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CHANNEL-NETWORK CASCADE FOR PACKET AND SYMBOL ERASURES IN BINARY LINEAR NETWORK

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This paper covers the problems of packet and symbol erasures in binary linear networks. A method based on sequential cascading of network and channel codes to resist symbol erasures is described. The channel code is considered to be a code based on equal-weight columns. A non-linear subcode of rank code is used as a network code. Encoding and decoding algorithms for cascading code are constructed. A simulation model has been developed. With the help of this model experiments have been carried out to show the effectiveness of the presented method.

Keywords: network encoding; code cascade; equal-weight columns; symbol erasure; packet erasure.

Introduction

In modern networks the speed of information transmission is one of the primary values. Construction of high-speed networks such as 4G or 5G poses a serious challenge to developers for optimizing information transmission processes, their protection from interference to achieve maximum data flow speeds. Thus, it is necessary to find fast ways to encode and decode data.

Deterministic linear networks are defined as networks where linear combinations of packets are calculated at internal nodes. This modification of data transmission networks is intended to increase the network bandwidth [1]. A random linear network is a linear network in which neither the recipient nor the sender is aware of both the network structure and the coefficients of linear combinations of packets calculated in the internal nodes of the network. It is convenient to consider a "foreign" deterministic network as random [2].

In linear networks, you can solve various problems of anti-interference and protection from unauthorized access. The main problems and published results are presented in [1]. The description of codes that effectively deal with batch errors and losses is presented in [3, 4]. The main tool for data encoding is rank codes, which were first described in [5]. Nonlinear sub-codes of rank codes are considered in [2]. However, in the case of character erasures, the entire packet is often considered lost, which reduces the likelihood of successful decoding. An alternative to this approach is considered in [6], where two basic theoretical models are constructed: in the case of a small number of erasures, it is proposed to use cascading network code with binary channel code, and in the case of a large number of erasures, the method of expanding the alphabet. It should be noted that in the case of a binary field, the choice of an encoding matrix to deal with erasures is somewhat difficult [2].

This paper provides the problems which are connected with packet and symbol erasures in binary linear networks. We developed a method dealing with character erasures based on a cascading connection of network and channel codes, and built a corresponding simulation model that allowed us to test the new method experimentally. The article is organized as follows: the first section provides the necessary information about channel and network encoding, the second section describes a method for constructing a cascade of network and channel codes, appropriates algorithms for encoding and decoding data, provides simulation model of cascading network encoding. The third section is devoted to the discussion of experimental results.

1. Preliminaries on Network and Channel Codes

The necessary information on network encoding is presented in section 1.1, raised rank codes and their nonlinear sub-codes are discussed in section 1.2, and channel encoding based on the equilibrium column method (section 1.3) is exposed below.

1.1. The Scheme of Network Coding

In [1, p. 3] it is noted that network coding is based on modification of the model of information flow of data transmission over the communication network. The communication network is formally understood as a finite directed graph in which edges are channels of information transmission, vertices are nodes, and communication nodes are divided into source nodes, recipient nodes and internal nodes. The alphabet of the transmitted messages is the Galois field F_q. There is a problem of transmitting data from all sources to all recipients as quickly as possible and with the minimum number of errors.

Linear network coding differs from the usual way of transmitting data over the network by making linear combinations of packets in internal nodes. This variant is more efficient for data transmission. The problem of anti-interference has a number of features in the case of network coding. In addition to classical errors and packet losses, we also consider packet acquisitions, deviations, symbol errors, and symbol erasures [7]. This paper is connected with the problems of symbol and packet erasures. Note, that a package with symbol erasure inside is usually excluded from further transformations, i.e. it is replaced with a packet erasure. This causes additional difficulties in the decoding process, because the number of packet erasures can be very large. However, it is quite difficult to track symbol erasures. It is often necessary to equip all internal nodes with codecs, which is quite expensive operation for the network and slows down the data transfer process.

