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OPTIMAL CONTROL IN THE MATHEMATICAL MODEL OF INTERNAL WAVES

K. Yu. Kotlovanov[1]***,*** kotlovanovki@susu.ru,
E. V. Bychkov[1]***,*** bychkovev@susu.ru,
A. V. Bogomolov[2]***,*** a.v.bogomolov@gmail.com
[1] South Ural State University, Chelyabinsk, Russian Federation
[2] St. Petersburg Institute for Informatics and Automation of RAS, Saint-Petersburg, Russian Federation

The paper presents the results of the study of the problem on the optimal control to solutions for a mathematical model of internal waves, which is based on a linear system of equations of hydrodynamics. This model describes the propagation of waves in a homogeneous incompressible stratified fluid. The mathematical model includes the Sobolev equation, the Cauchy and Dirichlet condition. We use a parallelepiped as a considered domain in the mathematical model. The paper shows existence and uniqueness of a strong solution to the Cauchy – Dirichlet problem for the Sobolev equation. Also, we obtain the sufficient conditions for existence and uniqueness of a solution to the problem on optimal control to such solutions in Hilbert spaces. Proof of existence and uniqueness of a strong solution is based on the theorem for an abstract incomplete inhomogeneous Sobolev type equation of the second order and the theory of relatively p-bounded operators. In this paper, we present the theorem on existence and uniqueness of the optimal control for the problem under study, which is based on the works of J.-L. Lyons.

Keywords: Sobolev type equations; relatively p-bounded operator; strong solution; optimal control.

Introduction

Dynamics of particle oscillations of a homogeneous incompressible fluid, which rotates with the constant angular velocity Ω, is described by the linear system of hydrodynamic equations (the system of Sobolev equations [1])

$$\begin{cases} \nu_t + \frac{1}{\rho_0}\nabla p + 2[\Omega \times \nu] = f(x,t), \\ \rho_t = 0, \\ \nabla \nu = 0, \end{cases} \tag{1}$$

where $\nu = \{u, v, w\}$, while $\rho_0 = \text{const}$ is the equilibrium density and the buoyancy frequency is equal to zero. If we direct the axis Oz collinearly to the vector Ω, then we obtain the equation for the vertical component of the velocity of fluid particles (the Sobolev equation [1])

$$\Delta w_{tt} + F^2 w_{zz} = f(x,t), \tag{2}$$

where $2[\Omega \times \nu] = \{-Fv, Fu, 0\}$, and $F = 2\Omega$ is the Coriolis parameter. Wave solutions that satisfy equation (2) are called inertial or gyroscopic waves propagating on the surface of the rotating fluid. In the paper [1], a solution to equation (2) is obtained in an unbounded domain by the Green's function method. The paper [2] describes the behavior of solutions to two-dimensional Hamiltonian systems arising in the theory of small oscillations of a rotating ideal fluid, and constructs a mathematical model of the generation of a vortex structure.

Let D be a bounded domain that belongs to $\mathbb{R}^3$ and has a smooth boundary ∂D. For the function $w = w(x,t)$, we impose the Dirichlet condition

$$w(x,t) = 0, \quad (x,t) \in D \times \mathbb{R} \tag{3}$$

at the boundary of the domain D, and the Cauchy condition

$$w(x,0) = 0, \quad w_t(x,0) = 0. \tag{4}$$

In this paper, we study the optimal control problem on finding the pair $(\hat{w}, \hat{u})$, where $\hat{w}$ is the solution to problem (2), (4), and $\hat{u} \in \mathfrak{U}_{ad}$ is the control that satisfies the relation

$$J(\hat{w}, \hat{u}) = \min_{(w,u)\in\mathfrak{X}\times\mathfrak{U}_{ad}} J(w,u). \tag{5}$$

Here $J(w,u)$ is the quality functional constructed in a special way, and $\mathfrak{U}_{ad}$ is the set, which is closed and convex in the control space $\mathfrak{U}$.

