SHORT NOTES

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EXPONENTIAL DICHOTOMIES OF A STOCHASTIC NON-CLASSICAL EQUATION ON A TWO-DIMENSIONAL SPHERE

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The article discusses a stochastic analogue of the linear Oskolkov equation, which is obtained from the Oskolkov system of equations. The existence of a solution to the Oskolkov stochastic equation in spaces of differential forms defined on a two-dimensional sphere. For some values of the parameters characterizing the properties of the liquid, the existence of exponential dichotomies of solutions is proved. To solve the question of the existence and stability of solutions, this equation is considered as a special case of a linear stochastic Sobolev type equation. The Nelson – Glicklikh derivative of the stochastic process is considered as a derivative. To visualize the results obtained, an algorithm was developed for calculating stable and unstable solutions of the Oskolkov stochastic equation in spaces of 0-forms on a two-dimensional sphere. This algorithm is implemented in the Maple environment. Graphs of solutions with exponential dichotomy are plotted in a spherical coordinate system.

Keywords: Sobolev type equation; stochastic equations; differential forms; exponential dichotomies.

Introduction

The Oskolkov equation

$$(\lambda - \Delta)\Delta u_t = \nu\Delta^2 u + \frac{\partial(u, \Delta u)}{\partial(x_1, x_2)} \tag{1}$$

is a model of a multipurpose flow of a high-performance, non-essential liquid [1]. Here the coefficients $\alpha,\ \nu \in \mathbb{R}$ characterize the parameters of the fluid. Consider the equation

$$(\lambda + \Delta)\Delta\alpha_t = -\nu\Delta^2\alpha \tag{2}$$

in spaces of differential forms defined on a two-dimensional sphere. Here Δ is the Laplace – Beltrami operator, α is the q-form, $q = 0, 1, 2$. Equations (1), (2) were previously considered in different aspects of [2] — [4]. We will study the stochastic analogue of the equation (2). For this, we reduce it to a stochastic linear Sobolev type equation

$$L \overset{\circ}{\eta} = M\eta, \tag{3}$$

where the operators $L,\ M$ are linear and continuous, η is a stochastic **K** is a process, its Nelson – Glicklikh derivative is a Nelson – Glicklikh derivative [5].

At present, a large number of works are devoted to the study of the Cauchy and Showalter – Sidorov problems for stochastic Sobolev type equations [6] — [9]. Our goal is to study the exponential dichotomies of the stochastic equation (2) on the 2-sphere. This work is most closely related to the work [12], but unlike these works, we do not go from clarifying the issue of stability of solutions on a manifold to clarifying the issue of stability on one of the maps. Besides the Introduction and the References, the article contains two parts. In the first part, we consider the Cauchy problem for the equation (3) in spaces of differential forms. Following the works of [10] — [11], the existence and stability of the solution to the equation (3) and the application of these results to the stochastic equation (2) are considered. The second chapter is devoted to the algorithm for constructing stable and unstable solutions of the stochastic equation (2) on the sphere, graphs of solutions for 0-forms are presented.

1. Exponential Dichotomies of the Stochastic Oskolkov Equation on a Sphere

Consider a two-dimensional unit sphere S^2. The Laplace – Beltrami operator on a sphere S^2 is defined by the formula

$$\Delta = d\delta + \delta d = (d+\delta)^2,$$

where d is the is the operator of external differentiation, $\delta = (-1)^{2q+3} * d*$ is the codifferential, $*$ is the Hodge star. The eigenvalues of the Laplace – Beltrami operator Δ are the numbers $\vartheta_l = l(l+1)$, $l \geq 0$.

Consider the Hilbert spaces of differential forms $E^q = E^q(S^2)$, $q = 0, 1, 2$

$$a = \sum_{i_1 < i_2} f_{i_1,i_2} dx_{i_1} \wedge dx_{i_2},$$

on the sphere S^2 with scalar products

$$< a, b >_0 = \int\limits_{S^2} a \wedge *b, \tag{4}$$

$$< a, b >_1 = < a, b >_0 + < \Delta a, b >_0, \ < a, b >_2 = < a, b >_1 + < \Delta a, \Delta b >_0 .$$

By H^q_j we denote the completion of E^q according to the norms $|| \cdot ||_j$ induced by scalar products $< \cdot, \cdot >_j$, $j = 0, 1, 2$. Continuous and dense embeddings $H^q_2 \subset H^q_1 \subset H^q_0$ are valid. The eigenfunctions of the Laplace – Beltrami operator ψ_l orthonormal with respect to the scalar product $< \cdot, \cdot >_j$ is a basis in the spaces H^q_j, $j = 0, 1, 2$.