The main tool for anti-interference is Gabidullins' rank codes [5], which were modified and applied in network coding in the work [4]. Note, that the using of rank codes is accompanied by an expansion of the field. Non-linear approaches of rank codes are also used to overcome packet erasures [2, 4, 5]. It is possible to combine channel and network codes (see, for example, [6]). In this paper, a nonlinear sub-code of the rank code from [2] is selected as the network code, and the code based on the equal-weight columns [8] is considered as the channel code.

Consider the model of a linear network with one sender and several recipients, which is described in [4,9]. In this model, the alphabet is the field F_q. Matrices of size $n \times m$ over the field F_q will be denoted as $F_q^{n\times m}$ and matrices of the same size of rank t will be denoted as $F_q^{n\times m,t}$. Suppose there is one source in a linear network that sends n packets of length

$m(X_i \in F_q^m), i = 1, \ldots, n$. From these packets the source creates matrix $X \in F_1^{n\times m,t}$, which is passed in packets across the columns. The internal node of the linear network uses its own coefficients to make linear combinations of the received packets. Thus, each recipient receives n packets $Y_1, \ldots, Y_n$ of length m, from which the matrix $Y \in F_q^{n\times m}$ is formed. The recipient's task is to determine the original matrix X from the matrix Y. It is assumed that the matrices X and Y are connected by the following relation:

$$Y = AX + DZ,$$

where $X, Y \in F_q^{n\times m}$. The matrix $A \in F_q^{n\times n}$ specifies linear transformations of the source packets over the entire time of network data transmission. The matrix $D \in F_q^{n\times t}$ specifies linear transformations with erroneous packets on the path to the receivers, and the rows of the matrix $Z \in F_q^{t\times m}$ are erroneous packets. It is assumed that the matrices D, Z are unknown, and the matrix A can be both known and unknown.

This network coding model also can be used when there are letter erasures in the network, in which case, the network alphabet is expanded with either an asterisk $*$ or a more extensive auxiliary alphabet, and characters from the alphabet extension appear in the matrix Z [6].

1.2. One Class of Network Codes

Consider network codes from [2], which are non-linear approaches of rank codes from [4], adapted to a binary network. Enter the necessary network parameters and network code. Assume that L is the length of each packet, n is the maximum number of packets sent simultaneously, and F_2 is the network alphabet. Let k is the length of the information vector, m is the code redundancy coefficient, and $n = mk$. If n is not representable as a product of mk, then some number $n_0 \le n$ is chosen such that $n_0 = mk$. Suppose that $m + n \le L$.

Consider the encoding of the information vector $x(\in F_2^k)$. Let

$$B_i = (O \ldots O \; E \; O \ldots O) \in F_2^{k\times n}, i = 1, \ldots, m, \tag{1}$$

where E is the identity $(k \times k)$ matrix located at the i-th place, O is the zero $(k \times k)$ matrix. Calculate the vectors $z_i = xB_i = (z_1, z_2, \ldots, z_n), z_i \in F_2$. Consider the standard basis $\{e_j\}$ of the space F_2^n. Generate the vector columns $X^j = (e_j | z_{1,j}, z_{2,j}, \ldots, z_{m,j})$ of length $n + m$ for $j = 1, ..., n$. Make a code matrix X from the obtained vector columns:

$$X = \begin{pmatrix} 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ z_{1,1} & z_{1,2} & \cdots & z_{1,n} \\ z_{2,1} & z_{2,2} & \cdots & z_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{m,1} & z_{m,2} & \cdots & z_{m,n} \end{pmatrix} = \begin{pmatrix} E \\ z_1 \\ \vdots \\ z_m \end{pmatrix}.$$

