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CONSTRUCTION OF OBSERVATIONS BASED ON DATA DISTORTED BY INTERFERENCE OF VARIOUS TYPES

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The article describes the preliminary stage of the optimal dynamic measurement problem. Namely, an algorithm for constructing observation values based on the values obtained during the experiment is given. We assume that the experimental data can be affected by various types of interference, including «white noise», which is understood as a derivative of Nelson – Gliklikh from the Wiener process. To construct the observation values, a priori information about the form of the function describing the observation values is used. The article consists of two parts. The first part contains an algorithm for constructing the observations values. And in the second part, the results of computational experiments are presented.

Keywords: useful part of the signal; convex up function; statistical hypothesis.

Introduction

The study of the optimal dynamic measurement problem [1, 2] is based on the search for the penalty functional optimum for the norm of the difference between real observation and virtual one. By the real observation we mean a values based on data obtained during a full-scale experiment. And by virtual observation we mean data obtained using a computational algorithm [3]. This optimum is declared an optimal dynamic measurement. To date, within the framework of the optimal dynamic measurements theory [1, 2, 3], cases have been investigated when the measurement is distorted by the inertia of the measuring device [4], resonances in its circuits [5] or its degradation [6]. The article [7] considers a model that takes into account the distortion of measurement by all three interferences simultaneously. However, due to the fact that the basis for solving this problem is the theory of optimal control by solutions of Leontief type equations [8], then real and virtual observation should be sufficiently smooth functions. And if for the virtual observation this means choosing the type of this function, then for the real observation it is rarely possible without preprocessing the data [9]. This leads to interest in the preliminary stage of the optimal dynamic measurement problem, which we call the stage of constructing observations from distorted data.

Let's consider the results of real observations distorted by interference, i.e. we will assume that they have the form

$$\eta(t) = y_e(t) + \nu(t),$$

where $y_e(t)$ is the useful part of observations and $\nu(t)$ is the distort part of measurements. Unlike the case discussed in [10], this article assumes that the noises can be different. For the sake of certainty, we have considered two cases. The first one is the case of the normal distribution of interference, i.e. $\nu(t)$ is a Wiener process. And the second one is the case of

$\nu(t)$ is a «white noise» [7], which is understood as a derivative of Nelson –Gliklikh [11, 12] from the Wiener process. When constructing the reconstructed values, we assume that the useful part of the signal is described by a smooth convex up function with a single maximum.

1. The Algorithm of Constructing the Useful Part of Observation

Suppose that, as a result of an experiment, we simultaneously observe variables characterizing the observed process at time interval $[a, b]$, $a > 0$. This interval has a sampling frequency of N, i.e. $\{t_j : j \in \mathcal{I}\}$, $\mathcal{I} = \{0, 1, \ldots, N\}$, $t_0 = a$, $t_N = b$. As a result of such observations, we obtain $\eta(t_j)$ $(j = \overline{0, N})$. In addition, we know a priori information about the useful part of observed variable $\eta(t)$. Namely, the useful part of the signal is described by a smooth upward convex function with a single maximum. However, due to the influence of random noise, the observed variables $\{\eta(t_j)\}_{j=0}^{N}$ do not have these properties. Taking in account these assumptions there is the algorithm of constructing the useful part of observation distorted by Wiener process or «white noise» (see Fig. 1).

