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INFORMATION–LOGICAL MODELLING IN STUDIES OF NON-CLASSICAL LINEAR MODELS OF MATHEMATICAL PHYSICS

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The article proposes an information-logical model for the study of non-classical linear models of mathematical physics. Information-logical modelling is based on the stages of research of mathematical models and methods of system analysis. The decomposition carried out takes into account the following: the peculiarities of analytical and numerical methods for studying various initial-boundary value problems for Sobolev-type equations, various applied problems that are solved using non-classical linear models of mathematical physics. When constructing an information-logical model for the study of non-classical linear models of mathematical physics with a random external influence, general structural elements were identified, which made it possible to represent a set of studied information objects, their attributes and relations between them.

Keywords: information-logical modelling; system analysis; non-classical models of mathematical physics.

Introduction

In theoretical and applied research related to the problems of information processing and analysis, identification and control, stochastic models are used to assess the state of complex physical and financial systems and their parameters. Despite the fact that, for describing and modelling a large number of physical, technical and technological processes, the stochastic Sobolev-type equations

$$Ld\eta = M\eta dt + N d\omega \tag{1}$$

are used, where L, M and N are linear continuous operators acting from the Hilbert space $\mathfrak{U}$ into the Hilbert space $\mathfrak{F}$; $\eta = \eta(t)$ is a required process, and $\omega = \omega(t)$ is a given stochastic K-process. In each case, equation (1) is endowed with either the Cauchy equation

$$\eta(0) = \xi_0, \tag{2}$$

the Showalter–Sidorov condition

$$P(\eta(0) - \xi_0) = 0, \tag{3}$$

or the initial-final value condition

$$P_0(\eta(0) - \xi_0) = P_1(\eta(t_1) - \xi_1) = 0. \tag{4}$$

Here P, P_0 and P_1 are relatively spectral projectors, and

$$\xi_0 = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\xi_{0k}\varphi_k, \quad \xi_1 = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\xi_{1k}\varphi_k,$$

$\xi_0, \xi_1 \in \mathbf{L}_2$ are pairwise independent Gaussian random variables such that $D\xi_{0k}, \quad D\xi_{1k} \leq C_j$, $k \in \mathbf{N}$, $j = \overline{0, m}$.

The purpose of this article is information-logical modelling of research of non-classical linear models of mathematical physics with a random external influence. The method of information-logical modelling, using the methods of structural system analysis, makes it possible to represent a set of information objects, their attributes and the relationship between them [1].

1. Main Blocks of Information–Logical Model

The process of constructing an information-logical model for the study of non-classical linear models of mathematical physics begins with constructing a context diagram (Fig. 1), which displays the process of work with the mathematical model as a whole.

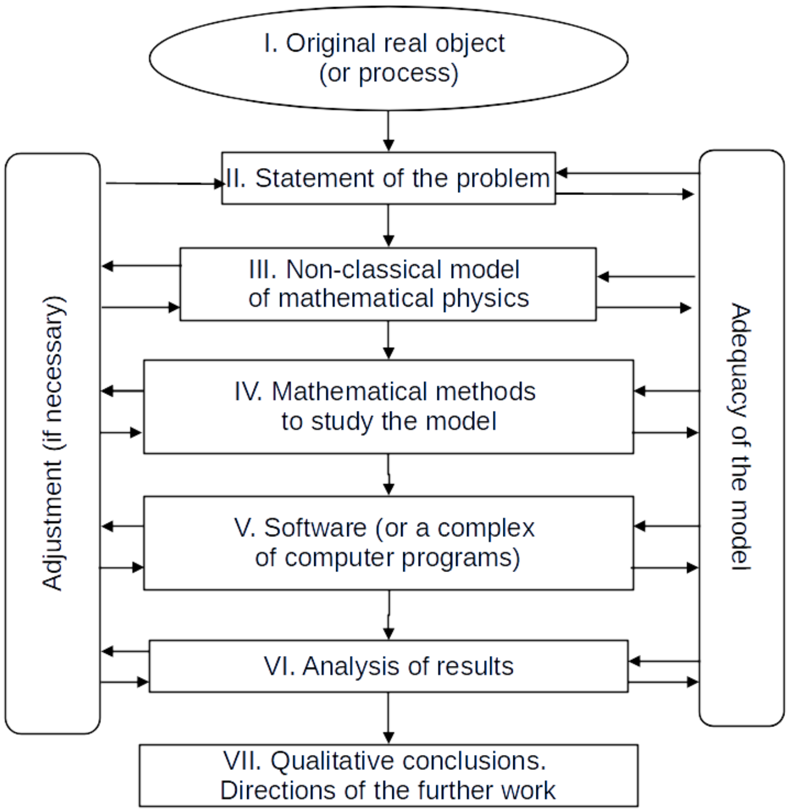

Fig. 1. Context diagram of work with non-classical models of mathematical physics

There are seven main stages: description of the original real object (or process), allowing to start work with it; statement of the problem, allowing its formalization; construction of a non-classical model of mathematical physics; application of mathematical methods to study the model; development and application of a software (or a complex of computer programs); analysis of results; conclusions on the result of work. Note that the seven stages are defined on the basis of generally accepted approaches to construction of a mathematical model (for example, [2]), taking into account its integration into the processes of systems analysis and synthesis (for example, [3]). In the context diagram, the stage numbers are indicated by Roman numerals.