The result mapping translates the information space F_2^k into the space of all code matrices Z. This is called the encoding mapping, and the set Z is called the network code. Note, that in the coding algorithm, the identity matrix is "glued" to the information vectors. The corresponding construction in the literature is called a lifting structure [3]. Using the lift construction allows us to get the matrix A at the output of the network, or some part of it, if errors occur in the network. If the matrix A is not completely obtained, it certainly complicates the process of decoding the information. Assume that $A \in F_2^{n\times r}$, where the parameter r corresponds to the network quality. Then the result of the network's action on the matrix X can be described as a matrix equality

$$Y = XA(\in F_2^{(n+m)\times r}).$$

1.3. Channel Encoding Method Based on the Equal-Weight Column

Consider the simplest model of data transmission in a binary noise-tolerant channel:

Source – Encoding – Channel – Decoding – Receiver

In this case, the physical channel generates interference such as erasure (see, for example, [7, p. 27]). Recall that erasure is an error in which the recipient accepts a word with the correct symbols and with the coordinate numbers in which the symbol was lost. For example, for a word $(a_1, a_2, a_3, a_4, a_5)$, getting the word $(a_1, *, a_3, *, a_5)$ means presence erasures in the second and fourth coordinates and the correct values in the other places. Thus, if there are erasures, the input alphabet is the field F_2, and the output alphabet is the set $F_2 \cup \{*\}$.

Encoding is the multiplication of the information message by the coding matrix G of full rank: $c = aG$.

The method based on equal-weight columns, presented in [8], is considered for constructing the coding matrix. Consider all vectors of fixed weight w and length k. Obviously that the number of all such vectors is equal to C_k^w. The method of constructing coding matrices is organized as follows: the parameter $n(\leq C_k^w)$ is chosen - the number of columns in the coding matrix, randomly generated n non-repeating vectors-columns of length k and weight w, from such vectors-columns of the matrix G. If $\mathrm{rank}(G) = k$, then the matrix is built, otherwise we repeat the procedure. Next, we use the method of decoding linear codes for information sets [10], adapted for channels with erasures [8]. As criterion for evaluating the quality of the constructed matrices, the probability vector of successful decoding from [11] is used (see [8]). Note, that the components of the probability vector of successful decoding for matrices constructed in the described way coincide and sometimes exceed the best similar results from [12]. This allows us to conclude that the described method is promising and can be applied to build cascading code. Encoding and decoding algorithms for the channel method based on equal-weight columns and experimental results are presented in [8].

2. The Cascade Model of Network Coding

2.1. General scheme

The network code in section 1.2 only can deal with packet erasures. For example, if there is even one symbol erasure in a package, we should exclude that package from future using. This creates additional difficulties if the decoding process, because the number of "non-erased" packets can be very small. The idea of our method is to build a channel-network cascade by connecting the network code from section 1.2 and the channel code from section 1.3 to simultaneously resisting with both packet and symbol erasures. Describe the general scheme of encoding and decoding such a cascade (see Fig. 1).

