MSC 05C38

THE ALGORITHMS FOR CONSTRUCTING OF CUTTER OPTIMAL PATH

T. A. Makarovskikh, South Ural State University, Chelyabinsk, Russian Federation, kwark@mail.ru

This paper considers methods of automatic cutter path choice for laser equipment. Graphs are often used as mathematical models for different control problems or automated design. Particularly, for automated system of cutting process preparation the mathematical model of a cutting plan should be a topological plane graph. Nowadays a branch of graph theory dealing with constructing some paths and trails with different restrictions is rapidly developing. This research considers the following cutting problem formulated in the terms of graph theory. We need define a shortest cutter trajectory so that part cut out a sheet does not require further cuttings. If one considers the cutter trajectory to be a trail of a plane graph, the requirement of eliminating the necessity of cutting of a piece separated from the sheet can be formalized as the condition that internal faces of any initial part of the given trail does not intersect the graph edges. The polynomial algorithms presented in the paper allow to solve the routing problem either for connected or disconnected plane graph.

Keywords: Plane graph, Effective algorithm, Cutting process, Ordered enclosing, Path.

Introduction

Great number of industrial branches is dealing with cutting the material such as metal, wood, plywood, glass and others. These materials are presented in industrial flow as sheets, boards, pipes, profiled rolls etc. Obviously, the usage of these materials implies their separation (or cutting) on parts of the given size and form (the samples of some details).

That is why the significance of industrial cutting be a source of economy and it is mentioned in technical literature and some industrial journals [1]. Theoretically this field is rather unexplored. There are some researches devoted to maximization of wood volume at lumbering process. The well known problem of circles placement on a plane is also similar to a cutting problem for a infinity sheet and equal round billets.

The experience of advanced engineering plants shows that accurate planning of cutting process allows to achieve the economy of materials [1].

Nowadays the problems of creation the high-effective technologies of social sphere development, flexible automated enterprises on the base of information technologies, particularly, clothes production for individuals are actual.

And the branch connected with flexible automated enterprises of customer goods on the base of information technologies is officially proclaimed as one of the priority branches of science development.

The process of any product development begins from a creation of some details being their constructive parts. By the way the stage of their production is to be one of the most laborious and complex of all production cycle. As a matter of fact the volume of details nomenclature is rather big and there exist some problems of technological projecting of cutting-and-preparing operations [2]. More than one half of technological equipment with computer numerical control (CNC) used for sheets cutting in Russia and some other countries are the thermal cutting machines (TCM). This class of machines includes also some equipment for gas (oxygen), plasma, laser and EDM cutting. So industrials and projecting enterprises dealing with cutting and packing problems need usage of some automated cutting systems for flat details. Usually such a system has unit structure and each its unit allows to automate some stages of cutting process.

While cutting the material into some figured details one of the significant stages of preparation the details is constructing of cutter movement trajectory. Optimization of this trajectory may considerably reduce the cost of a cutting. Usually this optimization means the minimization of idling length. One more optimization criterion is the number of incuts on a sheet. It is shown in [2] that according to the type of material and cutting technology the cost of one incut achieves 20–30% of cutting cost. Minimization of thermal deformations is also one of significant optimization criteria. Convent heuristic rules used for algorithm of automated generation of control programs for TCM are presented in [3].

A large number of technological equipment used for processing of a sheet material consists of CNC laser cutting machines. Computer-Aided Manufacturing (CAM) systems are used for development of control programs for these machines. They allow decreasing time of programs development, increase the accuracy of processing and decrease the cost of cutting. Modern CAM systems are oriented on automated development of control programs and as usual include two modes, interactive and automatic. The researches held nowadays in Russia concern basically optimization of a cutter moving path on a time criterion by minimizing the idle path of a cutter with use of standard way of cutting where the number of incuts is equal to number of details to be cut. However there is a lack of algorithms for minimization of cost criteria (for example, saving the energy of a cutter). Summary time of uncommon ways of cutting is also poorly explored. That is why this paper considers methods of automatic cutter path choice for laser equipment.