After describing the process of work with a mathematical model as a whole (context), its functional decomposition is carried out. The top-level decomposition diagram (Fig. 2) consists of seven main functional blocks.

Within the framework of constructing decomposition diagrams, both of the second and subsequent levels, the dashed lines of structural objects highlight those elements that are directly related to this study.

The analysis of the studied objects and processes using non-classical models of mathematical physics showed that the following processes are currently the most actively studied [4 – 13]:

– filtration of liquid in fractured porous and other media;

– distribution of the potential of the speed of movement of the free surface of the filtering liquid;

– evolution of the free surface of the fluid filtering in a reservoir of limited thickness;

– deformation of the I-beam and the I-beam structure;

– vibrations in a thin rod and in the construction of thin elastic rods;

– flow of a viscoelastic fluid through a pipeline;

– propagation of long waves in shallow water;

– small vibrations of the rotating fluid;

– motion of an incompressible viscoelastic fluid in the earth's magnetic field;

– dynamics of a weakly compressible viscoelastic fluid;

– propagation of a potential electric field in a semiconductor;

– propagation of Rossby waves or planetary waves, which have a long-wave character and a low frequency;

– propagation of linear waves in plasma in an external magnetic field;

– vibrations in the DNA molecule;

– propagation of a nerve impulse in the membrane sheath;

– processes of the type «reaction – diffusion».

Fig. 3 shows the indicated processes in the form of a hierarchical tree structure. The group of the first two digits clearly postulates the structural connections of each process with the objects of the decomposition diagram.

There are four goals of mathematical modelling:

– descriptor goal related to the description and understanding of the device of a specific object, its structure and basic properties, the laws of its dynamics and relationship with the surrounding world;

– optimization goal associated with the search for the best values of the parameters and (or) characteristics of the model under the given criteria;

– control goal related to the determination of the control capabilities of an object or

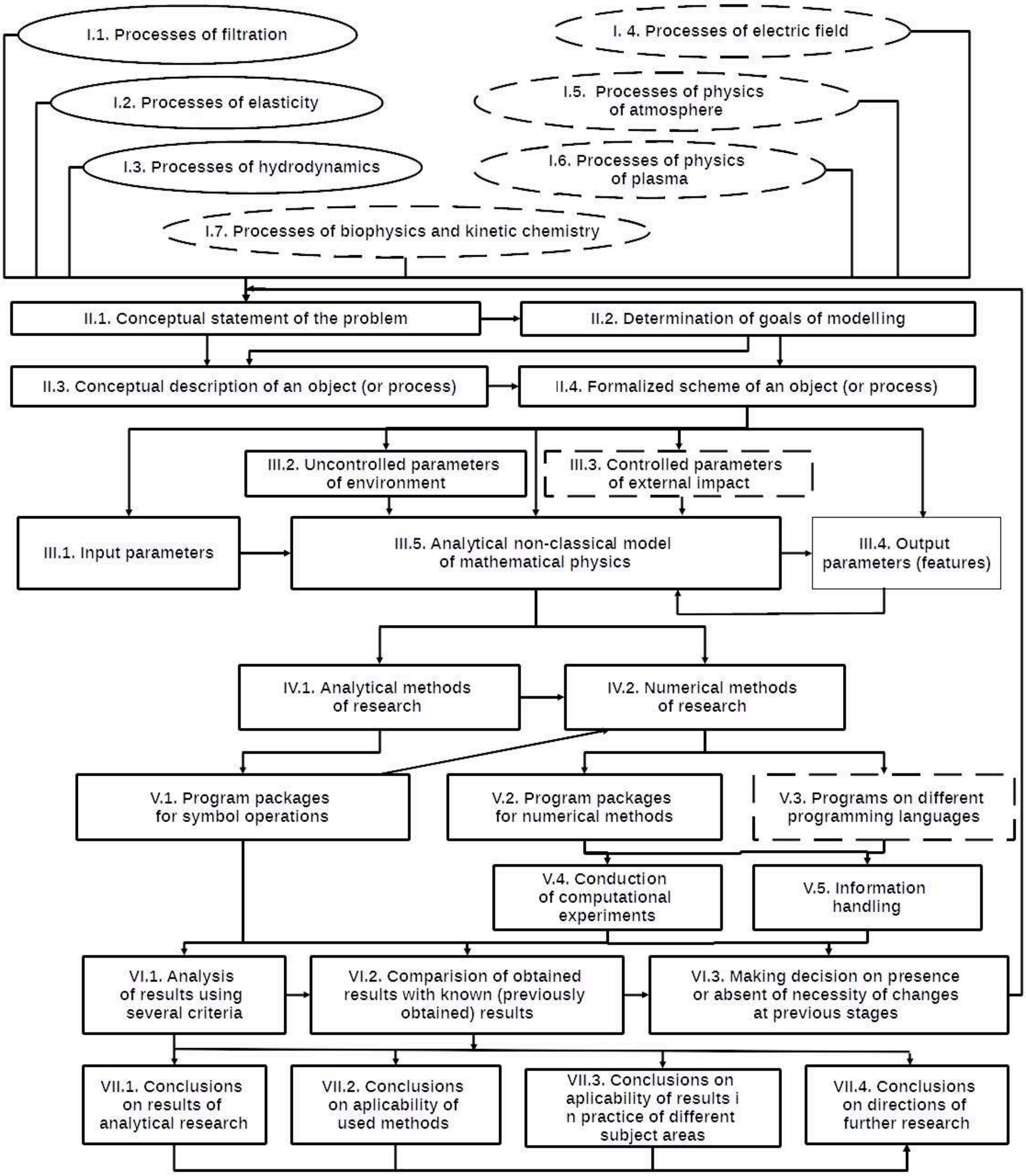

Fig. 2. Top-level decomposition diagram

process and finding the best control method for the given goals and control criteria;

– predictive goal associated with predicting the consequences of the implementation of specified methods and forms of impact on an object or process.

