MSC 93A30 DOI: 10.14529/jcem150201

MATHEMATICAL MODELING OF GROUP CONCURRENCY IN GAME THEORY

A. N. Ivutin*[1]*, alexey.ivutin@gmail.com,
E. V. Larkin*[1]*, elarkin@mail.ru,
A. S. Novikov*[1]*, alsnovikov@yandex.ru.
[1]Tula State University, Tula, Russian Federation.

The approach to an estimation of results of group "competitions" of intellectual agents is described. Petri-Markov models group "competition" are analyzed. Expressions for the determination of the probability density distribution and the participants or groups of participants win or lose "competition" are given. In general, the temporal and probabilistic characteristics of the game are obtained, The technique for an estimation of sequence of victories in "competition" of groups of subjects is offered, the estimation of efficiency of "competitions" of groups is resulted. Are considered two most often used principle of distribution of penalties: the lost group pays the penalty to the won group; each participant of the lost group pays the penalty to the won group, and sizes of penalties are distributed on time.

Keywords: competition, group concurrent games, Petri-Markov nets, effectiveness, the penalty of a participant.

Introduction

In recent years, game theory is becoming more widely used in industry, economy, military and cybernetics as a powerful mechanism for system modeling. Traditional game theory has been developed for static games with their matrices of values and strategies of the players, which can lead to a gain or loss of any resources. In computer science, game theory is used to model the interactions within the network between processors and computing modules, peripherals, etc. [1]

So far, the focus of this issue is given purely antagonistic games (eg, zero-sum games), useful for modeling systems, developing in "hostile" environment. Provisional aspects of the evolution of games is quite insufficient attention has been paid. In particular, it worked out the mathematical formalism to determine the price of "victory" ("loss"), if the price is reduced to the time factor, in group "competitions" agents [2].

This article deals with the use of Petri-Markov nets [3] for the mathematical description of the group "competitive" processes, which can be considered as parallel random processes with a time factor. Introduction to the formalism of Petri-Markov [4, 5, 6] allows considering setting time and determining payments participants in the game, and thus, its full price.

1. Petri-Markov nets

Concurrency may be investigated with various mathematical apparatus. One of it is Petri-Markov net (PMN). PMN is defined through system of sets

$$\Psi = (\Pi, M)\,; \tag{1}$$

$$\Pi = \left\{ A, Z, \tilde{\mathbf{R}}, \hat{\mathbf{R}} \right\}; \tag{2}$$

$$M = \{\mathbf{q}, \mathbf{h}(t), \mathbf{L}\}, \tag{3}$$

where Π – is the Petri net; M – is parallel semi-Markov process; $A = \left\{a_{1(a)}, \ldots, a_{j(a)}, \ldots, a_{J(a)}\right\}$ – is the finite set of places; $J(a)$ – is the size of the set of places; $Z = \left\{z_{1(z)}, \ldots, z_{j(z)}, \ldots, z_{J(z)}\right\}$ –is the finite set of transitions; $J(z)$ – is the size of the set of transactions; $\tilde{\mathbf{R}} = \tilde{r}\left[_{j(a)j(z)}\right]$ – is the $J(a) \times J(z)$ adjacency matrix, which represents places of A set to transactions of Z set; $\hat{\mathbf{R}} = \left[\hat{r}_{j(z)j(a)}\right]$ – is the $J(z) \times J(a)$ adjacency matrix, which represents transactions of Z set to places of A set; $\mathbf{q} = \left[q_{j(z)}\right]$ – is a vector of $J(z)$ size, which determine probabilities of start a process in one of places of Z set; $\mathbf{h}(t) = \left[h_{j(a)j(z)}(t)\right]$ –is the $J(a) \times J(z)$ semi-Markov matrix; t – is the time; $\mathbf{L} = \left[\sigma_{j(z)j(a)}\right]$ – is the $J(a) \times J(z)$ matrix of logical conditions of switching from the transition $z_{j(z)}$ to place $a_{j(a)}$;

$$\hat{r}_{j(z)j(a)} = \begin{cases} 1, \text{ when } a_{j(a)} \in O_A\left(z_{j(z)}\right); \\ 0, \text{ when } a_{j(a)} \notin O_A\left(z_{j(z)}\right); \end{cases} \tag{4}$$

where $O_A\left(z_{j(z)}\right)$ – is an output functions of transition $z_{j(z)}$;

$$\mathbf{h}(t) = \mathbf{p} \otimes \mathbf{f}(t) = \left[p_{j(a)j(z)} \cdot f_{j(a)j(z)}(t)\right] = \left[h_{j(a)j(z)}(t)\right]. \tag{5}$$

