R}_+) \cap C^p((0,\tau);\mathbb{R}_+)$ and $b \in C([0,\tau];\mathbb{R}_+)$, and the vector $x_0 \in \ker[R^L_\mu(M)]^p$. Then $\lim\limits_{k\to\infty} v_k = v$.*

2.2. The Second Approximation

Construct the next approximation. Under conditions of Lemma 1, we can write

$$P = \lim_{l\to\infty}(l(L-lM)^{-1}L)^{pl},\ Q = \lim_{l\to\infty}(lL(L-lM)^{-1})^{pl},$$

$$L_1^{-1} = \lim_{l\to\infty}\left(L - \frac{1}{l}M\right)^{-1}Q,\ H = M^{-1}L(\mathbb{I}_n - P).$$

Hence

$$x_{kl}(t) = \int_0^t \left(\left(L - \frac{1}{l}\int_s^t a(r)dr M\right)^{-1} L\right)^l \left(L - \frac{1}{l}M\right)^{-1}(lL(L-lM)^{-1})^{pl}u_k(s)ds +$$

$$+\sum_{q=0}^{p} H^q M((lL(L-lM)^{-1})^{pl} - \mathbb{I}_n)\left(\frac{1}{a(t)}\frac{d}{dt}\right)^q \frac{Du_k(t)}{a(t)}, \quad (11)$$

$$y_{kl}(t) = b(t)Nx_{kl}(t) + Fu_k(t). \quad (12)$$

Substituting (11) and (12) into the penalty functional J and taking the minimum of J on the set $\mathfrak{U}_\partial^k$, we obtain

$$J(v_{kl}) = \min_{u_k\in\mathfrak{U}_\partial^k} J(u_k).$$

The vector $v_{kl} \in \mathfrak{U}_\partial^k$ is called the *second approximate particular optimal measurement.*

Lemma 2. *Suppose that conditions of Lemma* 1 *hold. Then* $\lim\limits_{l\to\infty} v_{kl} = v_k$.

2.3. The Third Approximation

At the last step of the proposed algorithm, we note that the first term (the integral) and the second term (the sum of derivatives) of (11) can be calculated by any appropriate method at the discretion of a user. In order to calculate the first term, we can replace the integral with the Riemann sum or divide the interval $[0, t]$ by m parts and use Gaussian quadrature formula. In some cases, the integral can be calculated in explicit form. In order to calculate the second term, we can replace the derivatives with the differences and then calculate the sum by one of appropriate methods. Note that, in some cases, the derivatives can be calculated in explicit form. However, in any case, we obtain approximate values of the vectors $x_{klm} = x_{klm}(t)$ and $y_{klm} = y_{klm}(t)$ instead of formulas (11) and (12). Substituting the values into the penalty functional J and taking the minimum of J on the set $\mathfrak{U}_\partial^k$, we obtain

$$J(v_{klm}) = \min_{u_k\in\mathfrak{U}_\partial^k} J(u_k).$$

The vector $v_{klm} \in \mathfrak{U}_\partial^k$ is called the *third approximate particular optimal measurement.*

Lemma 3. *Suppose that conditions of Lemma* 1 *hold. Then* $\lim\limits_{m\to\infty} v_{klm} = v_{kl}$.

Proof of this statement depends on the way of approximation of the first and the second terms of (11). However, the proof is well-known and provides convergence of the sequence of the third approximate particular optimal measurements. Therefore, Lemma 1 and Lemma 2 lead to the following statement.