Note that, in the ongoing research of non-classical models of mathematical physics, only the following two types of goals were realized: descriptor ones, for example, [7, 10, 13] and control ones, for example, [14, 12, 15]. Fig. 4 shows the structure of non-classical models of mathematical physics in relation to various goals of mathematical modelling.

So, if the goal of modelling is control, then the mathematical model includes the penalty functional, control criteria.

Control problems were not considered within the framework of the ongoing studies of stochastic nonclassical linear models of mathematical physics. When the blocks of the

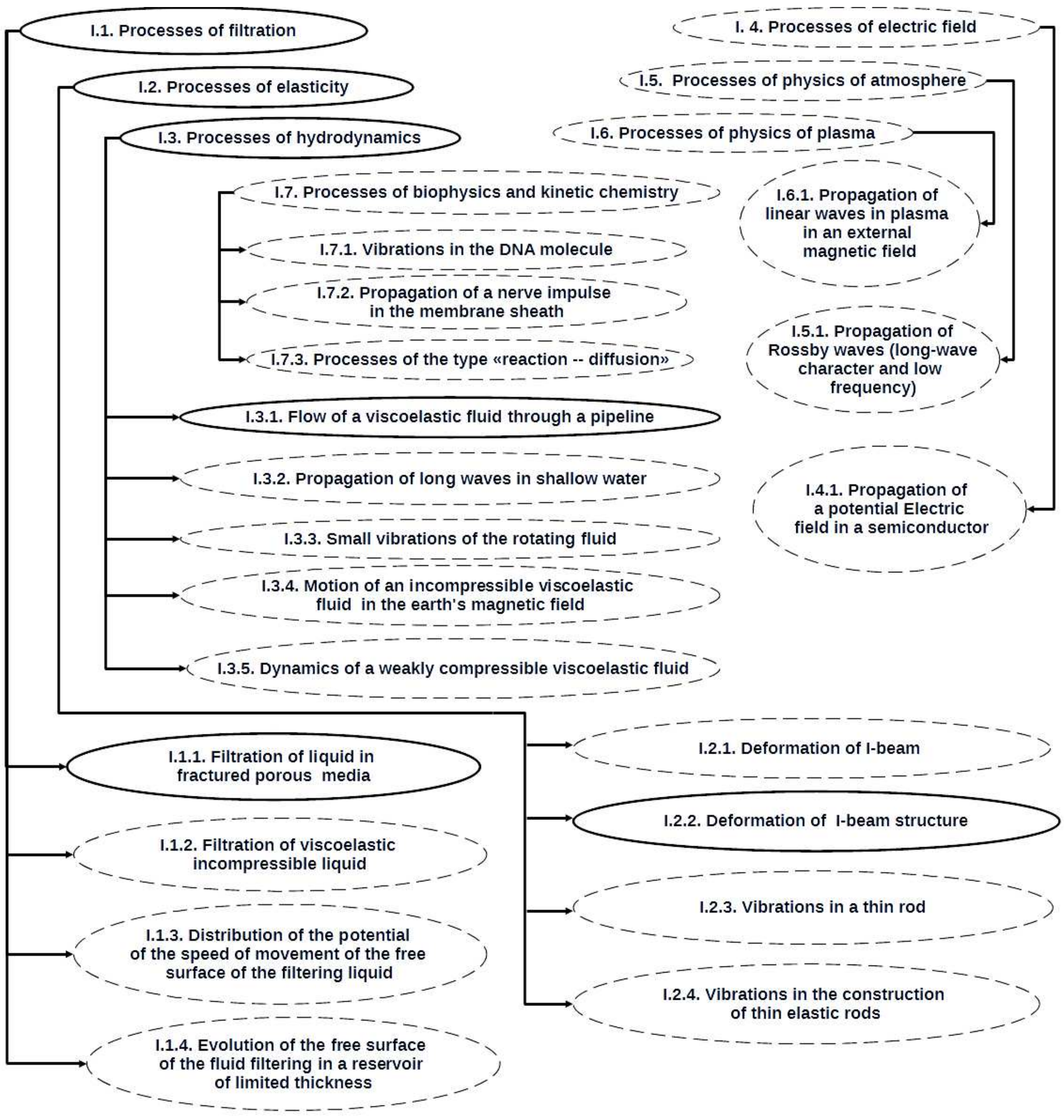

Fig. 3. Investigated processes using non-classical models of mathematical physics

mathematical model are decomposed (Fig. 5), it is assumed that the input parameters can be both stochastic and deterministic. For example, the initial state can be specified as a random variable, and the boundary condition can be specified as a deterministic zero value. In addition, in the block of initial and boundary conditions, the Cauchy condition, the Showalter–Sidorov condition [16], the initial–final value condition [17], and the multipoint initial–final value condition [11] are highlighted. Note that, if a mathematical model assumes consideration of a graph [18], then, in the block of initial and boundary value conditions, the condition of the balance of flows and the conditions of continuity at the vertices of the graph are distinguished. An important characteristic of a mathematical model is the presence of an external uncontrolled or controlled impact, while in the framework of the study, mathematical models are studied in the presence of an uncontrolled external impact of a stochastic nature.

