MSC 35G05 DOI: 10.14529/jcem210405

MACRO MODEL OF TRANSPORT FLOW AT THE CROSSROADS

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Currently, one of the important problems of the megalopolis is traffic management, and in connection with the problem of the formation of predatory and congestion situations in settlements, respectively, these studies are relevant. There are several approaches to mathematical modelling of the behaviour of vehicle traffic. The most common ones: microscopic, macroscopic, based on the theory of cellular automata. The third approach is macroscopic, with its help analog models are built, and the traffic flow is considered as a hydrodynamic, or gas-dynamic flow. Using this approach, you can find the time or traffic intensity, average speed, and the level of network load. One of the creators of this approach, which simulates the traffic flow by the Navier-Stokes system, which describes the flow of a viscous incompressible fluid, is A.B. Kurzhansky. A distinctive feature of this article is that the traffic flow model is built on the basis of the Oskolkov system of equations, which generalize the Navier-Stokes system. Here, in addition to the viscosity and incompressibility of the flow, elasticity is taken into account, due to which the retardation effect inherent in viscoelastic incompressible fluids appears.

Keywords: Oskolkov equation; geometric graph; traffic flows.

Introduction

Consider the crossroad at the initial moment of turning on the traffic light, when the crossroad mode changes from unregulated (yellow flashing traffic light mode) to regulated and denote this moment $t = \tau_0$. ≪Crossroad is a place of intersection, abutment or branching of roads at the same level, bounded by imaginary lines connecting, respectively, opposite, most distant from the center of the intersection, the beginning of the curvature of the carriageway. Exits from adjacent territories are not considered as crossroads≫[1].

Imagine a crossroad with a changing mode of its passage (traffic light is switched on) in the form of an eight-edge geometric graph $\mathbf{G_1}$ (fig. 1)[1], [4]. The length of the k-th edge l_k is measured in linear metric units (kilometers or miles), however, in the mathematical model of traffic flow, the value of l_k is dimensionless. The number of lanes on the carriageway in one direction d_k will be called the capacity; similarly, in the context of the mathematical model, the value of d_k is dimensionless. Suppose that all adjacent roads at the crossroad under consideration are equivalent, so we will assume that the capacity of each direction will be the same, i.e. $d_1 = d_2 = \ldots = d_8 = d$.

The traffic flow will be determined using the Oskolkov equations given on graph $\mathbf{G_1}$

$$\lambda u_{1kt} - u_{1ktxx} = \nu u_{1kxx} + f_{1k},\ \ k = \overline{1,8}. \tag{1}$$

[1]Decree of the Government of the Russian Federation of 23.10.1993 N 1090 (as amended on 04.12.2018) ≪On the Rules of the Road≫(together with the ≪Basic Provisions for the Admission of Vehicles to Operation and the Obligations of Officials to Ensure Road Safety≫).

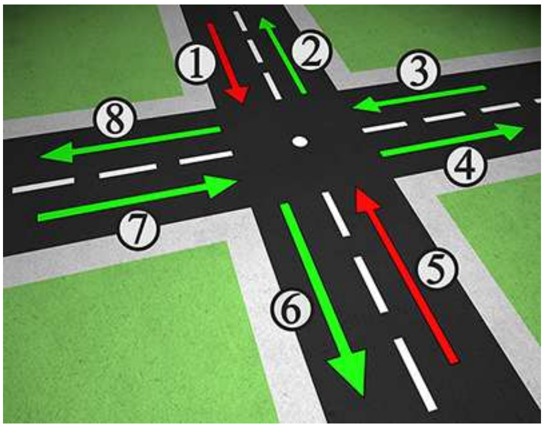

Fig. 1. Road map

Here $u_{1k} = u_{1k}(x,t)$, $x \in [0, l_k]$, $t \in \overline{\mathbb{R}}_+$ $(\equiv \{0\} \cup \mathbb{R}_+)$, $k = \overline{1,8}$, characterizes the average traffic speed on the set of edges E_k of graph $\mathbf{G_1}$. Coefficients $f_{1k} = f_{1k}(x,t)$, $(x,t) \in [0, l_{ik}] \times \overline{\mathbb{R}}_+$ corresponds to the average force that causes the wheels of vehicles to spin. Coefficient λ is equal to one divided by the coefficient of retardation, which can take negative values, therefore, we consider $\lambda \in \mathbb{R}$. Coefficient ν is responsible for the viscosity of the transport flow, namely, i.e. for its ability to «dampen» sharp speed differences, withing the physical meaning $\nu \in \mathbb{R}_+$.

