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DICHOTOMIES OF SOLUTIONS TO THE STOCHASTIC GINZBURG – LANDAU EQUATION ON A TORUS

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We consider a stochastic analogue of the Ginzburg – Landau equation in spaces of differential forms defined on a two-dimensional smooth compact oriented manifold without boundary. When studying the stability of solutions, the Ginzburg – Landau equation is considered as a special case of a stochastic linear Sobolev-type equation. All considerations are carried out in spaces of random K-variables and K-"noises" on the manifold. As a manifold, we consider a two-dimensional torus, which is a striking example of a smooth compact oriented manifold without boundary. Under certain conditions imposed on the coefficients of the equation, we prove the existence of stable and unstable invariant spaces and exponential dichotomies of solutions. We develop an algorithm to illustrate the results obtained. Since there exists a smooth diffeomorphism between a map and a manifold, we reduce the question of stability of solutions on a two-dimensional torus to the same question on one of its maps. The developed algorithm is implemented in the Maple software environment. The results of the work are presented in the form of graphs of stable and unstable solutions, which are obtained for various values of the parameters of the Ginzburg – Landau equation.

Keywords: Sobolev type equation; stochastic equations; differential forms; two-dimensional torus; exponential dichotomies.

Introduction

The equation

$$(\lambda - \Delta)\alpha_t = \nu\Delta\alpha - id\Delta^2\alpha \tag{1}$$

describes weakly linear effects in hydrodynamics in a particular case. Here the coefficients $\nu \in \mathbb{R}_+$, $\lambda,\ d \in \mathbb{R}$ describe the parameters of the system [1]. The work [2] proves solvability of equation (1) in the case when the right-hand side contains nonlinearity. The paper [3] considers the question of the stability of solutions and shows the existence of stable and unstable invariant spaces of the linear stationary Ginzburg – Landau equation. The study of solutions to this equation was carried out within the framework of the theory of Sobolev-type equations (see, for example, [4]).

The aim of this work is to study the behavior of solutions to the stochastic analogue of the Ginzburg – Landau equation in spaces of differential forms on a smooth two-dimensional manifold without boundary. To this end, we consider equation (1) as a special case of the linear stochastic Sobolev-type equation

$$L \overset{\circ}{\eta} = M\eta. \tag{2}$$

Here $\eta = \eta(t)$ is the required stochastic process, and $\overset{\circ}{\eta}$ is its Nelson – Gliklikh derivative [5]. In the works [6] — [9], equation (2) was studied in various aspects. The papers [10], [11] are devoted to the study of equation (2) in spaces of differential forms defined on a smooth compact oriented manifold without boundary. In [12], [13], computational experiments were carried out in order to represent the results obtained on two-dimensional and three-dimensional tori.

The paper is organized as follows. In Section 1, we consider random variables and stochastic processes, recall definition of the Nelson – Gliklikh derivative, and construct the spaces of random K-variables and K-"noises" on a two-dimensional manifold. Section 2 is devoted to the study of stability and unstability of solutions to stochastic equation (1) in spaces of smooth differential forms, the existence of exponential dichotomies is established. Section 3 illustrates the results obtained on a two-dimensional torus.

1. Spaces of K-variables and K-"noises" on Manifold

Let $\Omega = (\Omega, \mathcal{A}, P)$ be a complete probability space, $\mathbb{R}$ be the set of real numbers endowed with a Borel σ-algebra. The measurable mapping $\xi : \Omega \mapsto \mathbb{R}$ is called a *random variable*, the measurable mapping $\eta : \mathfrak{I} \times \Omega \mapsto \mathbb{R}$ is said to be a *stochastic process*, the function $\eta(t, \cdot), t \in \mathfrak{I}$ is a *trajectory of the stochastic process* η, and the random variable $\eta(\cdot, \omega), \omega \in \Omega$ is a *section of the stochastic process* η. Here $\mathfrak{I} \subset \mathbb{R}$ is some interval.