Routing problems are often used as mathematical models for different control problems or automated design. Particularly, for automated system of cutting process preparation the mathematical model of a cutting plan should be a topological plane graph. The interest to routing problems can be explained by their frequent usage as mathematical models of different control problems.

Nowadays a branch of graph theory dealing with constructing some paths and trails with different restrictions is rapidly developing.

Let's consider the following cutting problem. We need define a shortest path of cutter trajectory so that part cut out a sheet does not require further cuttings. If one considers the cutter trajectory to be a trail of a plane graph, the requirement of eliminating the necessity of cutting of a piece separated from the sheet can be formalized as the condition that internal faces of any initial part of the given trail does not intersect the graph edges. In [4], the author formulates and solves the problem of constructing (in a plane Eulerian graph) Eulerian cycles that satisfies the given condition. These are formally defined as cycles with v-ordered enclosing where v is a vertex incident to the external (infinite) face of the graph. This paper considers the most common case of this problem where graph is non-Eulerian and disconnected. The algorithms presented in the paper allow to solve the routing problem for any type of plane graph. As a matter of fact, there exist many models and algorithms for constructing Eulerian trails in a graph. Lots of them can be derived from one general algorithm [5] called Splitting algorithm. Speaking about constructing Eulerian trails for digraphs or trails with some restrictions, there are some other algorithms for solving these problems not based on Splitting algorithm. The restrictions on the order of vertices and edges in a trail can be classified as local (the next edge of a path is defined by conditions established at the current vertex or edge [5]–[8]), and global (Eulerian, Hamiltonian cycles, bidirectional double tracing etc.).

1. Main Definitions and Conceptions

Let's consider S as a plane; $G = (V, E)$ as a plane graph. Let f_0 be the outer face of G. Let for any part of this graph $J \subseteq G$ the set-theoretic union $\mathsf{Int}\,(J)$ of its inner faces be defined (i.e. the union of all connected components $S \setminus J$ not containing outside face f_0). The sets of vertices, edges, and faces of graph J let be designated as $V(J)$, $E(J)$, and $F(J)$ correspondingly, and $|M|$ be the number of elements of a set M.

To avoid loss of generality let's consider some basic definitions and proved earlier properties of Eulerian cycles and covers with ordered enclosing.

Definition 1. *[9] Let cycle $C = v_1e_1v_2e_2\dots v_k$ for Eulerian graph G be called as **a cycle with ordered enclosing** (or OE-cycle for short) if $\mathsf{Int}\,(C_l) \cap E = \emptyset$ for $C_l = v_1e_1v_2e_2\dots v_l$, $l \leq |E|$.*

For example, cycle $e_1e_3e_2e_4e_5e_6$ in fig.1 has ordered enclosing, and cycle $e_4e_5e_6e_1e_3e_2$ has not because its first three edges $e_4e_5e_6$ enclose the three edges that are not passed yet.

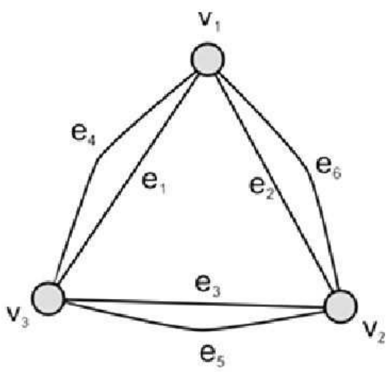

Fig. 1. The example of an Eulerian graph

Definition 2. *[10] Let minimal cardinality sequence of such edge-disjoint OE-trails*

$$C^0 = v^0e^0_1v^0_1e^0_2...e^0_{k_0}v^0_{k_0}, \quad C^1 = v^1e^1_1v^1_1e^1_2...e^1_{k_1}v^1_{k_1}, \quad \dots,$$

$$C^{n-1} = v^{n-1}e^{n-1}_1v^{n-1}_1e^{n-1}_2\dots e^{n-1}_{k_{n-1}}v^{n-1}_{k_{n-1}}$$

that

$$(\forall m : m < n) \quad \left(\bigcup_{l=0}^{m-1} \mathrm{Int}(C^l)\right) \cap \left(\bigcup_{l=m}^{n-1} C^l\right) = \emptyset$$

*be **an Eulerian cover with ordered enclosing** for plane graph $G = (V, E)$ (or OE-cover for short).*

The most interesting are the covers with minimal number of trails with minimal length of edges connecting these trails.