$\mathbf{p} = \left[p_{j(a)j(z)}\right]$ – is the stochastic matrix of semi-Markov process; $\mathbf{f}(t) = \left[f_{j(a)j(z)}(t)\right]$ — is the matrix of time densities of semi-Markov process; $\otimes$ – is a symbol of matrix direct multiplication.

2. "Competition" of J(a) participants

Let us consider the simplest case of "competition" of two participants, which is represented with PMN, being shown on fig. 1:

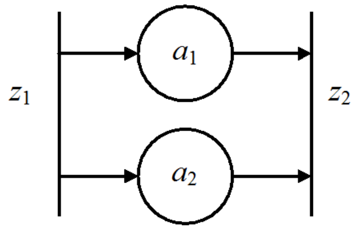

Fig. 1. "Concurrency" of two participants.

PMN being represented on fig. 1 is described with the next set of expressions:

$$\Pi = \left\{ \{a_1, a_2\}, \{z_1, z_2\}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\}; \tag{6}$$

$$M = \left\{ (1, 0), \begin{bmatrix} 0 & f_1(t) \\ 0 & f_2(t) \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\}, \tag{7}$$

where $f_1(t)$, $f_2(t)$ – are time densities of completion of their by first and second participants correspondingly;

$$f_{1,2}(t) = 0, when\ t < 0; \int_0^\infty f_{1,2}(t)\,dt = 1. \tag{8}$$

Let us split time intervals on periods dt. Then "competition" in PMN (6), (7) may be described with semi-Markov process, which is shown on fig. 2.

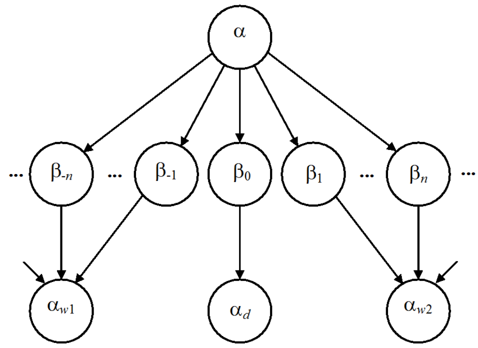

Fig. 2. Semi-Markov process inside PMN on fig. 1.

In semi-Markov process: the state α simulates the start of the competition; the state α_{w1} simulates the winning the competition by the first participant; the state α_{w2} simulates the winning the competition by the second participant; the state α_d simulates the draw of the competition; states β_{-n} , $-\infty < -n < 0$ simulate the situation in which the first participant wins the competition with the result $t = ndt$; states β_n, $0 < n < \infty$ simulate the situation in which the second participant wins the competition with the result $t = ndt$; state β_0 simulates a draw result, probability of which is less, than probabilities of any other results.

Let us determine time densities of achievement of an absorbing states α_{w1}, α_{w2}, α_d from initial state α.

- probability of the fact, that in time $t = n\Delta_t$ the state α_{w1} will be achieved with time lag, which is defined by the state β_{-n}, is equal to $P_{-n}(t = n\Delta_t) = [1 - F_2(n\Delta_t)] \cdot f_1(n\Delta_t)\,\Delta_t$, where $F_2(n\Delta_t)$ -is distribution function corresponding to density $f_2(t)$;
- probability of the fact, that in time $n\Delta_t$ the state α_{w2} will be achieved with time lag, which is defined by the state β_n, is equal to $P_n(t = n\Delta_t) = [1 - F_1(n\Delta_t)] \cdot f_2(n\Delta_t)\,\Delta_t$, where $F_1(t)$ - is distribution function corresponding to density $f_1(t)$;
- probability of the fact, that in time $n\Delta_t$ the state α_d will be achieved with time lag, which is defined by the state β_0, is equal to $P_0(t = n\Delta_t) = f_1(n\Delta_t) \cdot f_2(n\Delta_t)\,\Delta_t^2$, and is much less then probabilities $P_{-n}(t = n\Delta_t)$ and $P_n(t = n\Delta_t)$.