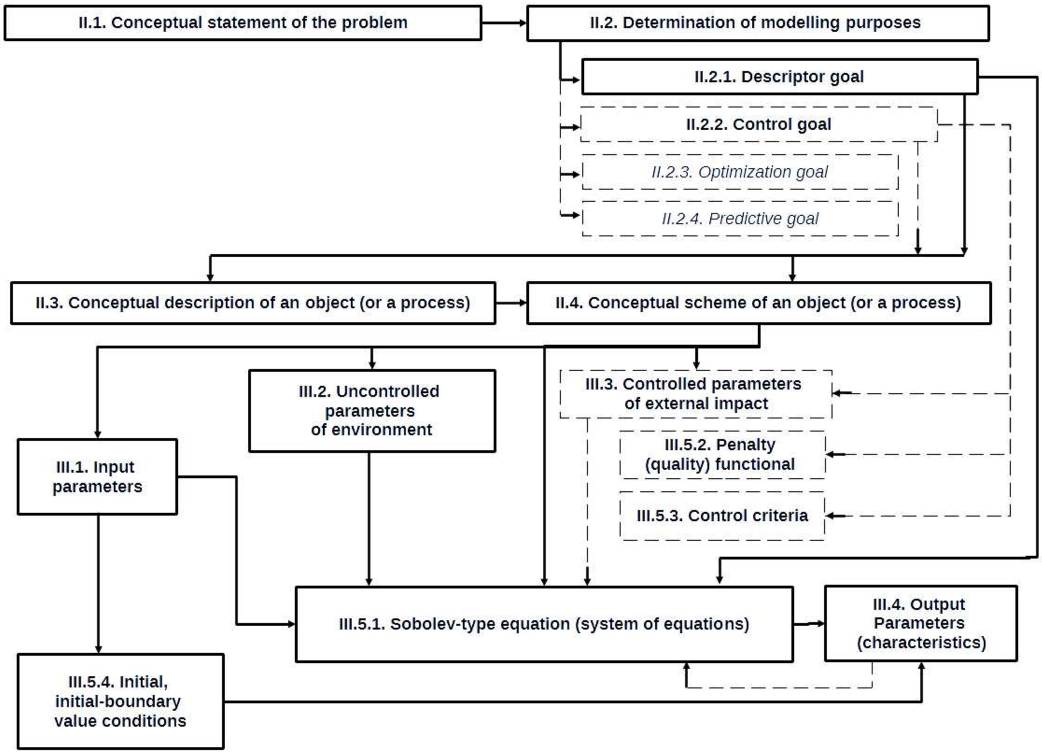

Fig. 4. Structure of non-classical models of mathematical physics, taking into account the goals of mathematical modelling

When decomposing the block of analytical research, the following two main directions are identified: the mathematical foundations of the research and theoretical research of the considered mathematical problems (Fig. 6).

Mathematical foundations involve the development of mathematical theories and methods that allow theoretical research. As the main ones, the following methods of classical theories and branches of mathematics are highlighted: functional analysis; equations of mathematical physics. For research of nonclassical models of mathematical physics, the following are used [19, 20]:

– theory of relatively p-bounded operators and the resolving groups generated by them;

– theory of relatively p-sectorial operators and the analytic resolving semigroups generated by them;

– theory of relatively p-radial operators and strongly continuous resolving semigroups generated by them.

When considering finite-dimensional mathematical models [21], the theory of descriptor (or algebraic – differential) systems [21, 23], is used, including the theory of Leontief-type systems.