Definition 3. *[11] Let minimal cardinality sequence of edge-disjoint trails with ordered enclosing in plane graph G be called **Eulerian cover with ordered enclosing** (Eulerian OE-cover).*

All algorithms presented below use the following representation of plane graph $G = (V, E)$ up to homeomorphism. Graph G is unambiguously defined by the following functions for each edge $e \in E$ [12]:

1) $v_1(e)$, $v_2(e)$ be the vertices incident to edge e;
2) $f_k(e)$ be a face laying at left when one moves by edge e from vertex $v_k(e)$ to vertex $v_{3-k}(e)$, $k = 1, 2$,
3) $l_k(e)$ be an edge belonging to face $f_k(e)$ and incident to vertex $v_k(e)$, $k = 1, 2$.

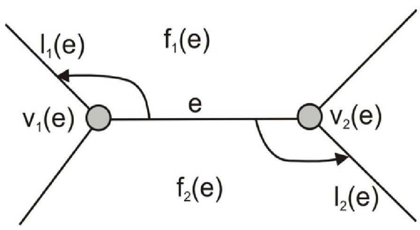

Fig. 2. The representation of plane graph

Illustration of defined functions is represented in fig. 2. Construction of these functions is rather easy. In fact they are defined and used on the stage of graph G projecting. Spatial complexity of this representation is $O\left(|E| \cdot \log_2 |V|\right)$.

The existence of Eulerian OE-cycles for plane Eulerian graphs is proved in papers [12], [9]. Recursive algorithms for constructing of such cycles are represented in paper [12]. The problem of constructing the OE-path for any plane graph is reviewed in [10, 13]. There the algorithm for constructing of such paths is presented and it is proved that the computing complexity of this algorithm is not more than $O\left(|E|^2\right)$. The paper [9] presents the effective algorithm for constructing OE-cycles for plane Eulerian graphs. This algorithm computing complexity is $O\left(|E| \cdot \log_2 |V|\right)$. Abstract [11] and paper [14] are devoted to construction of optimal Eulerian OE-cover for a plane connected graph. As for Eulerian OE-cover for a disconnected plane graph the polynomial algorithm for constructing of such trails was presented on the Seventh Czech-Slovak Symposium on Graph Theory, Combinatorics and Applications [15].

The aim of this paper is presentation of algorithms for constructing of OE-covers for any plane graph without end-vertices. But first of all let's consider optimal algorithm for connected plane graphs (for proofs of its correctness see [10]) because it is cited later at the text of algorithm for a disconnected graph.

2. The Shortest Length OE-Cover

An optimal Eulerian OE-cover can be constructed by algorithm OptimalCover. This algorithm uses three procedures: Initialisation, Ordering, and Forming. To review this algorithm we need the definition of edge e rank.

Definition 4. *The rank of edge e for graph $G(V, E)$ can be defined recursively:*

1. *All the edges restricting the outside face f_0 of graph $G(V, E)$ are forming a set of edges*
$$E_1 = \{e \in E : \quad e \subset f_0\}$$
with rank $(\forall e \in E_1)\,(rank(e) = 1)$.

2. *Edges with rank 1 for graph*
$$G_k\left(V, E \setminus \left(\bigcup_{l=0}^{k-1} E_l\right)\right)$$
are forming a set E_k of edges with rank k for initial graph G, i.e. $(\forall e \in E_k)\,(rank(e) = k)$.