Weighed time densities of achievement of states α_{w1} and α_{w2} is defined as follows:

$$h_{w1}(t) = \lim_{\substack{n \to \infty \\ \Delta_t \to 0}} \frac{P_{-n}(t = n\Delta_t)}{\Delta_t} = [1 - F_2(t)] \cdot f_1(t)\,; \tag{9}$$

$$h_{w2}(t) = \lim_{\substack{n \to \infty \\ \Delta_t \to 0}} \frac{P_n(t = n\Delta_t)}{\Delta_t} = [1 - F_1(t)] \cdot f_2(t) . \qquad (10)$$

The sum of weighted densities (9) and (10) is equal to 1, due to the fact, that other results of "competition" can not be:

$$h_{w1}(t) + h_{w2}(t) = \frac{d}{dt}\{1 - [1 - F_1(t)] \cdot [1 - F_2(t)]\} . \qquad (11)$$

"Competition" of $J(a)$ participants is defined by PMN:

$$\Pi = \left\{ \left\{a_{1(a)},\ \dots,\ a_{j(a)},\ \dots,\ a_{J(a)}\right\}, \{z_1, z_2\}, \begin{bmatrix} 0 & & 1 \\ & \dots & \\ 0 & & 1 \\ & \dots & \\ 0 & & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ & \dots & \\ 0 & 0 & 0 \end{bmatrix} \right\}. \qquad (12)$$

$$M = \left\{ (1, 0), \begin{bmatrix} 0 & f_{1(a)}(t) \\ \dots & \\ 0 & f_{j(a)}(t) \\ \dots & \\ 0 & f_{J(a)}(t) \end{bmatrix}, \begin{bmatrix} 1 & \dots & 1 & \dots & 1 \\ 0 & & 0 & & 0 \end{bmatrix} \right\}, \qquad (13)$$

where $f_{j(a)}(t)$ – is the time density of distance completion by participants $j(a)$, $1(a) \leq j(a) \leq J(a)$.

For a common case of "concurrency" $J(a)$ participants weighed sum of densities may be obtained from (11) with mathematical induction method:

$$\sum_{j(a)=1}^{J(a)} h_{wj(a)}(t) = \frac{d}{dt}\left\{1 - \prod_{j(a)=1}^{J(a)} \left[1 - F_{j(a)}(t)\right]\right\}, \qquad (14)$$

where $F_{j(a)}(t)$ – are distribution functions corresponding to densities $f_{j(a)}(t)$.

From (12) may be obtained probabilities of winning in competition of $j(a)$-s participant:

$$p_{wj(a)} = \int_0^\infty f_{j(a)}(t) \cdot \prod_{\substack{k(a) = 1(a) \\ k(a) \neq j(a)}}^{J(a)} \left[1 - F_{k(a)}(t)\right] dt. \qquad (15)$$

Time density of achievement in PMN (12), (13) of place z_2 by participant-winner $j(a)$ is defined as:

$$f_{wj(a)}(t) = \frac{f_{j(a)}(t) \cdot \prod_{\substack{k(a) = 1(a) \\ k(a) \neq j(a)}}^{J(a)} \left[1 - F_{k(a)}(t)\right]}{p_{wj(a)}}. \qquad (16)$$

In specific case, when $f_{j(a)}(t) = \lambda_{j(a)} \exp\left[-\lambda_{j(a)} t\right]$, where $\lambda_{j(a)}$ is the parameter of Poisson distribution,

$$p_{wj(a)} = \frac{\lambda_{j(a)}}{\sum\limits_{j(a)=1(a)}^{J(a)} \lambda_{j(a)}}; \tag{17}$$

$$f_{wj(a)}(t) = \sum_{j(a)=1(a)}^{J(a)} \lambda_{j(a)} \cdot \exp\left[-t \cdot \sum_{j(a)=1(a)}^{J(a)} \lambda_{j(a)}\right]. \tag{18}$$

Let us note, that (18) describes conditional time density of "winning" of participant $j(a)$ (all other participants lose "competition"). Due to the fact conditional time densities of achievement of transition z_2 for all $J(a)$ participants are quite equal, but probabilities, are quite different for different $\lambda_{j(a)}$.