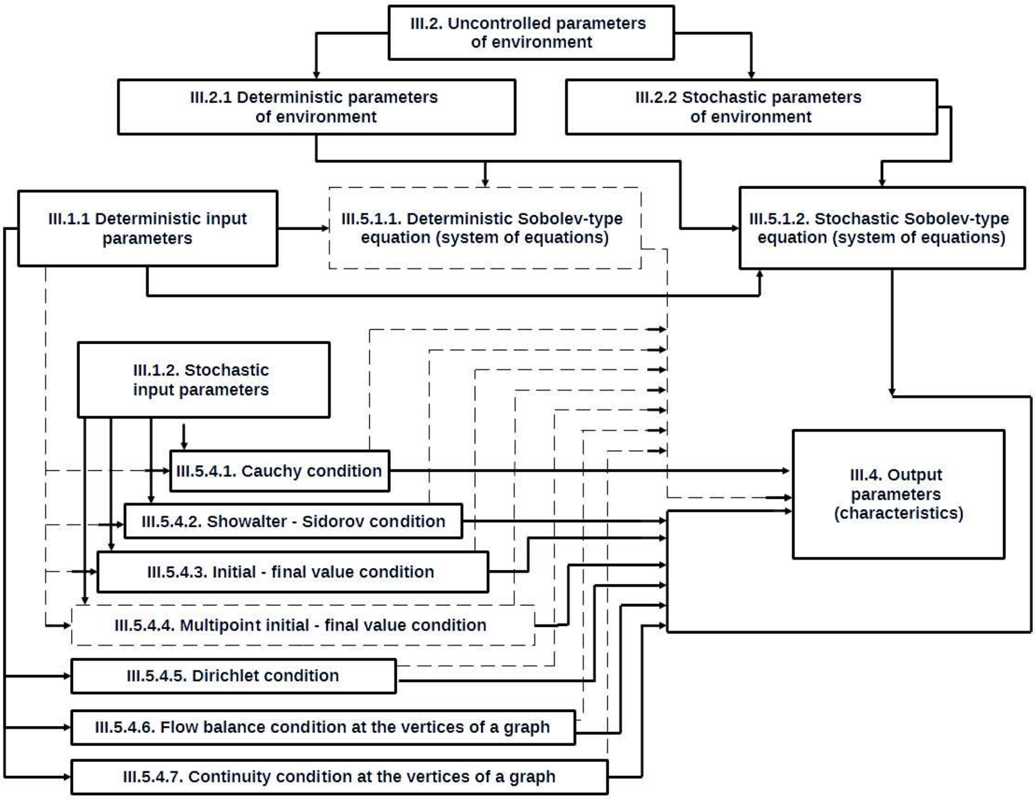

Fig. 5. Decomposition of the structure of non-classical models of mathematical physics

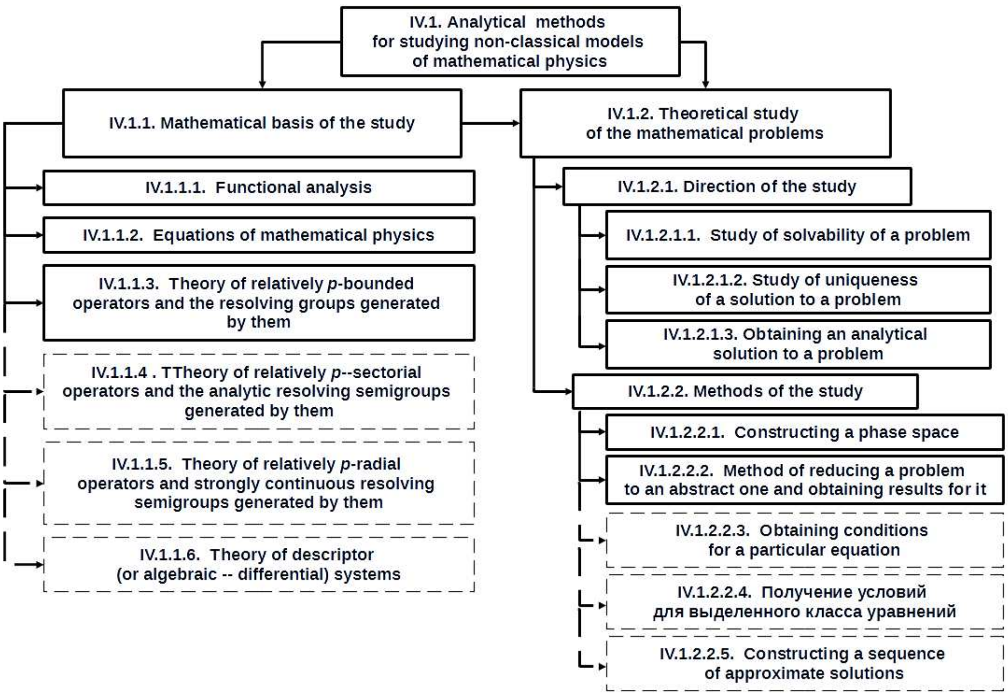

Fig. 6. Analytical methods of research of non-classical models of mathematical physics

When carrying out theoretical studies, directions are distinguished related to the study of the solvability of various problems for Sobolev-type equations, obtaining conditions for the uniqueness of a solution, and obtaining an analytical solution to the problem. As the research methods that are used in this case, let us single out those that are used in the works of the scientific school of Professor G.A. Sviridyuk, and in other scientific schools [5, 7, 8, 24, 25].

In the study of a mathematical model with the initial Cauchy condition, a method for constructing a phase space was developed in the Chelyabinsk scientific school [24]. Most of the mathematical models are reduced to an abstract problem, which is then investigated on the basis of the theory of degenerate (semi)groups. In line with these methods, stochastic nonclassical linear models of mathematical physics are studied, which are presented in the next paragraph of the article.

Fig. 7 shows the decomposition of the block of numerical research methods.