The rank of each edge (the example of ranking is shown in fig.3) of plane graph can be defined by the time $O(|E|)$ using procedure Ordering quoted in [10] and other algorithms for OE-trails constructing.

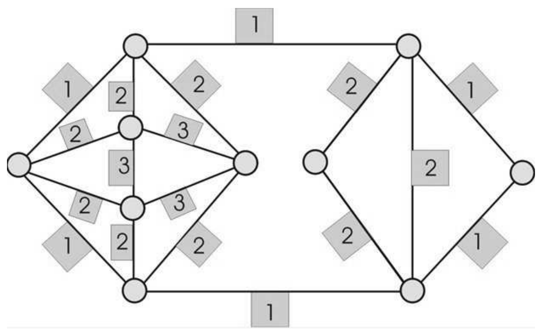

Fig. 3. Ranking the edges of graph

Let's designate a set of odd vertices as V_{odd}. Obviously, complexity of such set construction with used graph representation is not more than $O(|E|)$.

The functional aim of Forming procedure is construction of a trail with ordered enclosing ending in vertex $v \in V_{odd}$. As a result of this procedure one will have a simple trail $C = v_0 e_1 v_1 e_2 ... e_k v_k$ where

$$v_1, v_2, ... v_{k-1} \notin V_{odd}, \quad v_0, v_k \in V_{odd},$$

$$e_i = \arg \max_{e \in E(v_i) \setminus \{e_l | l < i\}} \text{rank}(e),\ v_{i+1} = \overline{v_1}(e_i),\ i = 1, 2, \ldots, k,$$

moreover for any its initial part $C_l = v^0 e_1 v_1 e_2 v_2 \dots e_l$, $l \le k$ and for any vertex $v \in V$ the following inequality takes place:

$$\min_{e \in E(v) \bigcap E(C_l)} \mathrm{rank}(e) > \max_{e \in E(v) \setminus E(C_l)} \mathrm{rank}(e).$$

The possibility for effective constructing of optimal OE-cover gives the following theorem.

Theorem 1. *Let $G = (V, E)$ be a plane connected graph topologically represented on a plane S. Let G has no end-vertices. For any such a set M being a matching on the set V_{odd} of G that M : $(M \cap S) \backslash V = \emptyset$ there exists such an Eulerian cycle $C = v_1 e_1 v_2 e_2 ... e_n v_1$, $n = |E| + |M|$ that for any its initial part $C_l = v_1 e_1 v_2 e_2 ... v_l$, $l \le |E| + |M|$ the condition $\mathsf{Int}(C_l) \cap E = \emptyset$ holds.*

The proof of this theorem is also constructive and consists of proof of effectiveness for the following algorithm. To construct the optimal cover it's enough to take the shortest matching on set V_{odd} as M.

Algorithm OptimalCover

Input:

- plane graph G represented by a list of edges with defined functions $v_k(e)$, $l_k(e)$, $f_k(e)$, $k = 1, 2$;
- the shortest matching M on a set of odd degree vertices V_{odd}.

Output: C_j, $j = 1, ..., |V_{odd}|/2$ be an Eulerian OE-cover of G by trails.

Step 1. Initialization

- Define the initial values of all variables (the first edge and vertex, all edges be unmarked, and all lists be empty).

Step 2. Ordering

- $\forall e \in E(G)$ define rank(e) (eg., fig.3);
- $\forall v \in V(G)$ form a list of adjacent edges. Sort the edges e by decreasing of rank(e).

Step 3. Let $j = 1$. Let vertex $v_0 \in f_0$ be the current one and let $v_0^j = v_0$.

Step 4. Proceed construction of trail C_j by procedure $\mathsf{Forming}$ using v_0 as initial: construct a trail beginning at this vertex and ending at another vertex of odd degree. Let v^0 be the last vertex of a trail constructed. If $v^0 \notin V_{odd}$ go to step 8; otherwise go to step 5.

Step 5. If v^0 is a dead end then go to step 7, otherwise go to step 6.