Denote by $\mathbf{L}_2$ the Hilbert space of random variables ξ with zero mathematical expectation and finite variance with the scalar product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\xi_2$, while $\mathbf{CL}_2$ is the Banach space of continuous stochastic processes η with the norm

$$\|\eta\|^2_{\mathbf{CL}_2} = \max_{t \in \mathfrak{I}} D\eta(t, \cdot).$$

By a *continuous stochastic process* we mean a stochastic process $\eta = \eta(t, \omega)$ whose trajectories are a.s. (almost sure) continuous. We fix $\eta \in \mathbf{L}_2$ and $t \in \mathfrak{I}$, denote by $\mathcal{N}^\eta_t$ the σ-algebra generated by η and $\mathbf{E}^\eta_t = \mathbf{E}(\cdot|\mathcal{N}^\eta_t)$. By the *Nelson – Gliklikh derivative* of the stochastic process η at the point $t \in \mathfrak{I}$ we mean the limit

$$\overset{o}{\eta}(\cdot, \omega) = \frac{1}{2}\left(\lim_{\Delta t \to 0+} \mathbf{E}^\eta_t\left(\frac{\eta(t + \Delta t, \cdot) - \eta(t, \cdot)}{\Delta t}\right) + \lim_{\Delta t \to 0+} \mathbf{E}^\eta_t\left(\frac{\eta(t, \cdot) - \eta(t - \Delta t, \cdot)}{\Delta t}\right)\right),$$

if the limit converges in the uniform metric on $\mathbb{R}$. Denote by $\mathbf{C}^l\mathbf{L}_2$ the space of stochastic processes, whose Nelson – Gliklikh derivatives are a.s. continuous on $\mathfrak{I}$ up to the order l inclusive.

Let us construct the spaces of random K-variables and K-"noises" on the two-dimensional manifold $\mathcal{M}$. Let $\mathcal{M}$ be a smooth compact oriented Riemannian manifold without boundary. Consider the vector space of q-forms $E^q = E^q(\mathcal{M}),\ q = 0, 1, 2$, of the form

$$a = \sum_{i_1 < i_2} a_{i_1,i_2}(t, x_{i_1}, x_{i_2}) dx_{i_1} \wedge dx_{i_2},$$

$a_{i_1,i_2} \in C^\infty$. In the spaces E^q, consider the Hodge operator $* : E^q \to E^{2-q}$, the exterior derivation operator $d : E^q \to E^{q+1}$ and the Laplace – Beltrami operator $\Delta = d\delta + \delta d$, where $\delta = (-1)^{2q+3} * d*$. In the spaces E^q, define the scalar products by the relations

$$(a, b)_0 = \int_{\mathcal{M}} a \wedge *b,\ (a, b)_2 = (a, b)_0 + (\Delta a, b)_0 + (\Delta a, \Delta b)_0$$

and denote by H^q_l the completion of E^q in the norms $\|\cdot\|_l,\ l=0,2$.

Denote by $\mathbf{K}=\{\lambda_k\}$ the eigenvalues of the Laplace – Beltrami operator. The spectrum of the Laplace – Beltrami operator is discrete finite-multiple and converges to the point $+\infty$. Let $\{\varphi_k\}$ be the eigenfunctions of the Laplace – Beltrami operator, which are orthonormal with respect to the scalar products $(\cdot,\cdot)_l,\ l=0,2$. The eigenfunctions are bases in the spaces H^q_l. Next, choose the sequence of random variables $\{\xi_k\}\subset\mathbf{L}_2$ such that $\mathbf{D}\xi_k\le const$. Denote by $\mathbf{H}^q_l$ the spaces of *random* $\mathbf{K}$-*variables* whose elements are the vectors

$$\xi=\sum_{k=1}^{\infty}\lambda_k\xi_k\varphi_k. \tag{3}$$

Define the norm in the spaces $\mathbf{H}^q_l$ as follows:

$$\|\xi\|^2_{\mathbf{H}^q_l}=\sum_{k=1}^{\infty}\lambda_k^2\mathbf{D}\xi_k.$$

Denote by $\mathbf{C}(\mathfrak{I};\mathbf{H}^q_l)$ the sets of *continuous stochastic* $\mathbf{K}$-*processes*

$$\eta(t)=\sum_{k=1}^{\infty}\lambda_k\eta_k(t)\varphi_k, \tag{4}$$

if series (3) converges uniformly on any compact set in $\mathfrak{I}$ ($\mathfrak{I}$ is an interval), while $\{\eta_k\}\subset\mathbf{CL}_2$. Let $\mathbf{C}^1(\mathfrak{I};\mathbf{H}^q_l)$ be the set of the *processes, which are continuously differentiable in the sense of Nelson – Gliklikh*

$$\overset{\circ}{\eta}(t)=\sum_{k=1}^{\infty}\lambda_k\overset{o}{\eta}_k(t)\varphi_k, \tag{5}$$

if series (5) converges uniformly on any compact set in $\mathfrak{I}$ and $\{\eta_k\}\subset\mathbf{C}^1\mathbf{L}_2$.