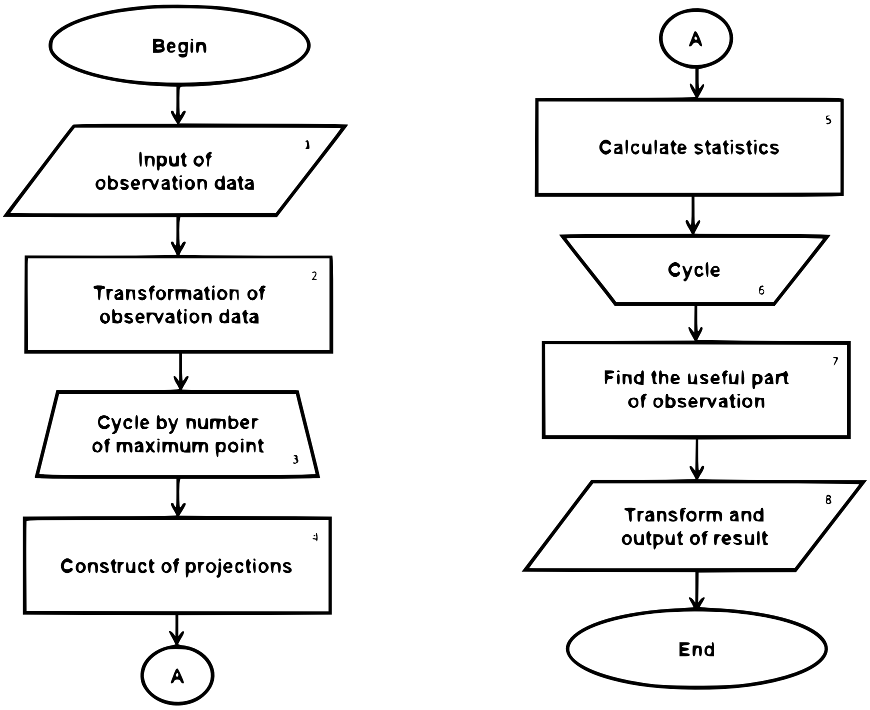

Fig. 1. Algorithm of constructing the useful part of observation

Let the shape of the useful signal depend on the parameter $k \in \mathcal{I}$, which indicates the position of the maximum point. We find with a given probability γ an estimate of the parameter k_0 from the measured signal

$$\eta(t) = y_e(t) + \nu(t), \quad y_e \in V_{k_0}, \quad \forall t \in [t_0; t_N], \tag{1}$$

where $\nu(t)$ is a Wiener process or «white noise» [7]. That is, we assume that at the useful component of the signal $y_e(t)$ has a maximum in the point k_0 of a uniform grid, i.e. $y_e \in V_{k_0}$.

Here

$$V_k = \left\{ f \in C[t_0, t_N] : \begin{array}{ll} 2f(t_j) \geq f(t_{j-1}) + f(t_{j+1}), & j = \overline{1, N-1}, \\ f(t_j) \leq f(t_{j+1}), & j = \overline{0, k-1}, \\ f(t_j) \geq f(t_{j+1}), & j = \overline{k, N-1} \end{array} \right\}. \quad (2)$$

is a class of functions of upward convex functions with a single maximum at the point t_k, which are defined on a uniform grid $\{t_j\}_{j=0}^N$.

Based on the results of [7], to estimate the parameter $k_0 \in \mathcal{I}$, we use either the statistics τ_k from [13] for the Wiener process $\nu(t)$, or the statistics τ_k from [10] for ≪white noise≫ $\nu(t)$. The value of the constructed statistics is used to find the value of the parameter k, at which the useful part of the signal (1) is closest in shape with its projection on the set of functions (2). The problem of constructing the values of observations on a uniform grid $\{t_j\}_{j=0}^N$ is considered as the problem of the best approximation by elements of the set V_k from (2) and its solution is given in [7].

The steps of the algorithm in Fig. 1 for interference $\nu(t)$ in the form of ≪white noise≫ are described in [10]. For the Wiener process in this algorithm, the calculation formulas of 2 and 5 blocks are changed to the standard [13], and in 8 block, the need for data transformation disappears.