Let us find a solution to problem (2)–(5) in the framework of the theory of Sobolev type equations. First, consider the Cauchy problem for the incomplete inhomogeneous Sobolev type equation of the second order

$$A\ddot{w} = Bw + y, \tag{6}$$

$$\dot{w}(0) = w_1, \; w(0) = w_1, \tag{7}$$

where the operators $A, B \in \mathcal{L}(\mathfrak{X}; \mathfrak{Y})$, the function $y : [0, \tau) \subset \mathbf{R}_+ \to \mathfrak{Y}$ $(\tau < \infty)$, and $\mathfrak{X}, \mathfrak{Y}$ are Hilbert spaces.

The paper is based on the theory of relatively bounded operators, degenerate semigroups of operators [3] and the theory of incomplete Sobolev type equations of the high order [4]. The monographs [3, 5] study in detail Sobolev type equations and equations that are not resolved with respect to the highest time derivative. In the monograph [5], various classes of Sobolev type equations are introduced and equation (2) is referred to simple Sobolev type equations. Sobolev type equations represent a significant field of non-classical equations of mathematical physics. Optimal control problems for models of mathematical physics represent a promising direction. New statements of optimal control problems arise. For example, we note optimal control of solutions to stochastic equations [6], [7], optimal control of solutions to a multipoint initial-final value problem [8]. Optimal control problems are considered in [9, 10] for first-order Sobolev type equations and in [11] for high-order equations.

1. Relatively p-bounded Operators

Definition 1. The set $\rho^A(B) = \{\mu \in \mathbb{C} : (\mu A - B)^{-1} \in \mathcal{L}(\mathfrak{Y}; \mathfrak{X})\}$ is said to be the *resolvent set* of the operator B with respect to the operator A (in short, A-*resolvent set* of the operator B). The set $\mathbb{C} \backslash \rho^A(B) = \sigma^A(B)$ is called the *spectrum* of the operator B with respect to the operator A (in short, A-*spectrum* of the operator B).

Definition 2. The operator-functions

$$(\mu A - B)^{-1}, \quad R^A_\mu = (\mu A - B)^{-1}A, \quad L^A_\mu = A(\mu A - B)^{-1}$$

with the domain $\rho^A(B)$ are called the *resolvent*, *right resolvent*, *left resolvent* of the operator B with respect to the operator A (in short, *A-resolvent, right A-resolvent, left A-resolvent of the operator B*), respectively.

Definition 3. The operator B is called spectrally bounded with respect to the operator A (in short, *(A, σ)-bounded*), if

$$\exists a > 0 \; \forall \mu \in \mathbb{C} : (|\mu| > a) \Rightarrow (\mu \in \rho^A(B)).$$

Lemma 1. [3] *Let the operator B be (A, σ)-bounded. Then the operators*

$$P = \frac{1}{2\pi i} \int\limits_\Gamma R^A_\lambda(B) d\lambda \text{ и } Q = \frac{1}{2\pi i} \int\limits_\Gamma L^A_\lambda(B) d\lambda$$

are projectors with $P : \mathfrak{X} \to \mathfrak{X}$ and $Q : \mathfrak{Y} \to \mathfrak{Y}$. Here $\Gamma = \{\lambda \in \mathbb{C} : |\lambda| = r > a\}$.

Let $\mathfrak{X}^0 = \ker P$, $\mathfrak{Y}^0 = \ker Q$, $\mathfrak{X}^1 = \operatorname{im} P$, $\mathfrak{Y}^1 = \operatorname{im} Q$. Denote by $A_k(B_k)$ the restriction of the operator A (B) to the subspace $\mathfrak{X}^k$, $k = 0, 1$.