2. Exponential Dichotomies of the Stochastic Ginzburg – Landau Equation

In the spaces $\mathbf{H}^q_0,\ q=0,1,2$, consider equation (1) as the stochastic linear Sobolev-type equation

$$L\overset{\circ}{\eta}=M\eta, \tag{6}$$

where the operators $L,\ M:\mathbf{H}^q_0\to\mathbf{H}^q_2$ are defined by the following formulas:

$$L=\lambda+\Delta,\ M=-\nu\Delta-id\Delta^2.$$

Lemma 1. *For any $\lambda,\ d\in\mathbb{R}$ and $\nu\in\mathbb{R}_+$, the operator M is strongly $(L,0)$-radial.*

By a *solution to equation (6)* we mean a stochastic $\mathbf{K}$-process $\eta\in\mathbf{C}^1(\mathfrak{I};\mathbf{H}^q_0)$, if substituting η in (6) a.s. converts the equation into identity.

Definition 1. *The set $\mathfrak{P}\subset\mathbf{H}^q_0$ is called the phase space of equation (6), if*

(i) a.s. each trajectory of the solution $\eta=\eta(t)$ to equation (6) belongs to $\mathfrak{P}$;

(ii) for a.a. $\eta_0 \in \mathfrak{P}$, there exists a solution to equation (6) that satisfies the condition $\eta(0) = \eta_0$.

Theorem 1. *(i) Let $\lambda \notin \{-\lambda_k\}$. Then the phase space of equation (6) coincides with the space $\mathbf{H}_0^q$.*

(ii) Let $\lambda \in \{-\lambda_k\}$. Then the phase space of equation (6) has the form

$$\mathfrak{P} = \{\eta \in \mathbf{H}_0^q : (\cdot, \varphi_k)\varphi_k = 0\}.$$

Definition 2. *The subspace $\mathbf{I_K} \subset \mathbf{H}_0^q$ is said to be the invariant space of equation (6), if for any $\eta_0 \in \mathbf{I_K}$ solution to the problem $\eta(0) = \eta_0$ for equation (6) is $\eta \in \mathbf{C}^1(\mathbb{R}; \mathbf{I_K})$.*

Definition 3. *We say that solutions to equation (6) have exponential dichotomy, if the following conditions hold:*

(i) the phase space $\mathfrak{P}$ of equation (6) can be represented as a direct sum of two invariant spaces $\mathfrak{P} = \mathfrak{I}_+ \oplus \mathfrak{I}_-$;

(ii) there exist the constants $N_1, \nu_1 \in \mathbb{R}_+$ such that the inequality

$$\|\eta^1(t)\|_{\mathbf{H}_0^q} \leq N_1 e^{-\nu_1(s-t)} \|\eta^1(s)\|_{\mathbf{H}_0^q} \qquad s \geq t$$

holds for $\eta^1 \in \mathfrak{I}_+$;

(ii) there exist the constants $N_2, \nu_2 \in \mathbb{R}_+$ such that the inequality

$$\|\eta^2(t)\|_{\mathbf{H}_0^q} \leq N_2 e^{-\nu_2(t-s)} \|\eta^2(s)\|_{\mathbf{H}_0^q} \qquad t \geq s$$

holds for $\eta^2 \in \mathfrak{I}_-$.

The solutions $\eta^1 \in \mathfrak{I}_+$ are called *exponentially stable*, and the solutions $\eta^2 \in \mathfrak{I}_-$ are called *exponentially unstable*. The subspaces $\mathfrak{I}_+$ and $\mathfrak{I}_-$ are called *stable* and *unstable* invariant spaces, respectively.