Step 6. If rank$(v_1) <$ rank(v^0) then let $v_0 = v^0$ and go to step 4 (keep constructing the trail C_j from the current vertex).

Step 7. Define v_1 : $(v^0, v_1) \in M$. Finish constructing the current trail: $v_1^j = v^0$, $j = j + 1$, $V_{odd} = V_{odd} \setminus \{v^0, v_1\}$, $M = M \setminus \{(v^0, v_1)\}$ let $v_0^j = v_1$ be a curret vertex of the next trail and go to step 4 (begin constructing a new trail C_j from vertex $v_0 = v_0^j$).

Step 8. End of algorithm.

Computing complexity of algorithm **OptimalCover** is less than $O(|V|^3)$ (i.e. matching problem complexity for a complete graph).

Let's consider the example of this algorithm execution (fig.4).

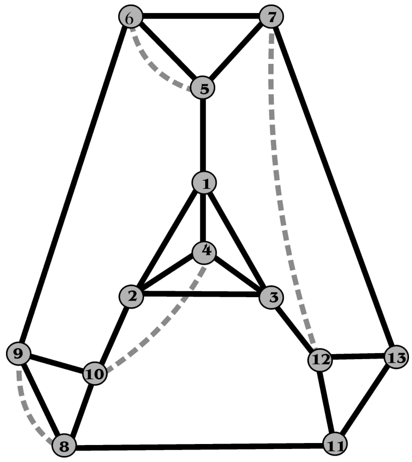

Fig. 4. Optimal Eulerian OE-cover for a connected graph. The trails are: $T_1 = \{11-12-3-1-2-3-4\}$; $T_2 = \{10-2-1-5-6\}$; $T_3 = \{5-7\}$; $T_4 = \{12-13-7-6-9\}$; $T_5 = \{8-10-9-8-11-13\}$.

3. Algorithms for a Disconnected Graph

The most interesting is the case of optimal cover with minimal length of additional edges connecting the ends of trails for a disconnected graph. Algorithm for constructing the allowed Elerian cover with ordered enclosing for this type of graphs is suggested in [16]. Let's present the algorithm for constructing such an OE-trail. This algorithm applies the concept of ranking as in earlier papers (for example, in [17]).

Definition 5. *Let the* ***rank of connected component*** *be the minimal rank of this component's edges.*

Definition 6. *Let the* ***enclosed union*** *n be a family of connected components S_n of plane graph where a component of rank k contains (encloses) only the components of rank more than k.*

So for graph in fig.5 the components bounded by cycles 1-2-3-4-1 and 8-9-10-11-8 have ranks equal to 1, and components 5-6-7-5 and 12-13-14-12 have ranks equal to 2.

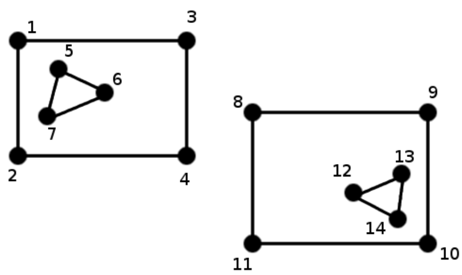

Fig. 5. The example of enclosed unions

Algorithm AllowedMultiComponentCover

Input:

plane graph G.

Output: Graph G covering C_j^s, $j = 1, ..., |V_{odd}|/2$ by trails with ordered enclosing, $s = 1, 2, ...$ be the index of component.

Step 1. Recognize a set S of all components of graph G and $\forall s \in S$ define the nesting value $K(s)$.

Step 2. Sort the elements of set S by decreasing of $K(s)$.

Step 3. Let $i(s)$ be an index of a component s, and $s(i)$ be the component of index i. Run algorithm OptimalCover $1 \le i \le |S|$ and get the OE-covering C_j^i of i-th component by trails.

Step 4. End of algorithm.

Computing complexity of this algorithm is $O(|V^2|)$.