Following article [11] we construct the spaces of $\mathbf{K}$-variables and $\mathbf{K}$-"noises" on a sphere S^2. Let the sequence $\mathbf{K} = \{\lambda_l\} \subset \mathbb{R}_+$ be such that $\sum\limits_{l=1}^{\infty} \lambda_l^2 < +\infty$, $\{\xi_l\}$ is a sequence of random variables with zero mathematical expectation and variance $\mathbf{D}\xi_l \leq const$, $l \in \mathbb{N}$. The elements of the space $\mathbf{H}^q_{j\mathbf{K}}$, q, $j \in \{0, 1, 2\}$ are *random* $\mathbf{K}$*-variables*

$$\eta = \sum_{l=1}^{\infty} \lambda_l \xi_l \psi_l. \tag{5}$$

Using the formula

$$\|\eta\|_{\mathbf{H}^q_{j\mathbf{K}}} = \sum_{l=1}^{\infty} \lambda_l^2 \mathbf{D}\xi_l, \tag{6}$$

we introduce the norm in Hilbert spaces $\mathbf{H}^q_{j\mathbf{K}}$.

A continuous stochastic process is a stochastic process $\eta : \mathbb{R} \times \Omega \to \mathbb{R}$, whose trajectories $\eta = \eta(\cdot, \omega)$ are continuous for almost all $\omega \in \Omega$. The set of continuous $\mathbf{K}$-*stochastic processes*

$$\eta(t) = \sum_{l=1}^{\infty} \lambda_l \xi_l(t)\psi_l, \tag{7}$$

denote by $\mathbf{C}(\mathfrak{I}, \mathbf{H}^q_{j\mathbf{K}})$, where $\{\xi_l(t)\}$ is a sequence continuous stochastic process, if the series (7) converge on any copact set in $\mathfrak{I}$. Let exist a Nelson – Glicklikh derivative [5] $\overset{\circ}{\eta}$ of a continuous stochastic process $\eta = \eta(t)$. The continuous stochastic $\mathbf{K}$-process $\eta = \eta(t)$ is called a *continuously differentiable by Nelson – Gliklikh* on $\mathfrak{I}$, if the series

$$\overset{\circ}{\eta}(t) = \sum_{k=1}^{\infty} \lambda_k \overset{o}{\xi}_k(t)\psi_k \tag{8}$$

converges uniformly on any compact set in $\mathfrak{I}$ and $\overset{\circ}{\eta} = \overset{o}{\eta}(t)$ is continuous stochastic process. The set of continuously differentiable by Nelson – Gliklikh process is denoted $\mathbf{C}(\mathfrak{I}, \mathbf{H}^q_{j\mathbf{K}})$ and called the space of differentiable $\mathbf{K}$-"noise".

Let the operators $L,\ M \in \mathcal{L}(\mathbf{H}^q_{0\mathbf{K}}, \mathbf{H}^q_{2\mathbf{K}})$. Consider the stochastic Sobolev type equation

$$L\overset{\circ}{\eta} = M\eta. \tag{9}$$

Definition 1. *A stochastic* $\mathbf{K}$*-process* $\eta \in \mathbf{C}^1(\mathbb{R}; \mathbf{H}^q_{0\mathbf{K}})$ *is called a solution to equation (9), if substitution of* η *in (9) gives an identity.*

Definition 2. *The set* $\mathbf{P} \subset \mathbf{H}^q_{0\mathbf{K}}\mathbf{L}_2$ *is called a phase space of equation (9), if*

(i) almost sure each trajectory of the solution $\eta = \eta(t)$ *to equation (9) belongs to* $\mathfrak{P}$*;*

(ii) for almost all $\eta_0 \in \mathbf{P}$ *there exists a solution to the Cauchy problem*

$$\eta(0) = \eta_0 \tag{10}$$

for equation (9).

Suppose the operator M is (L,p)-bounded, then there is an analytic group of operators

$$U^t = \frac{1}{2\pi i}\int_{\Gamma} (\mu L - M)^{-1} M e^{\mu t} d\mu, \tag{11}$$

where the contour Γ limits the region containing the L-spectrum of the operator M.

Theorem 1. *Suppose that the operator* M *is* (L,p)*-bounded, then phase space* $\mathfrak{P}$ *of the equation (9) is the image* $\mathrm{im} U^{\bullet}$ *of the group (11).*

Definition 3. *The subspace* $\mathbf{I} \subset \mathbf{H}^q_{0\mathbf{K}}$ *is called an invariant space of the equation (9), if the solution to problem (9), (10)* $\eta \in \mathbf{C}^1(\mathbb{R}; \mathbf{I})$ *for any* $\eta_0 \in \mathbf{I}$.

Remark 1. For the existence of invariant spaces of equation (9), it is sufficient for the equation (9) to represent the L-spectrum of the operator M in the form of two disjoint parts, and at least one of these parts is closed.