Suppose that at moment of time $t = \tau_0$ on the first and fifth edges a red traffic light is switched on, i.e. the boundary conditions for the Oskolkov equations (1) on graph $\mathbf{G_1}$ will have the form

$$u_{12}(0,t) = u_{13}(l_3,t) = u_{14}(0,t) = u_{16}(0,t) = u_{17}(l_7,t) = u_{18}(0,t), \tag{2}$$

$$u_{11}(l_1,t) = u_{15}(l_5,t) = 0, \tag{3}$$

$$\begin{gathered} -u_{12x}(0,t) + u_{13x}(l_3,t) - u_{14x}(0,t) - u_{16x}(0,t) + u_{17x}(l_7,t) - u_{18x}(0,t) = 0, \\ u_{11x}(0,t) = u_{13x}(0,t) = u_{15x}(0,t) = u_{17x}(0,t) = 0, \\ u_{12x}(l_2,t) = u_{14x}(l_4,t) = u_{16x}(l_6,t) = u_{18x}(l_8,t) = 0. \end{gathered} \tag{4}$$

Condition (2) means that the speed of the vehicle entering the crossroad should be equal to the exit speed, and it is a condition of «continuity». Condition (3) is a traffic ban condition, and conditions (4) require that the number of vehicles leaving the crossroad be equal to the number of vehicles leaving, and they are called «flow balance» conditions. We define the initial condition in the form of Showalter – Sidorov condition

$$P_1(u_1(x,\tau_0) - u_{10}(x)) = 0,$$

in this case, due to the change in terms f_{11}, f_{15} conditions $\lim\limits_{t\to\tau_0} u_{11}(l_1,t) = 0$, $\lim\limits_{t\to\tau_0} u_{15}(l_5,t) = 0$, required to stop before the red signal traffic light is switched on.

When time $t = \tau_1$ is reached, the traffic signal will change, therefore, the direction of traffic at the crossroad will change, therefore, instead of graph $\mathbf{G_1}$ (fig. 2) we will consider a new graph $\mathbf{G_2}$ (fig. 3). On this graph, Oskolkov's equations take the form

$$\lambda u_{2kt} - u_{2ktxx} = \nu u_{2kxx} + f_{2k}, \;\; k = \overline{1,8},$$

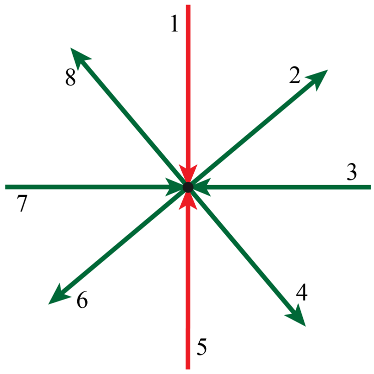

Fig. 2. Crossroads before the traffic signal changes, in time period $[\tau_0, \tau_1]$

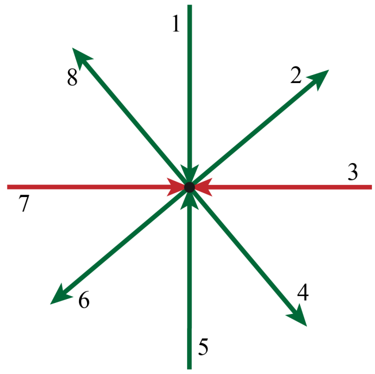

Fig. 3. Crossroads after changing the traffic signal, in time period $[\tau_1, \tau_2]$

and the conditions of «continuity», «traffic ban» and «balance of flow» are

$$u_{21}(l_1,t) = u_{22}(0,t) = u_{24}(0,t) = u_{25}(l_5,t) = u_{26}(0,t) = u_{28}(0,t),$$