Unfortunately, algorithm allows constructing only permissible cover with mentioned properties. It passes the components from the most inserted one and includes components to a cover set sequentially according to their rank (fig.6). Nevertheless, it is obvious that the better result can be received if one passes components not only according to their rank but also according to the distance between different components. That is why let's consider some algorithms optimizing the distance between components. So the main idea is that it's impossible to switch over to another enclosed union if there are some edges of the current one not included to any trail of being constructed covering. The problem of enclosed unions definition can be solved on the stage of components recognition. So it has the same computing complexity as well known wave algorithm.

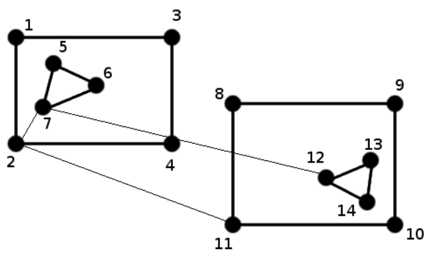

Fig. 6. Constructing of allowed Eulerian OE-cover for a disconnected graph

Let's introduce the algorithm allowing to get the better solution.

Algorithm DisconnectedCover

Input:

plane graph G.

Output: C_j^s, $j = 1, ..., |V_{odd}|/2$ be OE-covering of graph G by trails, $s = 1, 2, ...$ be an index of a component.

Step 1. Recognize a set S of all components of graph G and $\forall s \in S$ define the nesting value $K(s)$.

Step 2. $\forall s \in S$ define the shortest matching on the set of odd degree vertices.

Step 3. Construct an abstract graph $\mathrm{Im}:$ let its vertices be the components S of G, and lengths of edges are equal to the distance between the nearest vertices of corresponding components.

Step 4. Define minimal spanning $T(\mathrm{Im})$.

Step 5. Add a digon for each edge of MST to graph G: $G_{\mathrm{Im}} = G \cup T(\mathrm{Im})$.

Step 6. Run algorithm **OptimalCover** for graph G_{Im}.

End of algorithm DisconnectedCover.

The computing complexity of this algorithm is also $O(|V|^3)$. The example of this algorithm implementation is shown in fig.7. It is obvious that the length of additional edges is better than for allowed algorithm (fig.6).

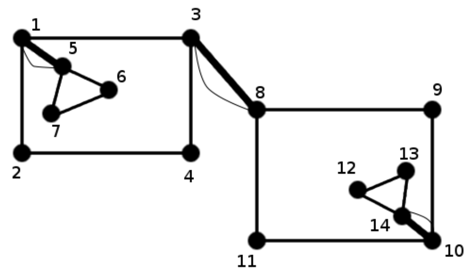

Fig. 7. The example how algorithm OptimalDisconnectedCover runs

Conclusion

So the research gives different polynomial algorithms for constructing the cover with ordered enclosing for a disconnected graph.

References

1. Kantorovich L.V., Zalgaller V.A. [*Rational Cutting of Industrial Materials*]. St. Peterburg, Nevskij Dialekt publ., 2012. (in Russian)

2. Verkhoturov M.A., Tarasenko P.Yu. [Mathematical Support for a Path Optimization Problem of a Plane Figure Circuit Cutting]. *Vestnik UGATU. Seriya "Upravlenie, Vychislitelnaya Tekhnika i Informatika" – Bulletin of USATU. Series "Control, Computing and Informatics"*, 2008, vol. 10, no. 2(27), pp. 123–130. (in Russian)

3. Petunin A.A. [About Some Heuristic Strategies Forming a Cutter Path in Development of Control Programs for Thermal Cutting Machines]. *Vestnik UGATU. Seriya "Upravlenie, Vychislitelnaya Tekhnika i Informatika" – Bulletin of USATU. Series "Control, Computing and Informatics"*, 2008, vol. 13, no. 2(35), pp. 280–286. (in Russian)

4. Pisanski T., Tucker T.W., Zitnik A. Straight-ahead walks in Eulerian graphs. *Discrete Mathematics*, 2004, no. 81, pp. 237–246. DOI: 10.1016/j.disc.2003.09.011