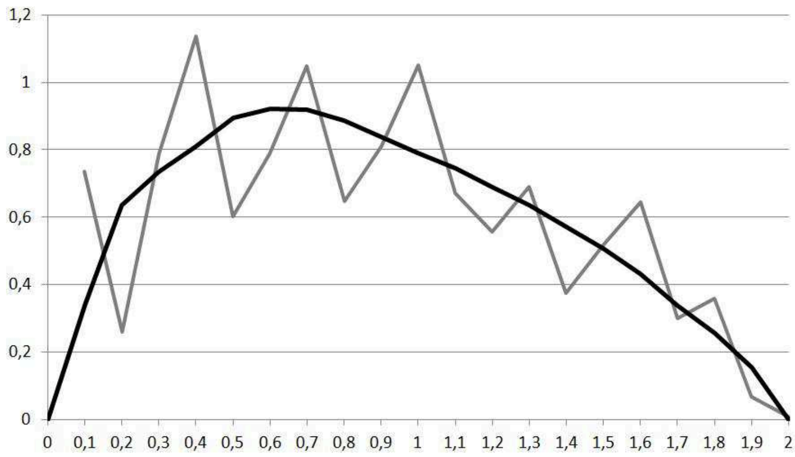

Fig. 2. Graphs of the noisy signal (grey line) and the reconstructed signal (black line)

2. The Computational Experiments

This algorithm was implemented in C++. The input data is a file with information about the interval $[a, b]$, its sampling frequency N, the values of observations $\{\eta_j\}_{j=0}^N$, where we understand $\eta_j = \eta(t_j)$ $(j = \overline{0, N})$. The program calculates an array of points $\{t_j\}_{j=0}^N$ and then according to the algorithm. The output data is a file containing an array $\{y_j = y_e(t_j)\}_{j=0}^N$.

Here are the results of the program.

Example 1. Let $a = 0$, $b = 2$, $N = 20$ and the results of observations are distorted by ≪white noise≫. The application of the algorithm gives the values, the graphs of which are shown in Fig. 2 and Fig. 3.

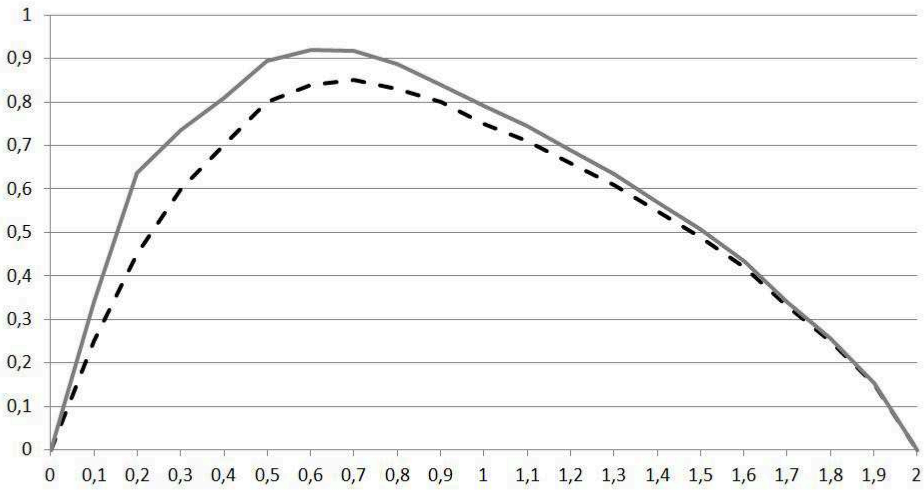

Fig. 3. Graphs of the original signal (black dotted line) and the reconstructed signal (gray solid line)

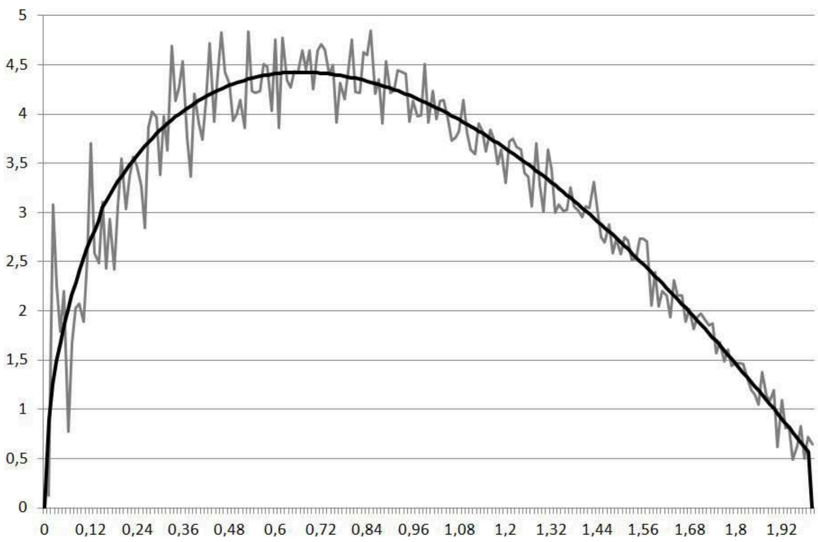

Fig. 4. Graphs of the noisy signal (grey line) and the reconstructed signal (black line)