Definition 4. *If the phase space* $\mathbf{P} = \mathbf{I}^1 \oplus \mathbf{I}^2$, *and there exist constants* $N_k \in \mathbb{R}_+, \nu_k \in \mathbb{R}_+,\ k = 1,\ 2,$ *such that*

$$\begin{array}{ll} \|\eta^1(t)\|_{\mathbf{U}} \leq N_1 e^{-\nu_1(s-t)}\|\eta^1(s)\|_{\mathbf{U}} & \textit{for} \quad s \geq t, \\ \|\eta^2(t)\|_{\mathbf{U}} \leq N_2 e^{-\nu_2(t-s)}\|\eta^2(s)\|_{\mathbf{U}} & \textit{for} \quad t \geq s, \end{array}$$

where $\eta^k = \eta^k(t) \in \mathbf{I}^k$ *for all* $t \in \mathbb{R}$, *and* $\mathbf{I}^k,\ k = 1,\ 2,$ *is invariant space of equation (9), then solutions* $\eta = \eta(t)$ *to the equation (9) have exponential dichotomy.*

Theorem 2. *Suppose that the operator* M *is* (L,p)*-bounded, and the* L*-spectrum of the operator* M $\sigma^L(M) = \sigma^L_+(M) \oplus \sigma^L_-(M)$, *where* $\sigma^L_+(M) = \{\mu \in \sigma^L(M) : \operatorname{Re}\mu > 0\} \neq \oslash$, $\sigma^L_-(M) = \{\mu \in \sigma^L(M) : \operatorname{Re}\mu < 0\} \neq \oslash$. *Then solutions of the equation (9) have exponential dichotomy.*

Define the operators L and M by the formulas

$$L = (\lambda + \Delta)\Delta,\ M = -\nu\Delta^2. \tag{12}$$

Then the stochastic equation (2) can be considered as the equation

$$L \overset{\circ}{\zeta} = M\zeta, \tag{13}$$

where operators $L,\ M \in \mathcal{L}(\mathbf{H}^q_{0\mathbf{K}}, \mathbf{H}^q_{2\mathbf{K}})$, and the operator M is $(L,0)$-bounded operator. The phase space of the equation (13) has the form

$$\mathbf{P} = \left\{ \begin{array}{c} \mathbf{H}^q_{0\mathbf{K}},\ \vartheta_l \neq \lambda, \\ \zeta \in \mathbf{H}^q_{0\mathbf{K}} : <\zeta, \psi_l>_0 = 0,\ \vartheta_l = \lambda. \end{array} \right.$$

Theorem 3. *For any* $\alpha \in \mathbb{R}\setminus\{0\}$, $\lambda \in \mathbb{R}\setminus\{0\}$, *and* $\zeta_0 \in \mathbf{P}$ *there exists the solution* $\zeta = \zeta(t)$ *to the problem* $\zeta(0) = \zeta_0$, *(13), and the solution has the form*

$$\eta(t) = {\sum_{l=1}^{\infty}}' \left[exp\left(\frac{-\nu\vartheta_l}{\lambda + \vartheta_l}t\right)\left(\sum_{k=1}^{\infty} \lambda_k \xi_k <\psi_k, \psi_l>_0 \psi_l\right)\right]. \tag{14}$$

Theorem 4. *For any* $\lambda \in \mathbb{R}_-$, $\nu \in \mathbb{R}$ *solutions of the equation (13) have exponential dichotomy.*

2. Computational Experiment

Consider the space of 0-form on a single sphere with a center in the initial order. The Laplace – Beltrami operator in the spheral system of coordinates (Θ, φ) is assigned with the formula

$$\Delta_{S^2} = (\sin\varphi)^{-1}\frac{\partial}{\partial\varphi}(\sin\varphi\partial\varphi) + (\sin\varphi)^{-2}\frac{\partial}{\partial\Theta}.$$

If ϑ_l are eigenvalues of the Laplace – Beltrami operator, then

$$Y^m_l(\Theta,\varphi) = \left\{ \begin{array}{c} P^m_l(\cos\Theta)\cos m\varphi,\ m = 0, 1, ..., l; \\ P^{|m|}_l(\cos\Theta)\sin|m|\varphi,\ m = -l, -(l+1), ..., -1 \end{array} \right. \tag{15}$$

are corresponding eigenfunctions, orthonormal with respect to the scalar product (4). Here

$$P_l(t) = \frac{1}{2^l l!}\frac{d^l}{dt^l}(t^2 - l)^l$$

is a Lagrange polynomial of degree l, and

$$P_l^{|m|}(t) = (1 - t^2)^{\frac{|m|}{2}}\frac{d^{|m|}}{dt^{|m|}}P_l(t) \tag{16}$$

are associated Lagrange polynomials. The scalar product is calculated by the formula

$$(Y_{l1}^{m1}, Y_{l2}^{m2}) = \int_0^{2\pi} \cos m_1\varphi \cos m_2\varphi d\varphi \int_{-1}^{1} P_{l1}^{m_1}(t) P_{l2}^{m_2}(t) dt. \tag{17}$$

Remark 2. Eigenfunctions of the Laplace – Beltrami operator For 1-forms, eigenvalues of the Laplace – Beltrami operator can be calculated using the formula $y_l = \delta Y_l^m(\Theta, \varphi)$. Since the space of 2-forms is isomorphic to the space of 0-forms, the eigenfunctions are determined by the formula (17).