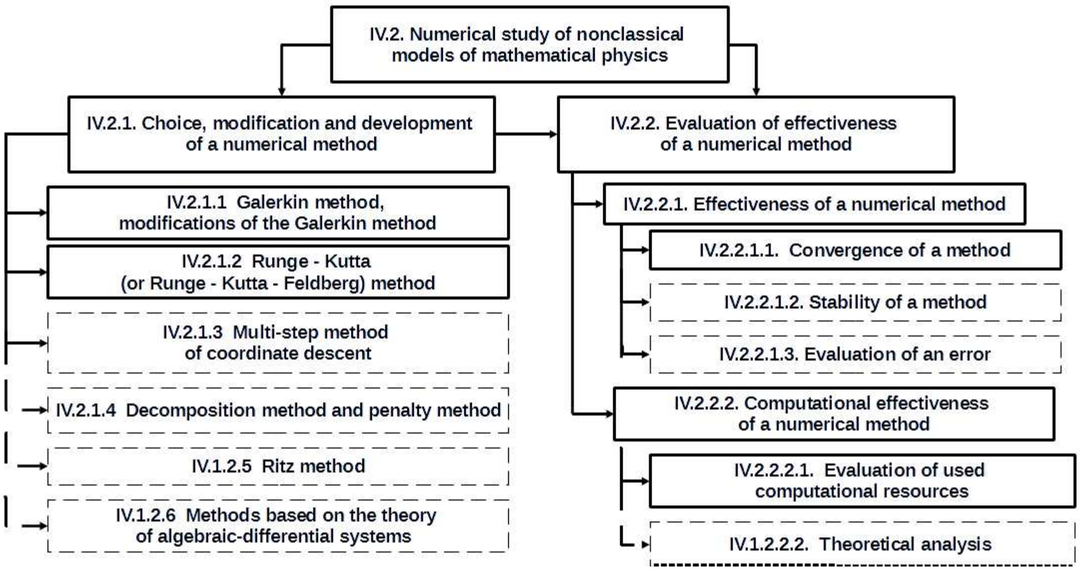

Fig. 7. Numerical methods for studying non-classical models of mathematical physics

The first step is the selection, modification or development of a numerical method. After analyzing various studies, the following approaches were identified:

– modifications of the Galerkin method [10–12];
– Runge – Kutta (or Runge – Kutta – Feldberg) method [26];
– multi-step method of coordinate descent and its modifications [27, 12];
– decomposition method and penalty method [12];
– Ritz method [27, 12];
– methods based on the theory of algebraic-differential systems [28, 29].

An important stage in the numerical study is the evaluation of the efficiency of the constructed algorithm. The main elements of such an assessment are to clarify issues related to the convergence and (or) stability of the numerical method. In addition, the estimation of the errors of the method for one or another of its characteristics gives an idea of the effectiveness of the numerical algorithm. It is important to estimate the computational resources used in the implementation of the algorithm.

The decomposition of the software block for the study of a mathematical model is shown in Fig. 8. The program can be implemented in such mathematical software packages as Matlab, Maple, Mathcad, etc., or written in various programming languages, for example, C++, Python, Fortran, etc. Undoubtedly, the important elements of software implementation are computational experiments and processing of information obtained on their basis.

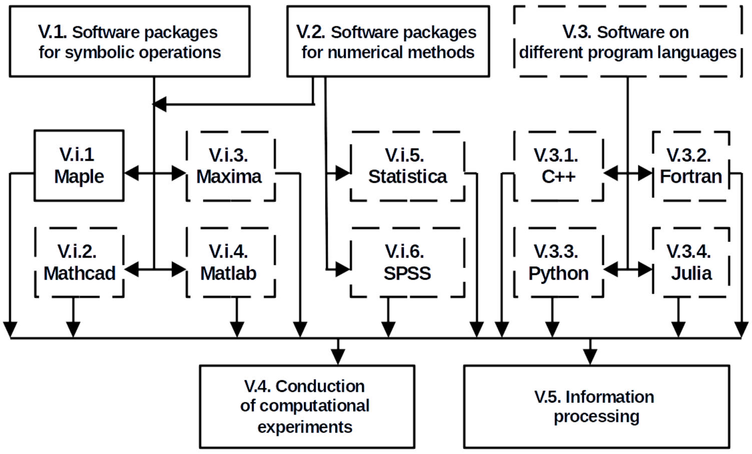

Fig. 8. Structural diagram of the software stage for work with a mathematical model

2. Information – Logical Model of Research of Stochastic Nonclassical Linear Models of Mathematical Physics

Let us apply the method of information-logical modelling to the study of three non-classical linear models of mathematical physics with a random external influence (Fig. 9).

Stochastic Barenblatt – Zheltov – Kochina model [30]. Let $G \subset \mathbb{R}^d$ be a bounded domain with the boundary ∂G of the class C^∞. Let us search for $\eta = \eta(x,t)$, satisfying, in the cylinder $G \times \mathbb{R}_+$, the equation

$$(\lambda - \Delta) d\eta = \alpha \Delta \eta dt + N d\omega, \tag{5}$$

the Dirichlet condition

$$\eta(x,t) = 0, \ (x,t) \in \partial G \times \mathbb{R}_+, \tag{6}$$

and the Cauchy condition

$$\eta(x,0) = \eta_0(x). \tag{7}$$

Here the parameter $\alpha \in \mathbb{R} \setminus \{0\}$, $\lambda \in \mathbb{R}$ characterizes the environment. Model (5), (6) describes the dynamics of fluid pressure, which is filtered in the fractured porous media. (Note that this equation has universal character, since it also simulates the process of moisture transfer in soil and the process of heat conduction with two temperatures).

In Fig. 9, consideration of this model begins with pp. I.1 Filtration processes – I.1.1. Filtration of liquids in fractured porous media – II.1.1. The Barenblatt - Zheltov - Kochina model is used to find the pressure of a liquid filtering in fractured porous media.