On the graph, you can see that at the beginning of the interval the large bursts give worse values of the restored observation than at the end of the interval.

Example 2. Let $a = 0$, $b = 2$, $N = 200$ and the results of observations are distorted by Wiener process. The application of the algorithm gives the values, the graphs of which are shown in Fig. 4 and Fig. 5.

This example illustrates that as the sampling rate increases, we get smoother data.

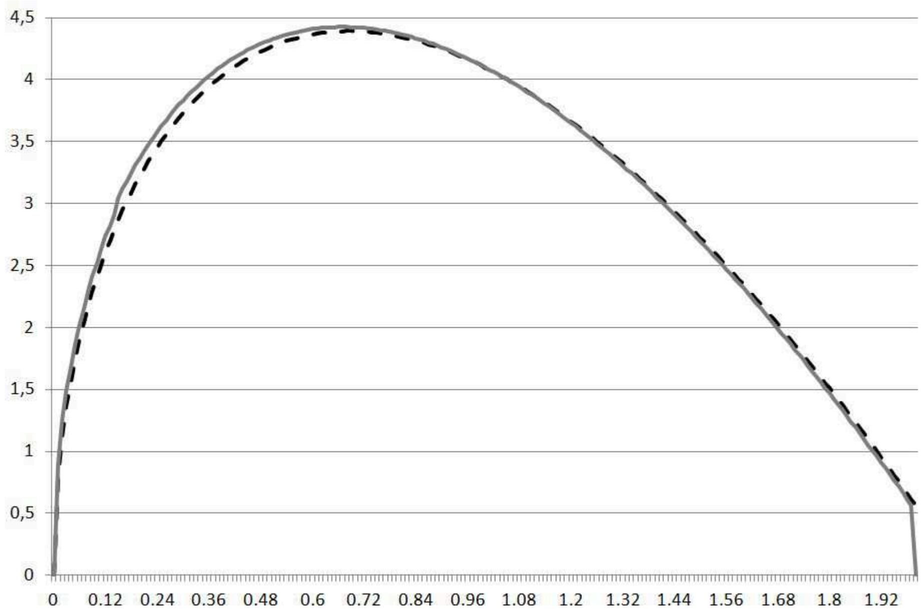

Fig. 5. Graphs of the original signal (black dotted line) and the reconstructed signal (gray solid line)

Conclusion

In the future, we plan to expand the class of functions describing the useful part of the signal. In addition, the described procedure will be used in the analysis of the dynamics of the solar activity [14].

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ПОСТРОЕНИЕ НАБЛЮДЕНИЙ ПО ДАННЫМ, ИСКАЖЕННЫМ ПОМЕХАМИ РАЗНОГО ВИДА

М.А. Сагадеева, О.В. Митин

В статье описывается предварительный этап задачи оптимального динамического измерения. А именно, приведен алгоритм построения значений наблюдения по значениям полученным в ходе эксперимента, которые предполагаются искаженными некоторыми случайными воздействиями. Предполагается, что на экспериментальные данные могут воздействовать помехи разного вида, в том числе «белый шум», который понимается как производная Нельсона – Гликлиха от винеровского процесса. Для построения значений наблюдения используется априорная информация о форме функции, описывающей значения наблюдения. Статья состоит из двух частей. Первая часть содержит алгоритм построения значений наблюдений. А во второй части приведены результаты вычислительных экспериментов.

Ключевые слова: полезная часть сигнала; выпуклая вверх функция; статистическая гипотеза.

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