Due to the fact that the relative spectrum has the form

$$\sigma^L_(M) = \left\{\mu \in \mathbb{C} : \mu_k = \frac{-\nu\lambda_k - id\lambda_k^2}{\lambda + \lambda_k}\right\} \tag{7}$$

the following theorem is true.

Theorem 2. *(i) Let λ, $\nu \in \mathbb{R}_+$ and $d \in \mathbb{R}$. Then the solutions to equation (6) are exponentially stable.*

(ii) Let $\lambda \in \mathbb{R}_-$, $\nu \in \mathbb{R}_+$ and $d \in \mathbb{R}$. Then for $-\lambda > \lambda_1$ the solutions to equation (6) have exponential dichotomy, and for $-\lambda < \lambda_1$ the solutions to equation (6) are exponentially stable.

3. Computational Experiment

Let us describe the algorithm for representing the results obtained in Section 2 on the two-dimensional torus $T^2 = [0, \pi] \times [0, \pi]$. The torus T^2 can be represented as a direct product $T^2 = S^1 \oplus S^1$, where S^1 is a circle of the radius π. Choose a square $Sq = \{(x, y) : 0 \leq x \leq \pi, 0 \leq y \leq \pi\}$, which is one of the maps of the torus T^2. The two-dimensional torus T^2 can be represented as "gluing" opposite sides of the square Sq.

Denote by δ a smooth diffeomorphism between the map Sq and the torus T^2. If $u = u(t)$ is a solution to this differential equation on the torus, then $z = z(t) = \delta(u(t))$ is a solution to this differential equation on the map. Then if the solution to the differential equation is stable on the torus T^2, then the solution is stable on the map Sq. The converse is also true. Due to the smoothness of the solutions, the nature of stability does not change at the places where the maps are "glued". Therefore, we reduce consideration of the question of stability of solutions to equation (1) on the torus T^2 to consideration of the same question on one of the maps Sq. Let us describe the algorithm.

Step 1. Enter the parameters of the equation λ, ν, d and the number of random variables K in representation (3).

Step 2. Calculate the eigenvalues λ_k and the eigenfunctions φ_k of the Laplace – Beltrami operator.

Step 3. Generate an array of the random variables

$$\xi_k \sim N(0,1), \quad k = 1, \ \dots \ , K$$

.

Step 4. Form the initial condition by the following formula:

$$\eta_0 = \sum_{k=1}^{K} \lambda_k \xi_k \varphi_k.$$

Step 5. Perform the procedure to check whether the initial condition belongs to the phase space.

Step 6. Find the points of the relative spectrum of the operator $L = \lambda + \Delta$

$$\mu_k = \frac{-\nu\lambda_k - id\lambda_k^2}{\lambda + \lambda_k}.$$

Step 7. Construct stable

$$\eta_1(t) = \sum_{l=1}^{M_1} e^{\mu_l t} \left(\int_0^{\pi} \int_0^{\pi} \eta_0 \varphi_l dx dy \right) \varphi_l$$

and unstable

$$\eta_2(t) = \sum_{l=M_2}^{K} e^{\mu_l t} \left(\int_0^{\pi} \int_0^{\pi} \eta_0 \varphi_l dx dy \right) \varphi_l$$

solutions to stochastic equation (1), where $M_1 = \max\{l : \lambda_l < -\lambda\}$ and $M_2 = \min\{l : \lambda_l > -\lambda\}$.

Step 8. Output a solution and plot a graph.

Fig. 1 shows a graph of the real part of the stable solution Re $\eta_1(t)$ to equation (1) for $\lambda = 4,2$, $\nu = 5$, $d = 2$ at times $t = 9, \ 9.1, \ 9.263$. Fig. 2 shows the exponentially dichotomous behavior of the real part of the solution to equation (1) for $\lambda = -4.2$, $\nu = 0.2$, $d = 2$ in the section $x = \dfrac{\pi}{2}$, $y = \dfrac{\pi}{2}$.