5. Fleischner H. *Eulerian Graphs and Related Topics: Part 1, Volume 2. Series "Annals of Discrete Mathematics", vol. 50.* Amsterdam, North Holland Publ., 1991. DOI: 10.1016/S0167-5060(08)70153-5

6. Fleischner H. *Eulerian Graphs and Related Topics: Part 1, Volume 1. Series "Annals of Discrete Mathematics", vol. 45.* Amsterdam, North Holland Publ., 1990. DOI: 10.1016/S0167-5060(08)70942-7
7. Fleischner H., Beineke L.W., Wilson R.J. *Eulerian Graphs, Selected Topics in Graph Theory 2.* London, New-York, Academic Press, 1983.
8. Kotzig A., Moves Without Forbidden Transitions in a Graph. *Mat.-Fiz. Casopis*, 1968, vol. 18 , no. 1, pp. 76–80.
9. Panyukov A.V., Panioukova T.A. The Algorithm for Tracing of Flat Euler Cycles with Ordered Enclosing. *Proceedings of Chelyabinsk Scientific Center*, 2000, no. 4(9), pp. 18–22.
10. Panyukova T. Cover with Ordered Enclosing for Flat Graphs. *Electronic Notes in Discrete Mathematics*, 2007, no. 28, pp. 17–24. DOI: 10.1016/j.endm.2007.01.004
11. Panyukova T. Ordered Enclosing Covers with Minimal Length of Additional Edges. *Abstracts of 8th French Combinatorial Conference: Orsay, France (June 28 – July 2, 2010).* Orsay, France, 2010, abstract no. 18.
12. Panioukova T.A., Panyukov A.V. Algorithms for Construction of Ordered Enclosing Traces in Planar Eulerian Graphs. *The International Workshop on Computer Science and Information Technologies' 2003, Proceedings of Workshop, Ufa, September 16–18*, Ufa, USATU, 2003, vol. 1, pp. 134–138.
13. Panyukova T. The special routes in plane graphs. *Special Issue: Sixth International Congress on Industrial Applied Mathematics (ICIAM07) and GAMM Annual Meeting, 2007.* Zurich, 2008, vol. 7, issue 1, pp. 2070015–2070016. DOI: 10.1002/pamm.200701066.
14. Panyukova T. Covering with Ordered Enclosing for a Multiconneted Graph. *Abstracts of Seventh Czech-Slovak Symposium on Graph Theory, Combinatorics and Applications: Koshice, Slovakia (July 7–13, 2013)*, Koshice, 2013, p. 65.
15. Panyukova T.A. [Optimal Eulerian Covers for Plane Graphs] *Discrete Analysis and Operation Research*, 2010, vol. 18, no. 2, pp. 64–74. (in Russian)
16. Panyukova T., Savitsky E. [Allowed Eulerian Covers with Ordered Enclosing for Multiconnected Graph]. *Statistika. Modelirovanie. Optimizatsiya. Vserossijskaya nauchno-prakticheskaya konferentsiya. CHelyabinsk. YUzhno-Ural'skij gosudarstvennyj universitet* [*Statistics. Modelling. Optimization. Proceedings of All-Russian conference, Chelyabinsk, South Ural State University*], Chelyabinsk, SUSU, 2011, pp. 154–159. (in Russian).
17. Panyukova T., Savitskiy E. Optimization of Resources Usage for Technological Support of Cutting Processes. *Proceedings of the Workshop on Computer Science and Information Technologies (CSIT'2010), Russia, Moscow – St.Petersburg, September 13–19, 2010*, Ufa, USATU, 2010, vol. 1, pp. 66–70.

Tatiana A. Makarovskikh, Candidate of Physics and Mathematics Science, Associate professor, Department of Economic and Mathematical Methods and Statistics, South Ural State University, Chelyabinsk, Russian Federation, kwark@mail.ru.

Received September 15, 2014