Theorem 1. [3] *Let the operator B* be *(A, σ)-bounded. Then*

(i) *the operators $A_k, B_k : \mathfrak{X}^k \to \mathfrak{Y}^k, k = 0, 1$;*

(ii) *there exists the operator $B_0^{-1} \in \mathcal{L}(\mathfrak{Y}^0, \mathfrak{X}^0)$;*

(iii) *there exists the operator $A_1^{-1} \in \mathcal{L}(\mathfrak{Y}^1, \mathfrak{X}^1)$;*

(iv) *the operator $B_1 \in \mathcal{L}(\mathfrak{X}^1, \mathfrak{Y}^1)$.*

Under the conditions of Theorem 1, we construct the operators $H = B_0^{-1} A_0 \in \mathcal{L}(\mathfrak{X}^0)$ and $S = A_1^{-1} B_1 \in \mathcal{L}(\mathfrak{X}^1)$. Then

$$(\mu A - B)^{-1} = \left(-\sum_{k=0}^{\infty} \mu^k H^k \right) B_0^{-1}(\mathbb{I} - Q) + \sum_{k=1}^{\infty} \mu^{-k} S^{k-1} A_1^{-1} Q. \tag{8}$$

Definition 4. An infinitely distant point of the A-resolvent of the operator B is called

(i) *a removable singular point*, if $H \equiv \mathbb{O}$;

(ii) *a pole of the order p*, if $H^p \neq \mathbb{O}, H^{p+1} \equiv \mathbb{O}$; $p \in \mathbb{N}$,

(iii) *an essentially singular point*, if $H^q \neq \mathbb{O}$, $\forall q \in \mathbb{N}$.

Definition 5. The (A, σ)-bounded operator B is called *(A, p)-bounded*, if the point ∞ is a pole of the order $p \in \{0\} \cup \mathbb{N}$ of its A-resolvent.

2. Abstract Problem

Let $\mathfrak{X}$, $\mathfrak{Y}$ be Hilbert spaces, $\mathcal{L}(\mathfrak{X})$ be the space of linear operators acting on the space $\mathfrak{X}$, the operators $A, B \in \mathcal{L}(\mathfrak{X}; \mathfrak{Y})$. Consider the inhomogeneous linear Sobolev type equation

$$A\ddot{w} = Bw + y. \tag{9}$$

Definition 6. The operator-function $V^\bullet \in C^\infty(\mathbb{R};\mathcal{L}(\mathfrak{X}))$ is called the propagator of inhomogeneous equation (9), if the vector function $w(t) = V^t v$ is a solution to (9) $\forall v \in \mathfrak{X}$.

Theorem 2. [4] *Let the operator B be (A,σ)-bounded. Then the formula*

$$V_m^t = \frac{1}{2\pi i}\int\limits_\Gamma \mu^{1-m}(\mu^2 A - B)^{-1} A e^{\mu t} d\mu,\ m = 0,1,$$

where the contour $\Gamma = \{\mu \in \mathbb{C} : |\mu| = R > a\}$, defines the propagators of equation (6) for all $t \in \mathbb{R}$.

Lemma 2. (i) $V_m^\bullet \in C^\infty(\mathbb{R};\mathcal{L}(\mathfrak{X};\mathfrak{X}^1))$, $(V_m^t)_t^{(l)} = V_{m-l}^t$, *where* $m = 0,1,\ l = 0,1$;

(ii) $(V_m^t)_t^{(l)}\Big|_{t=0} = \mathbb{O}$ *for* $m \neq l$, $(V_m^t)_t^{(m)}\Big|_{t=0} = V_0^0 = P$.

Definition 7. The subspace $\mathcal{P} \subset \mathfrak{X}$ is called the *phase space of homogeneous equation* (6) if

(i) any solution $w = w(t)$ to equation (6) belongs to $\mathcal{P}$, i.e. $w(t) \in \mathcal{P}, \forall t \in \mathbb{R}$;

(ii) there exists the unique solution to problem (6), (7) for any $w_0, w_1 \in \mathcal{P}$.