Let us return to the case of "competition" of two participants (6), (7) and consider case of "competition" lose. In this case

- probability of the fact, that in time $t = \bar{n}\Delta_t$ will be achieved the state $\alpha_{\bar{w}1}$ with time lag, which is defined by state $\beta_{-\bar{n}}$, is equal to $P_{-\bar{n}}(t = n\Delta_t) = F_2(\bar{n}\Delta_t) \cdot f_1(\bar{n}\Delta_t)\Delta_t$(first participant had finished his distance the last);

- probability of the fact, that in time $\bar{n}\Delta_t$ will be achieved the state $\alpha_{\bar{w}2}$ with time lag, which is defined by state $\beta_{\bar{n}}$, is equal $P_{\bar{n}}(t = \bar{n}\Delta_t) = F_1(\bar{n}\Delta_t) \cdot f_2(\bar{n}\Delta_t)\Delta_t$ (second participant had finished his distance the last).

Weighed time density of achievement of states $\alpha_{\bar{w}1}$ and $\alpha_{\bar{w}2}$ is defined as

$$h_{\bar{w}1}(t) = \lim_{\substack{\bar{n} \to \infty \\ \Delta_t \to 0}} \frac{P_{-\bar{n}}(t = \bar{n}\Delta_t)}{\Delta_t} = F_2(t) \cdot f_1(t); \tag{19}$$

$$h_{\bar{w}2}(t) = \lim_{\substack{\bar{n} \to \infty \\ \Delta_t \to 0}} \frac{P_{\bar{n}}(t = \bar{n}\Delta_t)}{\Delta_t} = F_1(t) \cdot f_2(t). \tag{20}$$

The sum of weighted time densities (19) and (20) is equal to 1, due to the fact, that other results of "competition" can not be:

$$h_{\bar{w}1}(t) + h_{\bar{w}2}(t) = \frac{d}{dt}\left[F_1(t) \cdot F_2(t)\right]. \tag{21}$$

For "competition" of $J(a)$ participants (12), (13), the sum of weighted time densities may be obtained from (21) by (9) and (10) with mathematical induction method:

$$f_{\bar{w}j(a)}(t) = \frac{f_{j(a)}(t) \cdot \prod\limits_{\substack{k(a) = 1(a) \\ k(a) \neq j(a)}}^{J(a)} F_{k(a)}(t)}{p_{\bar{w}j(a)}}. \tag{22}$$

$$p_{\bar{w}j(a)} = \int_0^\infty f_{j(a)}(t) \cdot \prod_{\substack{k(a)=1(a) \\ k(a) \neq j(a)}}^{J(a)} F_{k(a)}(t) dt. \tag{23}$$

Expressions (22) and (23) determine time density and probability of the fact, that participant $j(a)$ take the last place in the "competition".

Let us consider the case, when it is necessary to completion the "competition" by any $K(a)$ from $J(a)$, $K(a) < J(a)$ participants. Let us create the set $N_{J(a)}$ $J(a)$-digit binary natural codes and assign $j(a)$-s binary digit $\sigma_{j(a),2}$ of code to $j(a)$-s participant. States of digit $\sigma_{j(a),2}$ may accept two meanings

$$\sigma_{j(a),2} = \begin{cases} 0, \text{ when participant } j(a) \text{ finish the distance;} \\ 1, \text{ when participant } j(a) \text{ does not finish the distance.} \end{cases} \tag{24}$$

Let us select from the set $N_{J(a)}$ subset $N_{J(a)}^{K(a)} \subset N_{J(a)}$ binary $J(a)$-digit codes, which have $K(a)$ ones and $J(a) - K(a)$ zeros:

$$N_{J(a)}^{K(a)} = \left\{ n_1, \ \ldots, \ n_{c[J(a),K(a)]}, \ \ldots, \ n_{C[J(a),K(a)]} \right\}, \tag{25}$$

where $C[J(a), K(a)] = C_{J(a)}^{K(a)}$ – number of $J(a)$-digit codes with $K(a)$ units, which is equal to $K(a)$-s binomial coefficient; $C[J(a), K(a)]$ – number of code in subset (25);

$$C[J(a), K(a)] = \frac{J(a)!}{K(a)! \cdot [J(a) - K(a)]!}. \tag{26}$$

$$n_{c[J(a),K(a)]} = \left\langle \sigma_{1(a),2}^{c[J(a),K(a)]}, \ \ldots, \ \sigma_{j(a),2}^{c[J(a),K(a)]}, \ \ldots, \ \sigma_{J(a),2}^{c[J(a),K(a)]} \right\rangle. \tag{27}$$