Theorem 4. *Suppose that conditions of Lemma* 1 *hold. Then*

$$\lim_{k\to\infty}\lim_{l\to\infty}\lim_{m\to\infty} v_{klm} = v.$$

3. Stochastic Optimal Measurement

Let $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ be a complete probability space with the probability measure $\mathbf{P}$ associated with the σ-algebra $\mathcal{A}$ of subsets of the set Ω, and $\mathbb{R}$ be a set of real numbers endowed with a Borel σ-algebra. A measurable mapping $\xi : \Omega \to \mathbb{R}$ is called a *random variable.*

The set of random variables having zero mathematical expectation $\mathbf{E}$ and finite variances $\mathbf{D}$ forms the Hilbert space

$$\mathbf{L_2} = \{\xi : \mathbf{E}\xi = 0,\ \mathbf{D}\xi < +\infty\} \text{ with an inner product } (\xi_1, \xi_2) = \mathbf{E}\xi_1\xi_2$$

and the norm $\|\xi\|^2_{\mathbf{L_2}} = \mathbf{D}\xi$. In $\mathbf{L_2}$, the vectors ξ and η are orthogonal to each other (i.e. $(\xi, \eta) = 0$) if and only if the random variables ξ and η are uncorrelated. Indeed, $0 = \operatorname{cov}(\xi, \eta) = \mathbf{E}\xi\eta = (\xi, \eta) = 0$.

Consider the set $\mathfrak{I} \subset \mathbb{R}$ and the following two mappings. The first, $f : \mathfrak{I} \to \mathbf{L_2}$, associates each $t \in \mathfrak{I}$ with a random variable $\xi \in \mathbf{L_2}$. The second, $g : \mathbf{L_2} \times \Omega \to \mathbb{R}$, associates to each pair (ξ, ω) a point $\xi(\omega) \in \mathbb{R}$. A mapping $\eta : \mathfrak{I} \times \Omega \to \mathbb{R}$ of the form $\eta = \eta(t, \omega) = g(f(t), \omega)$ is called an *(one-dimensional) stochastic process.* For each fixed $t \in \mathfrak{I}$, the value of the stochastic process $\eta = \eta(t, \cdot)$ is a random variable, i.e. $\eta(t, \cdot) \in \mathbf{L_2}$, which is called a *section* of the stochastic process at the point $t \in \mathfrak{I}$. For each fixed $\omega \in \Omega$, the function $\eta = \eta(\cdot, \omega)$ is called a *(sample) trajectory* of a random process corresponding to the elementary outcome $\omega \in \Omega$. The trajectories are also called *implementations* or *sample functions* of a random process. Usually, when this does not lead to ambiguity, the dependence of $\eta(t, \omega)$ on ω is not indicated, and the random process is denoted simply by $\eta(t)$.

Let $\mathfrak{I} \subset \mathbb{R}$ be an interval. The stochastic process $\eta = \eta(t)$, $t \in \mathfrak{I}$, is called *continuous,* if a.s. (almost surely) all trajectories of the process are continuous (i.e. for almost all $\omega \in \mathcal{A}$, the trajectories $\eta(\cdot, \omega)$ are continuous functions). The set of continuous stochastic processes forms a Banach space denoted by $\mathbf{C}(\mathfrak{I}; \mathbf{L_2})$ with the norm

$$\|\eta\|_{\mathbf{CL_2}} = \sup_{t\in\mathfrak{I}} (\mathbf{D}\eta(t, \omega))^{1/2}.$$

Let $\mathcal{A}_0$ be a σ-subalgebra of the σ-algebra $\mathcal{A}$. Construct the subspace $\mathbf{L^0_2} \subset \mathbf{L_2}$ of random variables measurable with respect to $\mathcal{A}_0$. Denote by $\Pi : \mathbf{L_2} \to \mathbf{L^0_2}$ the orthoprojector. Let $\xi \in \mathbf{L_2}$, then $\Pi\xi$ is called the *conditional mathematical expectation* of the random variable ξ and is denoted by $\mathbf{E}(\xi|\mathcal{A}_0)$. Fix $\eta \in \mathbf{C}(\mathfrak{I}; \mathbf{L_2})$ and $t \in \mathfrak{I}$, and denote by $\mathcal{N}^\eta_t$ the *σ-algebra generated by the random variable* $\eta(t)$, and denote $\mathbf{E}^\eta_t = \mathbf{E}(\cdot|\mathcal{N}^\eta_t)$.