Theorem 3. [4] *Let the operator B* be *(A,p)-bounded. Let the vector function $y : (-\tau,\tau) \to \mathfrak{Y}$ be such that $y^0 \in C^2((-\tau,\tau);\mathfrak{Y}^0)$, and $y^1 \in C((-\tau,\tau);\mathfrak{Y}^1)$. Suppose that the initial values satisfy the relations*

$$(I - V_0^0)w_m = -\sum_{q=0}^{p} H^q B_0^{-1}\frac{d^{2q+m}}{dt^{2q+m}} y^0(0), \quad m = 0,1.$$

Then there exists a unique solution to problem (6),(7), *which can be represented in the form*

$$w(t) = -\sum_{q=0}^{p} H^q B_0^{-1}(\mathbb{I} - Q)y^{(2q)}(t) + \sum_{m=0}^{1} V_m^t w_m^1 + \int\limits_0^t V_1^{t-s} A_1^{-1} Q y(s) ds, t \in (-\tau,\tau). \quad (10)$$

Definition 8. The vector function $w \in H^2(\mathfrak{X}) = \{w \in L_2(0,\tau;\mathfrak{X}) :\ \ddot{w} \in L_2(0,\tau;\mathfrak{X})\}$ is said to be a *strong solution to equation* (6), if w converts (6) into identity almost everywhere on $(0,\tau)$. A strong solution $w = w(t)$ to equation (6) is called a *strong solution to problem* (6), (7) if w satisfies (7).

The concept of a "strong solution" used in Definition 6 is introduced to distinguish the solution to equation (6) in this sense and the solution to (6), which is usually called "classical" one. The embedding $H^2(\mathfrak{X}) \hookrightarrow C^1([0,\tau];\mathfrak{X})$ is continuous, therefore Definition 8 is correct. Note that the classical solution to (6), (7) is also a strong solution to the problem.

Let us construct the space $H^2(\mathfrak{Y}) = \{v \in L_2(0,\tau;\mathfrak{Y}) : \ddot{v} \in L_2(0,\tau;\mathfrak{Y})$. The space $H^2(\mathfrak{Y})$ is a Hilbert space with the scalar product

$$[v,w] = \sum_{q=0}^{2}\int\limits_0^\tau \left\langle v^{(q)}, w^{(q)}\right\rangle_{\mathfrak{Y}} dt.$$

Theorem 4. [11] *Let the operator B be (A,p)-bounded. Then there exists a unique strong solution to problem* (7) *for equation* (6) *for any $w_0, w_1 \in \mathfrak{X}$ and $y \in H^2(\mathfrak{Y})$.*

Let us consider the optimal control problem for the solutions to problem (6), (7) with the penalty functional of the form

$$J(w,u) = \sum_{q=0}^{2} \int_0^{\tau} ||w^{(q)} - \tilde{w}^{(q)}||^2 dt + \sum_{q=0}^{2} \int_0^{\tau} \left\langle N_q u^{(q)}, u^{(q)} \right\rangle_{\mathfrak{U}} dt, \tag{11}$$

where $N_q \in \mathcal{L}(\mathfrak{U})$, $q = 0,1,2$, are positive definite and self-adjoint operators, w is a solution to problem (6), (7), $\tilde{w}(t)$ is the desired state of the system, and the heterogeneity function y is a control denoted by $y = u$. The vector function $\hat{u} \in H^2_\partial(\mathfrak{U})$ minimizing functional (11) is called the optimal control to problem (6), (7).

Define the control space

$$H^2(\mathfrak{U}) = \{u \in L_2(0,\tau;\mathfrak{U}) : \ddot{u} \in L_2(0,\tau;\mathfrak{U})\}.$$

The space $H^2(\mathfrak{U})$ is Hilbert, due to the Hilbert property of $\mathfrak{U}$, with the scalar product

$$[v,w] = \sum_{q=0}^{2} \int_0^{\tau} \left\langle v^{(q)}, w^{(q)} \right\rangle_{\mathfrak{U}} dt.$$

In the space $H^2(\mathfrak{U})$, consider the closed and convex subset $\mathfrak{U}_{ad} = H^2_\partial(\mathfrak{U})$, which is the *set of admissible controls.*

In the paper [11], the following theorem on the uniqueness of optimal control is proved.