Let us define function $\Phi\left(f_{j(a)}, \sigma_{j(a),2}^{c[J(a),K(a)]}\right)$, which is the next meanings:

$$\Phi\left(f_{j(a)}, \sigma_{j,(a)2}^{c[J(a),K(a)]}\right) = \begin{cases} F_{j(a)}(t), \text{ when } \sigma_{j(a),2}^{c[J(a),K(a)]} = 1; \\ \left[1 - F_{j(a)}(t)\right], \text{ when } \sigma_{j(a),2}^{c[J(a),K(a)]} = 0. \end{cases} \tag{28}$$

When taking into account (28) common expression for time distribution of completion of "competition" by any $K(a)$ participants from $J(a)$ will be the next:

$$F_{J(a)}^{K(a)}(t) = \sum_{c[J(a),K(a)]=1}^{C[J(a),K(a)]} \prod_{j(a)=1(a)}^{J(a)} \Phi\left(f_{j(a)}, \sigma_{j(a),2}^{c[J(a),K(a)]}\right). \tag{29}$$

First derivative of (29) gives time density under investigation:

$$f_{J(a)}^{K(a)}(t) = \frac{d \sum\limits_{c[J(a),K(a)]=1}^{C[J(a),K(a)]} \prod\limits_{j(a)=1(a)}^{J(a)} \Phi\left(f_{j(a)}, \sigma_{j(a),2}^{c[J(a),K(a)]}\right)}{dt}. \tag{30}$$

It is obviously, that (30) is the time density (but not weighed density) due to the fact, that after finishing the distance by $K(a)$ participants number participants, who finish the distance should be only to increase.Expressions (14) and (21) being obtained above are the special cases of (30), when $K(a) = 1$ and $K(a) = J(a)$, respectively.

3. Corporative "competition"

Let us investigate the "competition" (12), (13), when $J(a)$ participants are united into K corporative groups on $J(a,k)$ participants in k-th group, so that $\sum\limits_{k=1}^{K} J\left(a,k\right) = J\left(a\right)$.

Petri-Markov model of "competition" is shown on fig. 3, where are pictured:

- set of places, which is divided on subsets $A = \left\{\left\{a_{1(a,1)}, \ldots, a_{j(a,1)}, \ldots, a_{J(a,1)}\right\}, \ldots, \left\{a_{1(a,k)}, \ldots, a_{j(a,k)}, \ldots, a_{J(a,k)}\right\}, \ldots, \left\{a_{1(a,K)}, \ldots, a_{j(a,K)}, \ldots, a_{J(a,K)}\right\}\right\}$, every subset simulates corporative group of competing participants;
- set $B = \{b_1, \ldots, b_k, \ldots, b_K\}$ of places, which simulate switches from "competition" inside group to "competition" of groups of participants (this places are included into PMN for structural integrity of the model);
- starting z_0 and finishing z_{K+1} transitions, which simulate begin and end of "competition", respectively;
- subset $Z = \{z_1, \ldots, z_k, \ldots, z_K\}$ of transitions, which simulate the ends of "competitions" in groups $1, \ldots, k, \ldots, K$.

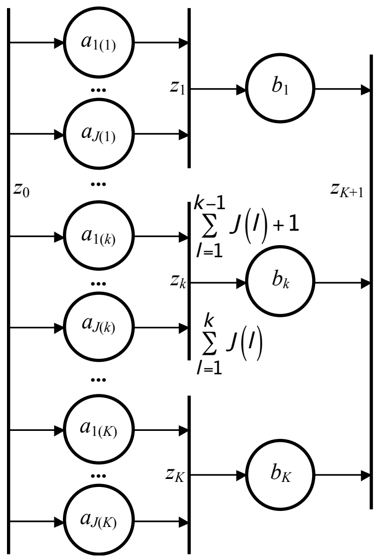

Fig. 3. PMN for a simulation of a corporative "competition".