$$u_{23}(l_3,t) = u_{27}(l_7,t) = 0$$

$$u_{21x}(l_1,t) - u_{22x}(0,t) - u_{24x}(0,t) + u_{25x}(l_5,t) - u_{26x}(0,t) - u_{28x}(0,t) = 0,$$

$$u_{21x}(0,t) = u_{23x}(0,t) = u_{25x}(0,t) = u_{27x}(0,t) = 0,$$
$$u_{22x}(l_2,t) = u_{24x}(l_4,t) = u_{26x}(l_6,t) = u_{28x}(l_8,t) = 0.$$

When the traffic signal changes at time $t = \tau_1$ the average speed on the third and seventh edges will tend to zero, i.e. $\lim\limits_{t\to\tau_2} u_{23}(l_3,t) = 0$, $\lim\limits_{t\to\tau_2} u_{27}(l_7,t) = 0$. In this case, on the remaining edges, the speed will be $u_{2k}(x,\tau_1) = u_{1k}(x,\tau_1) = u_{1k}(x)$, $k = 1,2,4,5,6,8$, it is reached on the corresponding edge by time τ_1, namely. In general terms, these conditions take the form

$$P_2(u_2(x,\tau_1) - u_{21}(x)) = 0.$$

Continuing the traffic light switching procedure at times $t = \tau_j$, $j = \overline{0,n}$, and for even n consider the crossroad as a graph $\mathbf{G_1}$, and for odd n consider the crossroad as a graph $\mathbf{G_2}$. In general, the multipoint initial-final condition takes the form

$$P_m(u_m(x,\tau_j) - u_{mj}(x)) = 0,\ j = \overline{0,n},\ m = 1,2,$$

where τ_j is the moment when the traffic light switches.

1. Abstract Model

In Banach spaces $\mathfrak{U}$ and $\mathfrak{F}$ consider operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$, $M \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$, and let operator M be (L,p)-bounded, $p \in \{0\} \cup \mathbb{N}$.

For a linear inhomogeneous Sobolev-type equation

$$L\dot{u} = Mu + f. \tag{5}$$

we formulate a multipoint initial-final condition [2], [3]

$$P_j(u(\tau_j) - u_j) = 0, \quad j = \overline{0,n}, \tag{6}$$

where $\tau_j \in \mathfrak{I} = [0,\tau]$, $(\tau_{j-1} < \tau_j)$, $j = \overline{1,n}$; $u_j \in \mathfrak{U}$, $j = \overline{0,n}$, $f \in C^\infty(\mathfrak{I};\mathfrak{F})$.

A vector function $u \in C^\infty(\mathfrak{I};\mathfrak{U})$ is called a *solution to equation* (5), if it satisfies this equation. A solution $u = u(t)$, $t \in \mathfrak{I}$, of equation is called (5) *a solution of a multipoint initial-final problem for equation* (5), if it additionally satisfies conditions (6).

Theorem 1. *If the operator M is (L,p)-bounded, $p \in \{0\}\cup\mathbb{N}$, then for any $f \in C^\infty(\mathfrak{I};\mathfrak{F}))$, $u_j \in \mathfrak{U}$, $j = \overline{0,n}$, problem* (5), (6) *is uniquely solvable, and the solution has the form*

$$u(t) = -\sum_{k=0}^{p} H^k M_0^{-1}(\mathbb{I}-Q)f^{(k)}(t) + \sum_{j=0}^{n} U_j^{t-\tau_j}u_j + \sum_{j=0}^{n}\int_{\tau_j}^{t} U_j^{t-s}L_{j1}^{-1}Q_j f(s)ds. \qquad (7)$$