Theorem 5. *Let the operator B be (A,p)-bounded. Then there exists the unique optimal control of solutions to problem* (7) *for equation* (6) *for any $w_0, w_1 \in \mathfrak{X}$ and $y \in H^2(\mathfrak{Y})$.*

3. Mathematical Model of Internal Waves

Let the domain D be the parallelepiped $[0,a] \times [0,b] \times [0,c]$. Mathematical model (2)–(4) can be reduced to Cauchy problem (7) for equation (6).

Introduce the spaces $\mathfrak{X} = W_2^{l+2}(D)$, $\mathfrak{Y} = W_2^l(D)$ and define the operators

$$A = \Delta, \quad B = -F^2 \frac{\partial^2}{\partial z^2}.$$

For any $l \in \{0\} \cup \mathbb{N}$, the operators $A, B \in \mathcal{L}(\mathfrak{X}, \mathfrak{Y})$. Denote by

$$-\lambda_{k,m,n}^2 = -\left(\frac{\pi k}{a}\right)^2 - \left(\frac{\pi m}{b}\right)^2 - \left(\frac{\pi n}{c}\right)^2$$

the eigenvalues of the Laplace operator Δ, which are numbered in non-increasing order with respect to multiplicity. Denote by

$$\varphi_{k,m,n} = \sin\left(\frac{\pi k x}{a}\right) \sin\left(\frac{\pi m y}{b}\right) \sin\left(\frac{\pi n z}{c}\right)$$

the orthogonal eigenfunctions that correspond to $\{-\lambda^2_{k,m,n}\}$ in the sense of the scalar product in $L^2(\Omega)$.

Since $\{\varphi_{k,m,n}\} \subset C^\infty(D)$, then

$$\mu A - B = \sum_{k,m,n=1}^{\infty} [-\lambda^2_{k,m,n}\mu - F^2\lambda^2_n] < \varphi_{k,m,n}, \cdot > \varphi_{k,m,n},$$

where $< \cdot, \cdot >$ is the scalar product in the space $L^2(D)$. Construct the equation to determine the relative spectrum:

$$\lambda^2_{k,m,n}\mu + F^2\lambda^2_n = 0,$$

and we obtain the relative spectrum in the form

$$\mu_{k,m,n} = -\frac{F^2\lambda^2_n}{\lambda^2_{k,m,n}}.$$

The relative spectrum $\sigma^A(B) = \{\mu_{k,m,n}\}$ is bounded, because $|\mu_{k,m,n}| \leq F$. Since the operator A is continuously invertible in the given spaces, then the point ∞ is a removable singular point of the A-resolvent of the operator B. As a result, the conditions of Lemma 1 hold. We construct propagators by Theorem 2. Since the relative spectrum of the operator B is discrete, we obtain

$$V_0^t w_0 = \sum_{k,m,n=1}^{\infty} \cos\left(\frac{F^2\lambda^2_n}{\lambda^2_{k,m,n}} t\right) < \varphi_{k,m,n}, w_0 > \varphi_{k,m,n},$$

$$V_1^t w_1 = \sum_{k,m,n=1}^{\infty} \frac{F^2\lambda^2_n}{\lambda^2_{k,m,n}} \sin\left(\frac{F^2\lambda^2_n}{\lambda^2_{k,m,n}} t\right) < \varphi_{k,m,n}, w_1 > \varphi_{k,m,n}.$$

Solution to problem (2)–(4) has the form

$$w(x,t) = \sum_{k,m,n=1}^{\infty} \cos\left(\frac{F^2\lambda^2_n}{\lambda^2_{k,m,n}} t\right) < \varphi_{k,m,n}, w_0 > \varphi_{k,m,n} +$$
$$+ \sum_{k,m,n=1}^{\infty} \frac{F^2\lambda^2_n}{\lambda^2_{k,m,n}} \sin\left(\frac{F^2\lambda^2_n}{\lambda^2_{k,m,n}} t\right) < \varphi_{k,m,n}, w_1 > \varphi_{k,m,n} + \int_0^t V_1^{t-s} A_1^{-1} Q y(s) ds. \quad (12)$$