It is considered, that k-th group finish a distance when distance finish a last participant of k-th group, independently of completion order. In particular case a group may consist on one participant. Also it can be considered, that k-th group includes participants with numbers $\sum\limits_{n=1}^{k-1} J\left(a,n\right) + 1 \le j\left(a\right) \le \sum\limits_{n=1}^{k} J\left(a,n\right)$. Indexes $j(a)$ may be recalculated from indexed $j\left(a,k\right)$ by the next way:

$$j\left(a\right) = j\left(a,k\right) + \sum_{n=1}^{k-1} J\left(a,n\right) \tag{31}$$

Time densities of distance completion by participants of k-th group belong to the set $\left\{f_{1(a,k)}(t), \ldots, f_{j(a,k)}(t), \ldots, f_{J(a,k)}(t)\right\}$. Time densities of switch PMN from places of subset B to transition z_{K+1} is defined by non-shifted Dirac δ-function: $f_{Bk}(t) = \delta(t)$, $1 \leq k \leq K$.

In accordance with Petri-Markov model time density of "completion" of distance by the whole k-th group is defined as

$$f_k(t) = \frac{d}{dt}\left[\prod_{\sum_{n=1}^{k-1} J(a,n)+1}^{\sum_{n=1}^{k} J(a,n)} F_{j(a)}(t)\right]. \tag{32}$$

"Competition" between groups is defined by expression:

$$\sum_{k=1}^{K} h_k(t) = \frac{d}{dt}\left\{1 - \prod_{k=1}^{K}\left[1 - F_k(t)\right]\right\}, \tag{33}$$

where $F_k(t)$ – distribution function corresponding to densities $f_k(t)$.

Probability of win of k-th group and time density of their "completion" of the distance is expressed as follows:

$$p_k = \int_0^{\infty} f_k(t) \cdot \prod_{\substack{l=1 \\ l \neq k}}^{K} [1 - F_l(t)]dt. \tag{34}$$

$$f_{wk}(t) = \frac{f_k(t) \cdot \prod\limits_{\substack{l=1 \\ l \neq k}}^{K} [1 - F_l(t)]}{p_k} \tag{35}$$

Sequence of wins in "competition" of groups may be evaluated on the next method.

1. In accordance to expression (34) probabilities of wins of all participants are calculated. From probabilities the highest one should be selected. Participant with the highest probability considered to have completed the distance.

2. For the remaining participants time density needed for completion the distance on expression $f_k(t) = f_{1\to k}(t)$, $2 \leq k \leq K$ is evaluated.

3. Paragraphs 1-2 are repeated until all participants complete the distance: $l = K - 1$.

4. Evaluation of group "competition" effectiveness

One important factor "competition" is the assessment of its effectiveness. Evaluating the effectiveness of the individual "competition" is given by the authors in [2].

For group "competition", which is modeled by Petri-Markov net shown in Fig. 3, the efficiency can be determined from the evaluation of the effectiveness of "competition" groups. Possible principles for the distribution of fines below are considered two of the most often used.

First, simplest principle consists in that losing l-th group pays to winning k-th group forfeit. In this case "density of forfeits" is evaluated as:

$$s_{kl}(t) = \sum_{j(a)=\sum_{n=1}^{k-1} J(a,n)+1}^{\sum_{r=1}^{k} J(a,n)} \sum_{i(a)=\sum_{r=1}^{l-1} J(a,n)+1}^{\sum_{r=1}^{l} J(a,n)} s_{j(a)i(a)}(t). \tag{36}$$

Group k wins from group l common forfeit:

$$s_{kl}^{+} = \int_{0}^{\infty} f_{k\to l}(t) \cdot s_{kl}(t)\, dt, \tag{37}$$

where $f_{k\to l}(t)$ is defined on expression

$$f_{1\to 2}(t) = \frac{\eta(t) \int_{0}^{\infty} f_1(\tau) f_2(t+\tau)\, d\tau}{\int_{0}^{\infty} F_1(t)\, dF_2(t)} \tag{38}$$

$f_k(t)$ and $f_l(t)$ in (38) calculated by expression (32) with the relevant indices and indices intervals determined by (31).