Example 1. The stochastic process describing the Brownian motion in the Einstein – Smoluchowski model (see [34])

$$\beta(t,\omega)=\sum_{k=0}^{\infty}\xi_k(\omega)\sin\frac{\pi}{2}(2k+1)t,\ \ t\in\{0\}\cup\mathbb{R}_+$$

is a continuous stochastic process. Here the coefficients $\{\xi_k=\xi_k(\omega)\}\subset\mathbf{L}_2$ are pairwise uncorrelated Gaussian random variables such that $\mathbf{D}\xi_k=\left[\frac{\pi}{2}(2k+1)\right]^{-2}$, $k\in\{0\}\cup\mathbb{N}$.

Let $\eta\in\mathbf{C}(\mathfrak{J};\mathbf{L_2})$. By the *Nelson – Gliklikh derivative* $\overset{\circ}{\eta}$ of the *stochastic process* η *at the point* $t\in\mathfrak{J}$ we mean a random variable

$$\overset{\circ}{\eta}(t,\cdot)=\frac{1}{2}\left(\lim_{\Delta t\to 0+}\mathbf{E}_t^{\eta}\left(\frac{\eta(t+\Delta t,\cdot)-\eta(t,\cdot)}{\Delta t}\right)+\lim_{\Delta t\to 0+}\mathbf{E}_t^{\eta}\left(\frac{\eta(t,\cdot)-\eta(t-\Delta t,\cdot)}{\Delta t}\right)\right),$$

if the limit exists in the sense of uniform metric on $\mathbb{R}$.

If the Nelson – Gliklikh derivatives $\overset{\circ}{\eta}(t,\cdot)$ of the stochastic process $\eta(t,\cdot)$ exist at all (or almost all) points of the interval $\mathfrak{J}$, then the Nelson – Gliklikh derivative $\overset{\circ}{\eta}(t,\cdot)$ exists on $\mathfrak{J}$ (a.s. on $\mathfrak{J}$). The set of continuous stochastic processes having continuous Nelson – Gliklikh derivatives $\overset{\circ}{\eta}$ forms the Banach space $\mathbf{C}^1(\mathfrak{J};\mathbf{L_2})$ with the norm

$$\|\eta\|_{\mathbf{C}^1\mathbf{L}_2}=\sup_{t\in\mathfrak{J}}\left(\mathbf{D}\eta(t,\omega)+\mathbf{D}\overset{\circ}{\eta}(t,\omega)\right)^{1/2}.$$

Further, by induction, we define the Banach spaces $\mathbf{C}^l(\mathfrak{J};\mathbf{L_2})$, $l\in\mathbb{N}$, of stochastic processes whose trajectories are a.s. differentiable by Nelson – Gliklikh on $\mathfrak{J}$ up to the order $l\in\{0\}\cup\mathbb{N}$ inclusively [16]. In these spaces, the norms are given by the formulas

$$\|\eta\|_{\mathbf{C}^l\mathbf{L}_2}=\sup_{t\in\mathfrak{J}}\left(\sum_{k=0}^{l}\mathbf{D}\overset{\circ}{\eta}{}^{(k)}(t,\omega)\right)^{1/2}.$$

Here we consider the Nelson – Gliklikh derivative of zero order as the initial random process, i.e. $\overset{\circ}{\eta}{}^{(0)}\equiv\eta$. For brevity, the spaces $\mathbf{C}^l(\mathfrak{J};\mathbf{L_2})$, $l\in\{0\}\cup\mathbb{N}$, are called the *spaces of "noises"* (see [8–11, 47]).

Example 2. The papers [15, 16] show that $\beta\in\mathbf{C}^l(\mathbb{R}_+;\mathbf{L_2})$, $l\in\{0\}\cup\mathbb{N}$, moreover, $\overset{\circ}{\beta}(t)=\frac{\beta(t)}{2t}$, $t\in\mathbb{R}_+$.