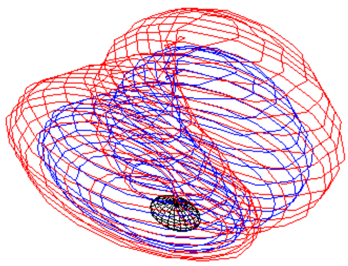

Fig. 1. Stable solutions (green color stands for $t = 0.1$, blue color stands for $t = 0.5$) for $\lambda = -3.6$, $\nu = 0.5$ on a two-dimensional sphere (black)

Let us construct an algorithm to study the stability of solutions to equation (12) in the space of differential forms $\mathbf{H}^0_{0\mathbf{K}}$.

Step 1. Input coefficients of the equation (2) $\lambda \in \mathbb{R}_-$, $\nu \in \mathbb{R}$, the number of eigenvalues of the Laplace – Beltrami operator L, the number of random **K**-variables K.

Step 2. Find the relative spectrum

$$\mu_l = \frac{-\nu\vartheta_l}{\lambda + \vartheta_l},$$

where $\vartheta_l = l(l + 1)$.

Step 3. Construct the array Y_l^m of eigenvalues of the Laplace – Beltrami operator by the formula (15) and the array $P_l^{|m|}$ by the formula (16). Legendre polynomials are formed using the function $P(l, \cos(\Theta))$ from the built-in module *Orthopoly*.

Step 4. Using the random number generator from the built-in module *Random* generate a sequence of random variables ξ_k with normal distribution, zero mathematical expectation and variance equal to one.

Step 5. If $\nu > 0$ then the number M_2 is equal to the number of eigenvalues ϑ_k satisfying the inequality $\vartheta_k < -\lambda$, $M_1 = K - M_2$. Find a stable solution

$$\zeta_1(t) = \sum_{l=M_1}^{K} e^{\mu_l t} \left(\sum_{k=1}^{K} (\vartheta_k)^{-1} \xi_k \sum_{m_1=1}^{k} \sum_{m_2=1}^{l} (Y_l^{m_1}, Y_k^{m_2}) Y_l^{m_1} \right) \tag{18}$$

and an unstable solution

$$\zeta_2(t) = \sum_{l=1}^{M2} e^{\mu_l t} \left(\sum_{k=1}^{K} (\vartheta_k)^{-1} \xi_k \sum_{m_1=1}^{k} \sum_{m_2=1}^{l} (Y_l^{m_1}, Y_k^{m_2}) Y_l^{m_1} \right) \tag{19}$$

to the equation (13). Here the dot product is calculated by the formula (17).

Step 6. If $\nu < 0$ then a stable solution has the form (19) and an unstable solution has the form (18).

Step 7. Plot solutions of the equation (13) in a spherical coordinate system.

Remark 3. The stability and instability of solutions of the equation (13) are understood here in the sense of Definition 4.

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ЭКСПОНЕНЦИАЛЬНЫЕ ДИХОТОМИИ ОДНОГО СТОХАСТИЧЕСКОГО НЕКЛАССИЧЕСКОГО УРАВНЕНИЯ НА ДВУМЕРНОЙ СФЕРЕ

О. Г. Китаева

В статье рассматривается стохастический аналог линейного уравнения Осколкова, которое получается из системы уравнений Осколкова. Показано существование решения стохастического уравнения Осколкова в пространствах дифференциальных форм, заданных на сфере. При некоторых значениях параметров, характеризующих свойства жидкости, доказано существование экспоненциальных дихотомий решений. Для решения вопроса о существовании и устойчивости решений, данное уравнение рассматривается как частный случай линейного однородного стохастического уравнения соболевского типа. В качестве производной рассматривается производная Нельсона – Гликлиха стохастического процесса. Для визуализации полученных результатов составлен алгоритм для вычисления устойчивого и неустойчивого решений стохастического уравнения Осколкова в пространствах 0-форм на двумерной сфере. Данный алгоритм реализован в среде Maple. Построены в сферической системе координат графики решений имеющих экспоненциальную дихотомию.

Ключевые слова: уравнения соболевского типа; стохастические уравнения; дифференциальные формы; экспоненциальные дихотомии.

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