• Equation (5) is reduced to the system

$$H\dot{u}^0 = u^0 + M_0^{-1}(\mathbb{I}-Q)f, \ \dot{u}^{1j} = S_j u^{1j} + L_{1j}^{-1}Q_j f, \qquad (8)$$

where $u^0 = (\mathbb{I}-P)u$, $u^{1j} = P_j u$, $j = \overline{0,n}$,

To find the first term, it is necessary to sequentially differentiate the first equation (8), while multiplying it by Hon the left. Using the nilpotency of the operator H, we obtain

$$u^0(t) = -\sum_{k=0}^{p} H^k M_0^{-1}(\mathbb{I}-Q)f^{(k)}(t). \qquad (9)$$

Noticing that the projector P_j is a unit operator on $\mathfrak{U}_j^1$, by virtue of (6) we formulate the Cauchy conditions at different times for the second equation (8)

$$\dot{u}^{1j} = S_j u^{1j} + L_{1j}^{-1}Q_j f, \quad u^{1j}(\tau_j) = P_j u_j, \quad j = \overline{0,n}. \qquad (10)$$

Successively solving problems (10), we obtain

$$u^{1j}(t) = U_j^{t-\tau_j}u_j + \int_{\tau_j}^{t} U_j^{t-s}L_{j1}^{-1}Q_j f(s)ds, \ j = \overline{0,n}. \qquad (11)$$

Adding (9) and (11), we obtain (7). The uniqueness of the solution to problem (5), (6) is obvious due to the above proof. •

2. Modification of the Mathematical Model of Traffic Flow at the Intersection

Consider a finite ordered set $\mathbf{\Gamma} = \{\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_i, \ldots\}$ of finite connected directed graphs $\mathbf{G}_i = \mathbf{G}_i(\mathcal{V}_i, \mathcal{E}_i)$. Each geometric graph $\mathbf{G}_i$ corresponds to a time period $[\tau_{i-1};\tau_i]$. Here $\mathcal{V}_i = \{V_{ij}\}$ is the set of vertices of graph $\mathbf{G}_i$, and $\mathcal{E}_i = \{E_{ik}\}$ is the set of edges of $\mathbf{G}_i$. Each edge E_{ik} of each graph $\mathbf{G}_i$ corresponds to two numbers: the «length» of the edge$l_{ik} \in \mathbb{R}_+$ and its «width» $d_{ik} \in \mathbb{R}_+$.

It is necessary to find the solutions of the Oskolkov equations

$$\lambda_i u_{ikt} - u_{iktxx} = \nu_i u_{ikxx} + f_{ik}, \qquad (12)$$

given on each edge E_{ik} of each graph $\mathbf{G}_i$ on time interval $[\tau_{i-1}, \tau_i]$, where coefficients $\lambda_i \in \mathbb{R}$ and $\nu_i \in \mathbb{R}_+$.

Let us consider the first condition for the speed limit when driving through a crossroad is the speed of a vehicle entering the crossroad must be equal to the exit speed, otherwise traffic jams or road accidents are possible at the crossroad. This condition in the mathematical model is the condition of «continuity»

$$\begin{aligned} u_{ik}(0,t) = u_{im}(l_{im},t) = u_{il}(0,t) = u_{in}(l_{in},t), \\ \forall E_{ik}, E_{il} \in E^{\alpha}(V_{ij}),\ \forall E_{im}, E_{in} \in E^{\omega}_{op}(V_{ij}). \end{aligned} \tag{13}$$

Here $E^{\alpha}(V_{ij})$ denotes the set of edges of graph $\mathbf{G}_i$, emerging from vertex V_{ij}, and by $E^{\omega}_{op}(V_{ij})$ denotes the set of edges of graph $\mathbf{G}_i$, that correspond to the entering the crossroad into vertex V_{ij} at the permitting traffic signal.

The second condition for the speed limit when driving through a crossroad is that the number of vehicles leaving the crossroad was equal to the number of those leaving. In the mathematical model, it is formulated as a condition for the «balance of flows»

$$\sum_{E_{ik}\in E^{\alpha}(V_{ij})} d_{ik}u_{ikx}(0,t) - \sum_{E_{im}\in E^{\omega}_{op}(V_{ij})} d_{im}u_{imx}(l_{im},t) = 0. \tag{14}$$

The third condition of the speed limit when driving through a crossroad is the condition of a «ban on traffic»

$$u_{ik}(l_{ik},t) = 0, \quad \forall E_{ik} \in E^{\omega}_{cl}(V_{ij}), \tag{15}$$

where $E^{\omega}_{cl}(V_{ij})$ is denoted as the set of edges of graph $\mathbf{G}_i$, corresponding to the entering to vertex V_{ij} at the prohibiting traffic signal.