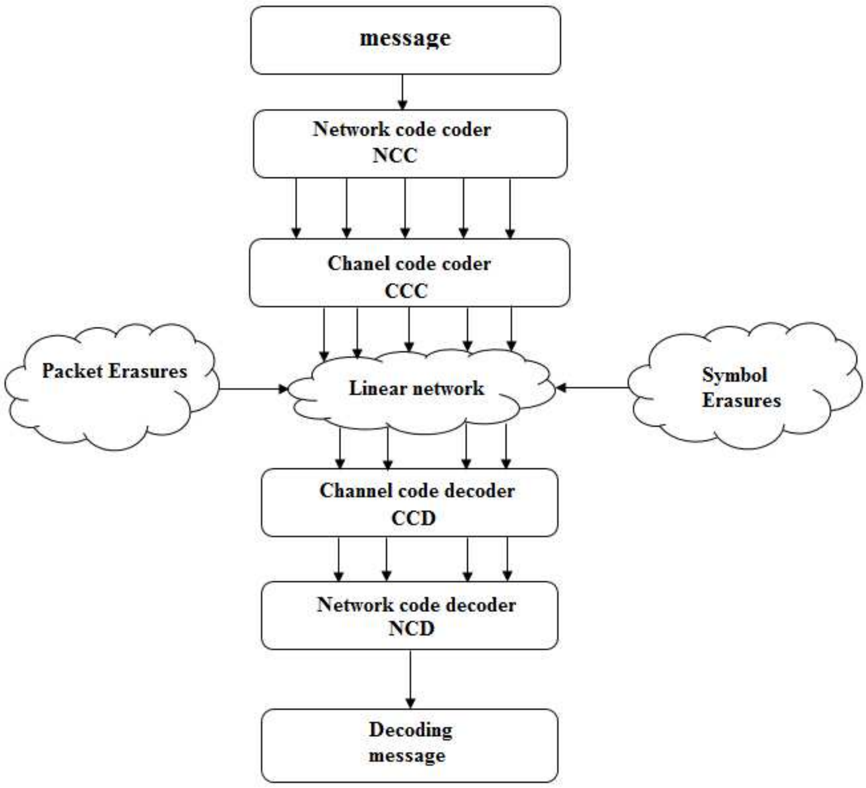

Fig. 1. The transmitting data process over a network using a cascade method

Consider a network code from the 1.2, with parameters (k, m), where k is the length of the information messages, m Џ the redundancy of code, and channel code from 1.3 with the coding matrix G of size $(n+m)\times(n+m+p)$, where p is the redundancy of the channel code, $n = km$ is the parameter of the network code. In **NCC** an information message of length k is transformed into a matrix X consisting of $n + m$ rows and n columns using the network code encoding algorithm. In **CCC** channel coding is applied to all columns of the matrix X. The result is the matrix $V(\in F_2^{(n+m+p)\times n})$, which is presented the encoded message. Then it is sent to the linear network (**LN**). Both symbol and packet erasures occur in the network. Channel decoding is applied to the received packets with symbol erasures in the **CCD**. If the channel code is not able to correct a certain number of character erasures, the packet is deleted. N-prefix-dependent packages are also deleted, i.e. only packages for which the set of prefixes of length n is linearly independent remain. Then, a matrix is formed from the remaining packets by columns. Then **NCD** is applied to this matrix. The output is a decoded information message.

Algorithms for encoding and decoding cascading code are given in section 2.2.

The main advantage of the described code cascade is flexibility when configuring for a specific network, which allows us to adjust the redundancy of the network and channel codes independently of each other. Thus, if a large number of symbol erasures occur in the network, we can separately increase the redundancy of the channel code. To resist packet erasures, the redundancy of the network code is separately increased by the user.

2.2. Encoding and Decoding Algorithms

The algorithms presented below for the channel-network cascade are based on a modification of the previously developed algorithms for the network and channel codes [2, 8].

Algorithm 1. The coding of cascade code.

Input: set of matrices $B_i(\in F_2^{k\times km}), i = 1,\dots,n$ from 1; encoding matrix $G(\in F_2^{(n+m)\times(n+m+p)})$ channel code; information message $x = (x_1, x_2, \dots, x_k)$.

Output: encoded message $V(\in F_2^{(n+m)\times m})$.

1. Calculate $z_i = xB_i = (z_{i,1}, z_{i,2}, \dots, z_{i,n}), z_{i,j} \in F_2, i = 1,\dots,m$.
2. Generate vectors $Z^j = (e_j|z_{1,j}, z_{2,j}, \dots, z_{m,j})$ of length $n+m$ for $j = 1,\dots,n$, where $\{e_j\}$ is the standard basis of the space F_2^n.
3. Calculate $V^j = Z^jG = (v_{j,1}, v_{j,2}, \dots, v_{j,n+m+p}), j = 1,\dots,n$.
4. Generate and return the encoded message V by placing the vectors V^j in columns:

$$V = \begin{pmatrix} v_{1,1} & \cdots & v_{n,1} \\ v_{1,2} & \cdots & v_{n,2} \\ \vdots & \vdots & \vdots \\ v_{1,n+m+p} & \cdots & v_{n,n+m+p} \end{pmatrix}.$$

Algorithm 2. The decoding of cascade code.