Second, more complex principle consists in that every participant of losing l-th group pays forfeit to winning k-th group individually. In this case time distribution of waiting by k-th group "completion" of distance by participants from l-th group is determined with expression, obtainable from (38):

$$f_{k\to j(a)}(t) = \frac{\eta(t) \int_{0}^{\infty} f_k(\tau) f_{j(a)}(t+\tau)\, d\tau}{\int_{0}^{\infty} F_k(t)\, dF_{j(a)}(t)}, \tag{39}$$

where $\sum_{n=1}^{l-1} J(a,n) + 1 \le j(a) \le \sum_{n=1}^{l} J(a,n)$; $f_k(\tau)$ - time density, which is defined on expression:

$$f_k(t) = \frac{d}{dt}\left[\prod_{\sum_{n=1}^{k-1} J(a,n)+1}^{\sum_{n=1}^{k} J(a,n)} F_{j(a)}(t) \right]. \tag{40}$$

Group k wins from group l common forfeit

$$s_{kl}^{+}(t) = \sum_{j=\sum_{n=1}^{l-1} J(a,n)+1}^{\sum_{n=1}^{l} J(a,n)} \int_{0}^{\infty} f_{k\to j(a)}(t) \sum_{i=\sum_{n=1}^{k-1} J(a,n)+1}^{\sum_{n=1}^{k} J(a,n)} s_{i(a)j(a)}(t) dt. \tag{41}$$

Conclusion

The paper presents a mathematical model of group "competitions" intelligent agents on the basis of mathematical formalism of Petri-Markov nets, who has shown to be effective for modeling the processes of this class. In the general form of a temporarily and probabilistic characteristics of the game, the technique for evaluating the sequence of victories in the "competition" stakeholder groups, give an estimate of the effectiveness of "competition" groups.

The results obtained can be used for planning the strategy of the "competition" with the number of participants more than two, and "competition" group of subjects, if the strategy and tactics can change their density distribution during the game. The proposed method can be a basis for the creation of mathematical models for solving the classical game theory: optimization strategy games, generating objective functions, etc.

References

1. Von Neumann J., Morgenstern O. *Theory of Games and Economic Behavior.* Princeton, NJ, USA, Princeton University Press, 2007.

2. Ivutin A.N., Larkin E.V. Simulation of Concurrent Games. *Bulletin of the South Ural State University. Series "Mathematical Modelling, Programming & Computer Software"*, 2015, vol. 8, no. 2, pp. 43–54. doi: 10.14529/mmp150204

3. Ivutin A.N., Larkin E.V., Lutskov Y.I., Novikov A.S. Simulation of Concurrent Process with Petri-Markov Nets. *Life Science Journal*, 2014, vol. 11, pp. 506–511.

4. Petri C.A. Nets, Time and Space. *Theoretical Computer Science*, 1996, vol. 153, no. 1–2, pp. 3–48. doi: 10.1016/0304-3975(95)00116-6

5. Jensen K. *Coloured Petri Nets: Basic Concepts, Analysis Methods and Practical Use: Volume 1.* London, UK, Springer-Verlag, 1996.

6. Ramaswamy S., Valavanis K.P. Hierarchical Time-Extended Petri Nets (H-EPN) Based Error Identification and Recovery for Hierarchical System. *IEEE Trans. on Systems, Man, and Cybernetics – Part B: Cybernetics*, 1996, vol. 26, no. 1, pp. 164–175. doi: 10.1109/3477.484450

Alexey N. Ivutin, Candidate of Engineering Sciences, Docent, Head of the Department of Computer Technology, Tula State University, Tula, Russian Federation, alexey.ivutin@gmail.com

Eugene V. Larkin, Doctor of Engineering Sciences, Full Professor, Head of the Department of Robotics and Automation, Tula State University, Tula, Russian Federation, elarkin@mail.ru

Alexander S. Novikov, Candidate of Engineering Sciences, Associate Professor at the Department of Computer Technology, Tula State University, Tula, Russian Federation, alsnovikov@yandex.ru

Received May 15, 2015