Oskolkov's stochastic model on a graph [31]. Let $\boldsymbol{G} = \boldsymbol{G}(\mathfrak{V}; \mathfrak{E})$ be a finite connected oriented graph. Here, denote by $\mathfrak{V} = \{V_i\}$ the set of vertices, and by $\mathfrak{E} = \{E_j\}$ the set of edges. On the edges E_j of the graph $\boldsymbol{G}$, we define the one-dimensional linear stochastic Oskolkov equations

$$\lambda_j du_j - du_{jxx} = \alpha_j u_j dt + N d\omega, \tag{8}$$

with the Showalter–Sidorov condition

$$P(u(0) - \xi_0) = 0, \tag{9}$$

and, at the vertices V_i of the graph $\boldsymbol{G}$, we set the continuity conditions

$$\begin{gathered} u_j(0,t) = u_k(0,t) = u_m(l_m,t) = u_n(l_n,t), \\ E_j, E_k \in E^{\alpha}(V_i), E_m, E_n \in E^{\omega}(V_i) \end{gathered} \tag{10}$$

and the flow balance conditions

$$\sum_{E_j \in E^{\alpha}(V_i)} d_j u_{jx}(0,t) - \sum_{E_k \in E^{\omega}(V_i)} d_k u_{kx}(l_k,t) = 0, \tag{11}$$

where P is a relatively spectral projector, $E^{\alpha(\omega)}(V_i)$ is the set of edges having begin (end) at the vertex V_i. It is important to note several separate cases for the vertices and edges of the graph.

(i) If a graph has two vertices (i.e., the graph is represented as one non-cyclic edge), then condition (10) is absent, and condition (11) is transformed into the Neumann condition.

(ii) If a graph has one vertex (i.e., the edge is cyclic), then conditions (10), (11) turn into a matching condition.

Equation (8) simulates the pressure and velocity dynamics of a viscoelastic incompressible fluid moving in the j-th section of the pipeline. As an example of such a liquid, we can use highly paraffinic oil grades, which are produced in the fields of Western Siberia. The parameter $\alpha \in \mathbb{R} \setminus \{0\}$ characterizes the elasticity of the fluid, the parameter $\lambda \in \mathbb{R}$ describes the viscosity of the fluid, the random process $u_j = u_j(x,t), (x,t) \in (a,b) \times \mathbb{R}$, characterizes the change in the velocity and pressure of a viscoelastic incompressible fluid in the j-th section of the pipeline.

In Fig. 9, consideration of this model begins with pp. I.3. Hydrodynamic processes - I.3.1. Flow of a viscoelastic fluid through a pipeline – II.1.1. The Oskolkov model is used to find the pressure of a viscoelastic incompressible fluid in pipeline sections.

Stochastic Hoff model on a graph [32]. Let $\boldsymbol{G} = \boldsymbol{G}(\mathfrak{B}; \mathfrak{E})$ be a finite connected oriented graph. Denote by $\mathfrak{B} = \{V_i\}$ the set of vertices, and denote by $\mathfrak{E} = \{E_j\}$ the set of edges. In addition, each edge E_j has two parameters. First, it has a length, which we denote by $l_j > 0$. Second, it has a cross-sectional area, which we denote by $d_j > 0$. Next, consider the Hoff linear stochastic model on the graph $\boldsymbol{G}$