Denote by $\mathbf{L}_2^n$ the space of n-dimensional random variables, which are called *random n-"variables"*, i.e. $\mathbf{L}_2^n=\{\xi:\xi=\mathrm{col}(\xi_1,\xi_2,\ldots,\xi_n),\xi_k\in\mathbf{L}_2,k=\overline{1,n}\}$. By analogy with the spaces of "noises", construct the spaces of *n-"noises"* $\mathbf{C}(\mathfrak{J};\mathbf{L}_2^n)=\bigoplus_{k=1}^{n}\mathbf{C}_k(\mathfrak{J};\mathbf{L}_2)$ and $\mathbf{C}^l(\mathfrak{J};\mathbf{L}_2^n)=\bigoplus_{k=1}^{n}\mathbf{C}_k^l(\mathfrak{J};\mathbf{L_2})$, where $\mathbf{C}_k(\mathfrak{J};\mathbf{L_2})\equiv\mathbf{C}(\mathfrak{J};\mathbf{L_2})$ and $\mathbf{C}_k^l(\mathfrak{J};\mathbf{L_2})\equiv\mathbf{C}^l(\mathfrak{J};\mathbf{L_2})$, k, $l\in\mathbb{N}$. In the space $\mathbf{C}^l(\mathfrak{J};\mathbf{L}_2^n)$, the norms are given by the formulas

$$\|\eta\|_{\mathbf{C}^l\mathbf{L}_2^n}=\sum_{k=1}^{n}\left(\sup_{t\in\mathfrak{J}}\sum_{j=0}^{l}\mathbf{D}\overset{\circ}{\eta}{}_k^{(j)}(t,\omega)\right)^{1/2},\ \ l\in\{0\}\cup\mathbb{N}.$$

Example 3. [15, 16] The Wiener n-process is given by formula

$$W_n(t) \equiv W_n(t,\omega) = \mathrm{col}(\beta_1(t,\omega), \beta_2(t,\omega), \ldots, \beta_n(t,\omega)),$$

where $\beta_k(t,\omega)$ is the Brownian motion, $k = \overline{1,n}$. It is easy to see that $W_n \in \mathbf{C}^l(\mathbb{R}_+; \mathbf{L}_2^n) \cap \mathbf{C}(\overline{\mathbb{R}}_+; \mathbf{L}_2^n)$, where $l \in \mathbb{N}$ and $\overline{\mathbb{R}}_+ = \{0\} \cup \mathbb{R}_+$.

Consider SMM of MT

$$\overset{\circ}{\xi}(t) = a(t)M\xi(t) + D\omega(t), \quad \eta(t) = b(t)N\xi(t) + F\omega(t), \tag{13}$$

where M, D, N and F are square matrices taken from (1), $a = a(t)$ and $b = b(t)$ are non-negative functions of a real variable, $\omega = \omega(t)$, $\eta = \eta(t)$ and $\xi = \xi(t)$ are stochastic processes simulating measurement, observation and state of MT. Endow SMM of MT (13) with the initial Showalter – Sidorov condition

$$\lim_{t\to 0+} [R_\mu^L(M)]^p(\xi(t) - \xi_0) = 0. \tag{14}$$

Theorem 5. *Let the matrix M be (L,p)-regular, $p \in \{0,1,\ldots,n\}$, and $\det M \neq 0$. Let the functions $a \in C([0,\tau];\mathbb{R}_+) \cap C^{p+1}((0,\tau);\mathbb{R}_+)$ and $b \in C([0,\tau];\mathbb{R}_+)$. Then, for any $\xi_0 \in \mathbf{L}_2^n$ and $\omega \in \mathbf{C}([0,\tau];\mathbf{L}_2^n) \cap \mathbf{C}^l((0,\tau);\mathbf{L}_2^n)$, there exists a solution $\eta \in \mathbf{C}([0,\tau];\mathbf{L}_2^n)$ to problem (13), (14) given by*

$$\eta(t) = b(t)N\xi(t) + F\omega(t), \tag{15}$$

where

$$\xi(t) = X(t,0)\xi_0 + \int_0^t X(t,s)L_1^{-1}QD\omega(s)ds + \sum_{q=0}^{p} H^q M_0^{-1}(Q - \mathbb{I}_n)\left(\frac{1}{a(t)}\frac{\delta}{\delta t}\right)^q \frac{D\omega(t)}{a(t)}. \tag{16}$$