Input: set of matrices $B_i(\in F_2^{k\times km}), i = 1,\dots,n$ from 1; matrix $Y(\in F_2^{(n+m)\times r})$ coming from the linear network; coding matrix $G(\in F_2^{(n+m)\times(n+m+p)})$.

Output: information vector $x = (x_1, x_2, \dots, x_k)$ or decoding error message.

1. Remove n-prefix-dependent columns of the Y matrix.
2. Create a set $\widehat{Y} = (Y^{i_1}, Y^{i_2}, \dots, Y^{i_b})$ columns of the matrix Y, in which there were character erasures.
3. For each vector from $\widehat{Y}$:
   - 3.1. Make a set of $J = (j_1, j_2, \dots, j_m)$ coordinates of the vector Y^{i_j} obtained without erasures. If $m < k$, then go to the next vector.
   - 3.2. Make an ordered set $L = \{l_1, l_2, \dots, l_s\}, s = C_m^k$ of all combinations of k elements of the set J. $p := 1$.

3.3. If $p > s$, go to step 4. Construct a $(k \times k)$-matrix H_p whose columns are columns of the matrix G with numbers from l_p.

3.4. Calculate $\text{rank}(H_p)$. If $\text{rank}(H_p) < k$, then $p := p + 1$ and go to the next vector.

3.5. Calculate $a = b_p H_p^{-1}$, where $b_p = (b_{i_1}, b_{i_2}, \ldots, b_{i_k}), i_m \in l_p$.

4. Exclude all vectors Y^{i_j} for which step 3 of the algorithm did not work from further consideration. The remaining number of columns is r.

5. If $mr > k$, then return the decoding error and exit the algorithm.

6. From the first n rows of the matrix Y, form the matrix $A(\in F_2^{n\times r})$.

7. Calculate $A_i = B_i A, i = 1, \ldots, m$, form $\widetilde{A} = (A_1|A_2|\ldots|A_m)$.

8. Compute $\widetilde{r} = \text{rank}(\widetilde{A})$. If $\widetilde{r} < k$, then return the decoding error and exit from the algorithm.

9. Solve the following matrix equation with respect to z:

$$z\widetilde{A} = (Y_{n+1}|Y_{n+2}|\ldots|Y_{n+m}).$$

10. Return $z = (z_1, z_2, \ldots, z_k)$.

Note, that algorithm 2 uses a channel decoder for messages with symbol erasures. Further, all corrected packets and packets received without erasures are submitted to the input of the network code decoding algorithm, where Y_j is the j-th row of the Y matrix.

2.3. Simulation Model of Cascade Coding

In the course of the study, a simulation model of cascade encoding with packet and symbol erasures was constructed.

This model implements algorithms for encoding and decoding cascading code, simulation of data transmission over the network, and algorithms for implementing erasures. An information message is received at the input of the encoding algorithm. To simulate the transmission of a message over the network, a random matrix of the full-rank network is created, the encoded message is multiplied by this matrix, then packet and character erasures occur randomly, from which an erasure matrix is formed that simulates external influences. The input of the decoding algorithm receives the result of the information message passing through the network, taking into account external influences. However, erased symbols are not used during decoding.

The constructed model has several modes of erasures simulation. Erasures can occur with some probability, i.e. the erasure of a particular package or symbol occurs with some configurable probability p. There is a mode of simulation of erasures, in which the number of erasures is set by the user, and the places are determined randomly.

The simulation model is implemented using a graphical interface, so the user can observe the main stages of the encoding and decoding process. In addition, the choice of encoding type is implemented: channel, network, combination.

The software implementation for conducting experiments was created in C++ using the QT framework and the library to work with Galois fields NTL [14].

The simulation model allows us to explore the cascading code and conduct experiments. Some experimental results are presented in the next section.

3. Experimental Study of the Simulation Model

This section presents the results of experiments using a simulation model for cascading and classical network methods in the presence of symbol and packet erasures.