$$\lambda_j du_j + du_{jxx} = \alpha_j u_j dt + N_j d\Omega_j, \tag{12}$$

with the initial-final value condition

$$P_0(u(\tau_0) - \xi_0) = P_1(u(\tau_1) - \xi_1) = 0, \tag{13}$$

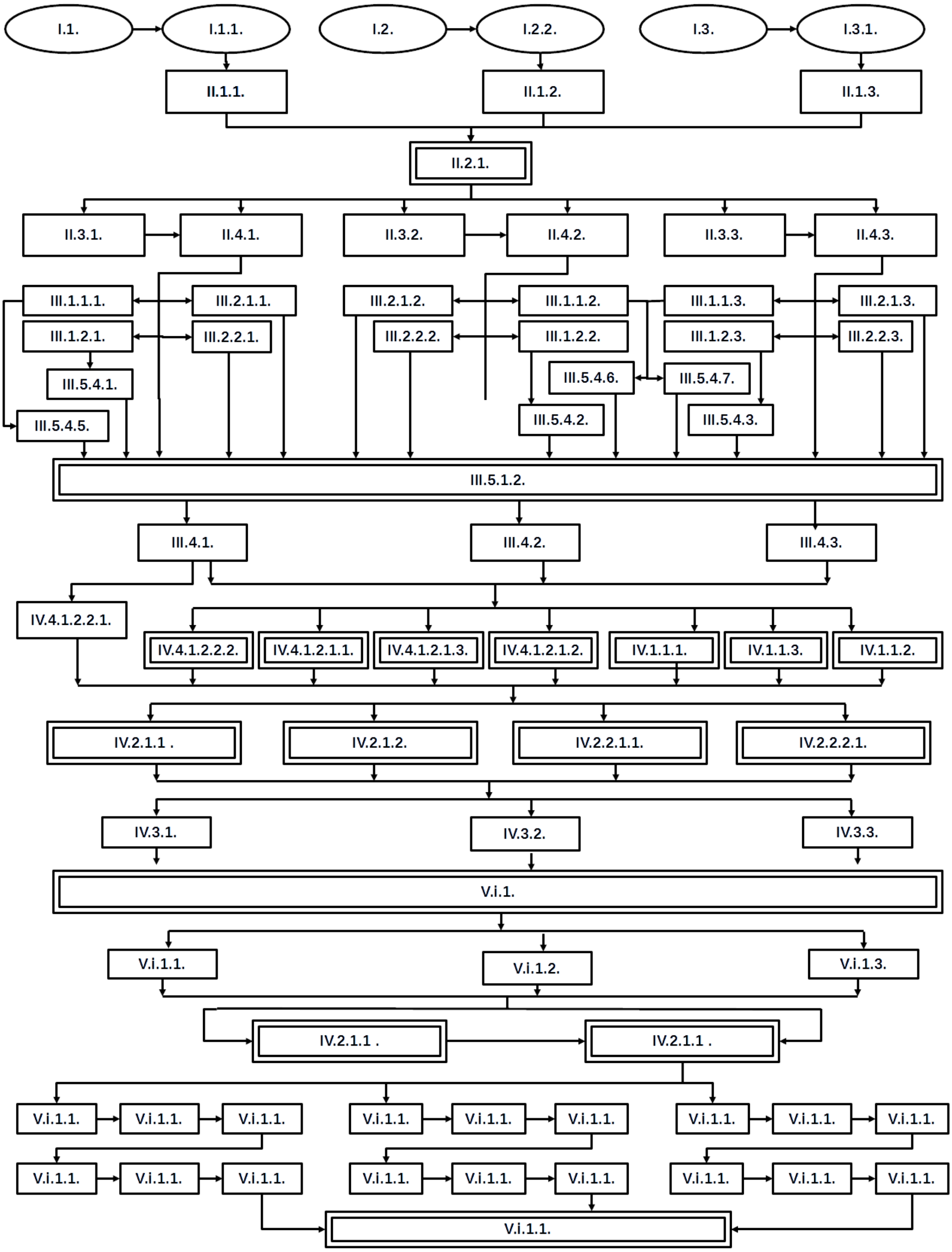

Fig. 9. Information-logical research model

and, at the vertices $\mathfrak{B}$ of the graph $\boldsymbol{G}$, we set the continuity condition

$$\begin{gathered} u_j(0,t) = u_k(0,t) = u_m(l_m,t) = u_n(l_n,t), \\ E_j, E_k \in E^{\alpha}(V_i), E_m, E_n \in E^{\omega}(V_i), \end{gathered} \tag{14}$$

and the flow balance condition

$$\sum_{j:E_j \in E^{\alpha}(V_i)} d_j u_{jx}(0,t) - \sum_{k:E_k \in E^{\omega}(V_i)} d_k u_{kx}(l_k,t) = 0. \tag{15}$$

Equation (13) describes the buckling dynamics of I-beams in a structure under a constant load with a random external action. Here, the parameter $\lambda \in \mathbb{R}_+$ characterizes the load on the j-th beam, the parameter $\alpha \in \mathbb{R}$, in turn, describes the properties of the j-th beam material, the random process $u_j = u_j(x,t), (x,t) \in (a,b) \times \mathbb{R}$ characterizes the deviation of the j-th beam from the position equilibrium, $dW_j = dW_j(t)$ corresponds to a random load on the j-th beam, P_0, P_1 are relatively spectral projectors.

In Fig. 9, consideration of this model begins with pp. I.2. Elasticity processes – I.2.2. Deformation of an I-beam structure – II.1.2. Hoff's model is used to find the buckling dynamics of I-beams in a structure.

In the information-logical model of the study of all three models, there are structural elements inherent in all three models, they are marked with double lines in Fig. 9, for example, pp. II.2 – Descriptor purpose of mathematical modelling. Common structural elements are as follows: stochastic linear Sobolev type equation; the theory of relatively p-bounded operators and the theory of degenerate groups generated by them; a study of the solvability and uniqueness of the solution with obtaining an analytical solution; in the numerical study, modifications of the Galerkin method are used, and the Matlab program is used to implement them. These structural elements create an unified concept for the study of different three mathematical models. Differences in other structural elements are due to different parameters and characteristics of the models, different initial conditions and other features of the investigated mathematical models.

In conclusion, we note that the construction of an information-logical model allows achieving the goals of cross-disciplinary research developments in various subject areas.

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ИНФОРМАЦИОННО–ЛОГИЧЕСКОЕ МОДЕЛИРОВАНИЕ В ИССЛЕДОВАНИЯХ НЕКЛАССИЧЕСКИХ ЛИНЕЙНЫХ МОДЕЛЕЙ МАТЕМАТИЧЕСКОЙ ФИЗИКИ

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В статье предложена информационно–логическая модель исследования неклассических линейных моделей математической физики. В основе информационно–логического моделирования лежат этапы исследования математических моделей и методы системного анализа. Проводимая декомпозиция учитывает: особенности аналитических и численных методов исследования различных начально-краевых задач для уравнений соболевского типа, различные прикладные задачи, которые решаются с использованием неклассических линейных моделей математической физики. При построении информационно–логической модели исследования неклассических линейных моделей математической физики со случайным внешним воздействием выделены общие структурные элементы, что позволило представить совокупность изучаемых информационных объектов, их атрибутов и отношений между ними.

Ключевые слова: информационно–логическое моделирование; системный анализ; неклассические модели математической физики.

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