Here the matrices $X(t,s)$, L_1^{-1}, Q, H and M_0^{-1} are taken from Section 1, $\frac{\delta}{\delta t}$ is the Nelson – Gliklikh derivative of the stochastic n-process $\frac{D\omega(t)}{a(t)}$. Note that the stochastic observation $\eta = \eta(t)$ is found "along the trajectories", therefore, there is no reason to talk about uniqueness of such a solution. Consider the penalty functional in this case:

$$J(\omega) = \varepsilon \int_0^\tau ||\eta(t) - \widetilde{\eta}(t)||^2_{\mathbf{L}_2^n} dt + (1-\varepsilon)\int_0^\tau \langle C\xi(t), \xi(t)\rangle_{\mathbf{L}_2^n} dt.$$

Here $\widetilde{\eta} = \widetilde{\eta}(t)$ is a stochastic observation on real MT (so maybe virtual observation), C is a positively defined symmetric matrix of order n. Denote by $\mathbf{C}^p_\partial \mathbf{L}_2^n$ a closed and convex set in the space $\mathbf{C}([0,\tau];\mathbf{L}_2^n) \cap \mathbf{C}^p((0,\tau);\mathbf{L}_2^n)$. Find

$$J(\zeta) = \min_{\omega \in \mathbf{C}^p_\partial \mathbf{L}_2^n} J(\omega). \tag{17}$$

Corollary 1. *Suppose that conditions of Theorem 5 hold. Then, for any $\xi_0 \in \mathbf{L}_2^n$, there exists $\zeta \in \mathbf{C}^p_\partial \mathbf{L}_2^n$ such that (17) holds.*

Here, as well as above, uniqueness of the stochastic process ζ is impossible due to the "trajectory" nature of the solution of (17).

Conclusion

The results of Section 3 of this paper are proved similarly to the results of Section 1. However, there exists another way to restore signals distorted by random noises. This way was outlined in [34], and we intend to pave the way to our n-dimensional situation. As for this problem, we will embody the approaches and algorithms proposed here in patents for inventions. Thankfully, our team has great experience to receive such patents [2, 3, 5–7].

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ТЕОРИЯ ОПТИМАЛЬНЫХ ИЗМЕРЕНИЙ КАК НОВАЯ ПАРАДИГМА МЕТРОЛОГИИ

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Статья носит обзорный характер и содержит изложение краткой истории теории оптимальных измерений как одной из парадигм в метрологии. Во введении приводятся основные положения парадигмальной концепции Т. Куна и ее критика П. Фейерабендом с анархистских позиций. Делается вывод о сосуществовании в рамках одной науки противоречащих друг с другом парадигм. В первой части описана математическая модель измерительного устройства и даны условия существования единственного точного оптимального измерения. Во второй части предложены различные приближенные оптимальные измерения и указаны условия сходимости последовательности приближенных оптимальных измерений к точному оптимальному измерению. Третья часть содержит подход к изучению стохастической математической модели измерительного устройства, основанный на производной Нельсона – Гликлиха стохастического процесса. В заключении намечены пути дальнейших возможных исследований. Список публикаций содержит все доступные источники, относящиеся к данной проблематике.

Ключевые слова: детерминированная математическая модель измерительного устройства; стохастическая математическая модель измерительного устройства; точное оптимальное динамическое измерение; приближенное оптимальное измерение; вырожденный поток; стохастическое оптимальное измерение; производная Нельсона – Гликлиха; винеровский процесс; «белый шум».

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