Consider Hilbert space

$$\mathbf{L_2}(\mathbf{G}_i) = \{g_i = (g_{i1}, g_{i2}, \dots, g_{ik}, \dots) : g_{ik} \in L_2(0, l_{ik})\}$$

with inner product

$$\langle g, h\rangle_i = \sum_{E_{ik}\in\mathcal{E}_i} d_{ik}\int_0^{l_{ik}} u_{ik}v_{ik}dx.$$

Moreover, consider space

$$\mathfrak{U}(\mathbf{G}_i) = \big\{u_i = (u_{i1}, u_{i2}, \dots, u_{ik}, \dots) : u_{ik} \in W^1_2(0, l_{ik})$$

$$\text{and (13), (15) are satisfied in each vertex } V_{ij} \in \mathcal{V}_i\}$$

with inner product

$$[u, v]_i = \sum_{E_{ik}\in\mathcal{E}_i} d_{ik}\int_0^{l_{ik}} (u_{ikx}v_{ikx} + u_{ik}v_{ik})\, dx.$$

We identify $\mathbf{L_2}(\mathbf{G}_i)$ with its dual and denote by $\mathfrak{F}(\mathbf{G}_i)$ the space dual to $\mathfrak{U}(\mathbf{G}_i)$ with respect to the duality $\langle\cdot,\cdot\rangle$. Note the dense and continuous embeddings $\mathfrak{U}(\mathbf{G}_i) \hookrightarrow \mathbf{L_2}(\mathbf{G}_i) \hookrightarrow \mathfrak{F}(\mathbf{G}_i)$and note that, by virtue of the Sobolev embedding theorems $W^1_2(0, l_{ik})$, functions from a.e. on $[0, l_{ik}]$ coincide with absolutely continuous functions, therefore, the spaces $\mathfrak{U}(\mathbf{G}_i)$ are well defined.

Take $\lambda_i \in \mathbb{R}_+$ and by the formula

$$\langle L_i u_i, v_i \rangle_i = \sum_{E_{ik} \in \mathcal{E}_i} d_{ik} \int_0^{l_{ik}} (u_{ikx} v_{ikx} + \lambda_i u_{ik} v_{ik}) dx, \ u_i, v_i \in \mathfrak{U}(\mathbf{G}_i),$$

define an operator $L_i \in \mathcal{L}(\mathfrak{U}(\mathbf{G}_i); \mathfrak{F}(\mathbf{G}_i))$. Consider space

$$\mathfrak{A}(\mathbf{G}_i) = \big\{ u_i = (u_{i1}, u_{i2}, \dots, u_{ik}, \dots) : u_{ik} \in C^2(0, l_{ik}) \cap C^1[0, l_{ik}]$$
$$\text{and (13), (15) are satisfied at each vertex } V_{ij} \in \mathcal{V}_i \big\}.$$

Dense and continuous embeddings $\mathfrak{A}(\mathbf{G}_i) \hookrightarrow \mathfrak{U}(\mathbf{G}_i)$ are obvious, and $\langle (\lambda_i u_i - u_{ixx}, v_i) \rangle_i = \langle L_i u_i, v_i \rangle_i$ for all u_i, $v_i \in \mathfrak{A}(\mathbf{G}_i)$. Thus, the flow balance conditions ((14) are «hidden» in the sense of O.A. Ladyzhenskaya in the definition of the operators L_i.