3.1. Experimental Study of Cascading Code with Packet and Character Erasures

For each set of parameters, 100 iterations of imitating the transmission of messages over the network were performed. At the end of each iteration, the message received after decoding was compared with the information message. Table 1 shows the percentage of decoding success that depends on two characteristics of symbol erasures: the total number of packets that received symbol erasures, and the maximum number of symbol erasures that occurred per packet.

Table 1

The percentage of successful decoding of cascade code

Symbol erasures		Successful decoding, %
Erased packets	Erasures in one packet	
0-5	1-4	100
6	1-3	100
6	4	99
7	1	98
7	2	99
7	2	99
7	3	97
7	4	98
8	1	98
8	2	90
8	3	69
8	4	60
9	1	83
9	2	63
9	3	33
9	4	0

The last column of table 1 shows the success rate of decoding the cascade code, calculated with the following parameters of the cascade code: the length of the transmitted message $k = 5$ characters, the redundancy coefficient of the network code $m = 3$ and the channel code $p = 4$, while the number of packet erasures was always 5. The analysis shows that the cascading code can successfully deal with packet and character erasures, and decoding is possible even in the case of the maximum number of character erasures in the packet for the considered redundancy of the channel code.

3.2. Experimental Comparison of the Constructed Cascade Method with the Classical Network Method

The experiment was performed for two codes with identical erasures. 100 iterations were performed for each set of parameters for one randomly generated network matrix. Note, that in cascading code, the length of each vector column transmitted over the network is greater by 4 elements.

Table 2

The comparing results of successful decoding of network and cascade codes

Erasures		Successful decoding, %	
Erased packets	Erasures in one packet	Network code	Cascade code
0-5	1-4	100	100
6	1-4	98	100
7	1	92	99
7	2	91	99
7	3	94	94
7	4	90	90
8	1	63	98
8	2	62	86
8	3	56	69
8	4	53	53
9	1	0	93
9	2	0	69
9	3	0	25
9	4	0	0

In table 2, the last two columns show the success rates of decoding network and cascade methods, respectively, with the same character and packet erasures. The experiments were performed for the following parameters of the cascade code: the length of the transmitted message $k = 5$ characters, the redundancy coefficient of the network code $m = 3$ and the channel code $p = 4$, while the number of packet erasures was always equal to 5. The network code was taken with a similar redundancy ratio. The analysis shows that the success rate of decoding cascading code is always greater than or equal to the success rate of decoding network code. This observation leads to the conclusion that the cascade method is better suited for using in networks with packet and symbol erasures.

Conclusion

A cascading method for building code over the F_2 field has been developed to protect against character and packet erasures in networks. An experimental study has shown that the cascading method allows us to deal with both the symbol and the packet erasures.

The presented method is flexibly configurable, has a wide degree of freedom in choosing code redundancy coefficients and can be configured for a network with specific restrictions, which is very useful for solving some application problems.

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О КАНАЛЬНО-СЕТЕВОМ КАСКАДЕ ДЛЯ БОРЬБЫ С ПАКЕТНЫМИ И СИМВОЛЬНЫМИ СТИРАНИЯМИ В БИНАРНОЙ ЛИНЕЙНОЙ СЕТИ

Е. Е. Айдаркин, В. М. Деундяк

В настоящей работе рассматриваются проблемы борьбы со стираниями, как с пакетными, так и символьными в бинарных линейных сетях. Описан способ борьбы с символьными стираниями с помощью последовательного каскадного соединения сетевого и канального кодов. В качестве канального кода рассматривается код, основанный на равновесных столбцах. В качестве сетевого Џ нелинейный подкод рангового кода. Построены алгоритмы кодирования и декодирования для каскадного кода. Разработана имитационная модель, с помощью которой проведены эксперименты, показывающие эффективность представленного метода.

Ключевые слова: сетевое кодирование; каскад кодов; равновесные столбцы; символьное стирание; пакетное стирание.

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