By virtue of Theorem 4, the solution given by formula (12) is strong. Define the control space

$$H^2(\mathfrak{U}) = \{u \in L_2(0,\tau;\mathfrak{U}) : \ddot{u} \in L_2(0,\tau;\mathfrak{U})\},$$

and consider the closed and convex subset $\mathfrak{U}_{ad} = H^2_\partial(\mathfrak{U})$, which is the set of admissible controls. The main result of the paper is the proof of the existence of the unique control $\hat{u} \in H^2_\partial(\mathfrak{U})$ minimizing the functional $J(w,u)$. Fix $w_0, w_1 \in \mathfrak{X}$ and consider (13) as the map $D : u \to w(u)$. Then the map $D : H^2(\mathfrak{U}) \to H^2(\mathfrak{X})$ is continuous. Therefore, the quality functional depends only on u, i.e. $J(w,u) = J(u)$.

Rewrite quality functional (11) in the form

$$J(u) = \|w(t,u) - \tilde{w}\|^2_{H^2(\mathfrak{X})} + [v,u],$$

where $v^{(q)}(t) = N_q u^{(q)}(t),\ q = 0, 1, 2$. Hence

$$J(u) = \pi(u, u) - 2\lambda(u) + \|\tilde{w} - w(t, 0)\|^2_{H^2(\mathfrak{X})},$$

where

$$\pi(u, u) = \|w(t, u) - w(t, 0)\|^2_{H^2(\mathfrak{X})} + [v, u]$$

is a bilinear continuous coercive form on $H^2(\mathfrak{U})$, and

$$\lambda(u) = \langle \tilde{w} - w(t, 0), w(t, u) - w(t, 0)\rangle_{H^2(\mathfrak{X})}$$

is a linear continuous form on $H^2(\mathfrak{U})$. Therefore, the conditions of Theorem 1.1 proved in [12] are satisfied.

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Konstantin Yu. Kotlovanov, PhD Student, Department of Mathematical Equations Physics, South Ural State University, Chelyabinsk, Russian Federation, kotlovanovki@susu.ru.

Evgeniy V. Bychkov, PhD (Math), Associate Professor, Department of Mathematical Equations Physics, South Ural State University, Chelyabinsk, Russian Federation, bychkovev@susu.ru.

Alexey V. Bogomolov, DSc (Techn), Professor, Intelligent Systems Laboratory, St. Petersburg Institute for Informatics and Automation of RAS, Saint-Petersburg, Russian Federation, a.v.bogomolov@gmail.com.

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ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ В МАТЕМАТИЧЕСКОЙ МОДЕЛИ ВНУТРЕННИХ ВОЛН

К. Ю. Котлованов, Е. В. Бычков, А. В. Богомолов

В статье представлены результаты исследования задачи оптимального управления решениями для математической модели внутренних волн, построенной на основе линейной системы уравнений гидродинамики. Данная модель описывает распространения волн в однородной несжимаемой стратифицированной жидкости. Математическая модель включает в себя уравнение Соболева, условие Коши и Дирихле. В качестве рассматриваемой области в математической модели используется параллелепипед. В данной работе показано существование и единственность сильного решения задачи Коши – Дирихле для уравнения Соболева. Получены достаточные условия существования и единственности решения задачи оптимального управления такими решениями в гильбертовых пространствах. Доказательство существования единственного сильного решения основано на теореме для абстрактного неполного неоднородного уравнения соболевского типа второго порядка и теории относительно p-ограниченных операторов. Приведенная в данной работе теорема существования и единственности оптимального управления для исследуемой задачи основана на работах Ж.-Л. Лионса.

Ключевые слова: уравнения соболевского типа; относительно p-ограниченный оператор; сильное решение; оптимальное управление.

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Котлованов Константин Юрьевич, аспирант, кафедра уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), kotlovanovki@susu.ru.

Бычков Евгений Викторович, кандидат физико-математических наук, кафедра уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), bychkovev@susu.ru.

Богомолов Алексей Валерьевич, доктор технических наук, профессор, Лаборатория автоматизации научных исследований, СПИИРАН (г. Санкт-Петербург, Российская Федерация), a.v.bogomolov@gmail.com.

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