Take $\nu_i \in \mathbb{R}_+$, and put $M_i = \nu_i(\lambda_i \mathbb{I}_i - L_i)$, where $\mathbb{I}_i : \mathfrak{U}(\mathbf{G}_i) \to \mathfrak{F}(\mathbf{G}_i)$ is the embedding operator. Consider the equation

$$L_i u_{it} = M_i u_i + f_i. \tag{16}$$

Lemma 1. *Operators $L_i : \mathfrak{U}_\mathrm{i} \to \mathfrak{F}_\mathrm{i}$ are linear and continuous, spectrum $\sigma(L_i)$ is real, discrete, finite, and condenses only to $-\infty$. Operators $M_i : \mathfrak{U}_\mathrm{i} \to \mathfrak{F}_\mathrm{i}$ are linear and continuous.*

Corollary 1. *Operators L_i are Fredholm, and* $\ker L_i = \{0\}$*, если $0 \notin \sigma(L_i)$.*

Lemma 2. *Let parameters $\nu_i \in \mathbb{R}_+$, $\lambda_i \in \mathbb{R}_+$, then operator M_i be $(L_i, 0)$-bounded.*

Take $\tau_j \in \overline{\mathbb{R}_+}$, $j = \overline{0, n}$, such that $\tau_{j-1} < \tau_j$ for $j = \overline{1, n}$, $u_j \in \mathfrak{U}$, $j = \overline{0, n}$. A vector function $u_i \in C^1((\tau_{i-1}, \tau_i); \mathfrak{U}(\mathbf{G}_i))$, satisfying (16) for some $f_i \in \mathfrak{F}(\mathbf{G}_i)$, is called a solution of equation (16), satisfying the multipoint initial-final condition

$$P_i(u_i(\tau_j) - u_{ij}) = 0, \ j = \overline{0, n}. \tag{17}$$

where P_i are relatively spectral projectors, and at moment of time τ_j the velocity that was the flow at this moment becomes the initial one.

Lemma 3. *For any λ_i, $\nu_i \in \mathbb{R}_+$, $f_i \in \mathfrak{F}(\mathbf{G}_i)$ and $u_{0i} \in \mathfrak{U}(\mathbf{G}_i)$ there exists a unique solution of problem (16), (17).*

Now we use conditions $u_{m+1}(\tau_m) = u_m(\tau_m)$, $m = 1, 2, \dots, i, \dots$, «to glue» the solutions of problems (16), (17), the existence and uniqueness of which follows from lemma 3. On the one hand, by definition, $u_m(\tau_m) \in \mathfrak{U}(\mathbf{G}_m)$; on the other hand, lemma 3 requires that $u_m(\tau_m) \in \mathfrak{U}(\mathbf{G}_{m+1})$. Therefore, by Lemma 3 the following theorem holds.

Theorem 2. *For any λ_i, $\nu_i \in \mathbb{R}_+$, $f_i \in \mathfrak{F}(\mathbf{G}_i)$ and $u_0 \in \mathfrak{U}(\mathbf{G}_1)$, such that $u_m(\tau_m) \in \mathfrak{U}(\mathbf{G}_{m+1}), m = 1, 2, \dots, i, \dots,$ there exists a unique solution of problem (12) – (15).*

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Received November 6, 2021

УДК 517.9 **DOI: 10.14529/jcem210405**

МАКРОМОДЕЛЬ ТРАНСПОРТНОГО ПОТОКА НА ПЕРЕКРЕСТКЕ

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В настоящее время одной из важных проблем мегаполиса является управление дорожным движением, а в связи с проблемой образования предзаторных и заторных ситуаций в населенных пунктах, соответственно, эти исследования являются актуальными. Существует несколько подходов математического моделирования поведения движения автотранспорта. Наиболее распространенные из них: микроскопический, макроскопический, на основе теории клеточных автоматов. Третий подход – макроскопический, с его помощью строятся модели-аналоги, и транспортный поток рассматривается как гидродинамический, или газодинамический поток. При применении данного подхода можно найти время или интенсивность движения, среднюю скорость, уровень загрузки сети. Одним из создателей данного подхода является А.Б. Куржанский, который транспортный поток моделирует системой Навье – Стокса, описывающей течение вязкой несжимаемой жидкости. Отличительная черта данной статьи заключается в том, что модель транспортного потока строится на основе системы уравнений Осколкова, которые обобщают систему Навье – Стокса. Здесь помимо вязкости и несжимаемости потока, учитывается упругость, из-за которой появляется эффект ретардации, свойственный вязкоупругим несжимаемым жидкостям.

Ключевые слова: уравнения Осколкова; геометрический граф; транспортные потоки.

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Поступила в редакцию 6 ноября 2021 г.