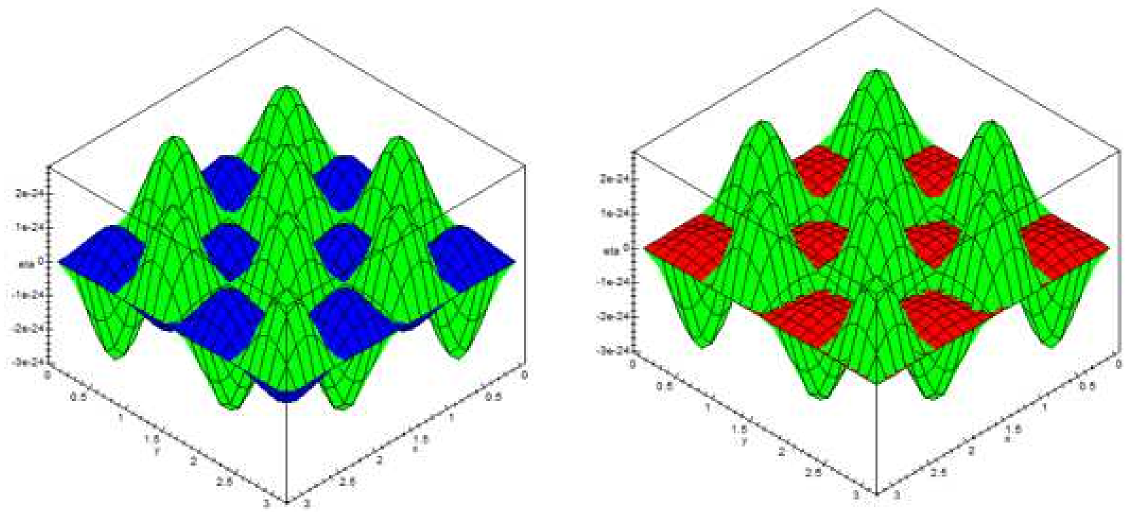

Fig. 1. Stable solutions for $t = 9$ (green), $t = 9,1$ (blue) and $t = 9,263$ (red)

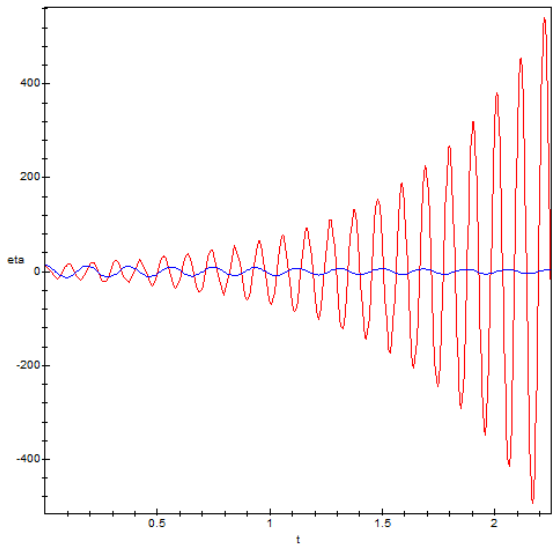

Fig. 2. Exponential dichotomy

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ДИХОТОМИИ РЕШЕНИЙ СТОХАСТИЧЕСКОГО УРАВНЕНИЯ ГИНЗБУРГА – ЛАНДАУ НА ТОРЕ

О. Г. Китаева

Рассматривается стохастический аналог уравнения Гинзбурга – Ландау в пространствах дифференциальных форм, заданных на двумерном гладком компактном ориентированном многообразии без края. При изучении устойчивости решений уравнение Гинзбурга – Ландау рассматривается как частный случай стохастического линейного уравнения соболевского типа. Все рассмотрения проводятся в пространствах K-величин и K-«шумов» на многообразии. В качестве многообразия рассматривается двумерный тор, являющийся ярким примером гладкого компактного ориентированного многообразия без края. При определенных условиях, накладываемых на коэффициенты уравнения, доказывается существование устойчивого и неустойчивого инвариантных пространств и экспоненциальных дихотомий решений. Разработан алгоритм для иллюстрации полученных результатов. Так как существует гладкий диффеоморфизм между картой и многообразием, то от рассмотрения устойчивости решений на двумерном торе переходим к рассмотрению данного вопроса на одной из его карт. Разработанный алгоритм реализован в программной среде Maple. Результаты работы представлены в виде графиков устойчивых и неустойчивых решений, которые получаются при различных значениях параметров уравнения Гинзбурга – Ландау.

Ключевые слова: уравнения соболевского типа; стохастические уравнения; дифференциальные формы; экспоненциальные дихотомии.

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