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CONSTRUCTION OF A MATHEMATICAL MODEL OF JSC «CHELYABINSKGORGAZ» ACTIVITIES

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The article considers a mathematical model of the production activity of the main gas supplier to the residents of Chelyabinsk. The model is constructed on the basis of the modified Cobb–Douglas production function for 2011–2020 years. We present the author's algorithm to construct a model, according to which a computer program for modelling the operating activities of the enterprise is written in the Java language and registered in the State Register of Computer Programs. The adequacy of the constructed model was checked by the coefficient of determination, the value of which shows its high reliability. The analysis of the constructed mathematical model allows to draw the following conclusions: the efficiency of operating activities does not meet modern requirements. Namely, there exists a decreasing effect of scale throughout the entire analyzed period; starting from 2016, the enterprise has no economic stability and the bankruptcy procedure becomes inevitable.

The results of the study can be recommended to the scientific community when carrying out similar work aimed at modelling operational activities, which is possible, since the author's computer modelling program is registered in the State Register of Computer Programs and is open source.

Keywords: mathematical model; analysis; gas supplier; software; economic stability.

Introduction

In the scientific community, the study of the optimal use of resources in the economic activity of firms using the apparatus of production functions remains relevant [1–3]. Under the conditions of high volatility of gas prices in the foreign market, the study of the operating activities of JSC «Chelyabinskgorgaz» is of scientific and practical interest.

JSC «Chelyabinskgorgaz» is engaged in the transportation of natural gas, the sale of liquefied gas to the population of Chelyabinsk, the provision of other paid services and emergency cover for gas facilities. At present, «Chelyabinskgorgaz» transports gas to industrial consumers, including such large energy and industrial enterprises as PJSC «Fortum», PJSC «Chelyabinsk Pipe Rolling Plant», PJSC «Chelyabinsk Tractor Plant», annually delivering up to 4 billion cubic meters of natural gas; provides gas to more than 250 thousand apartments; serves over 1850 km of gas distribution pipelines and facilities on them; participates in the regional program of gasification of settlements of individual development in the Chelyabinsk city district, having invested 210 million rubles in its implementation since 2005; includes a single city emergency gas service 04, which around the clock is engaged in the prevention, localization and elimination of emergencies, manages the mode of operation of the gas distribution system.

In order to analyze the performance results of JSC «Chelyabinskgorgaz», a mathematical model of the enterprise operating activities for 2011–2020 was constructed and studied.

1. Modelling Algorithm

In order to construct a mathematical model of the enterprise activity based on the modified Cobb–Douglas production function, taking into account the autonomous technical progress neutral according to Hicks (1), the author's algorithm described in [1] is used.

$$CP = A \cdot N^{\alpha} \cdot FA^{\beta} \cdot CA^{\gamma} \cdot e^{\lambda t}, \tag{1}$$

where $e^{\lambda t}$ is a factor that allows to take into account the «autonomous» technical progress neutral according to Hicks (here e is a base of the natural logarithm). Production function (1) is an economic and mathematical model of production that reflects the impact of resource provision of production on output. On the basis of retrospective data on revenue CP, wages of employees FOT, non-current assets FA, current assets CA, and the corresponding time t, in the most general form, the parameters of the production function $A, \alpha, \beta, \gamma, \lambda$ are found as a solution to system of equations (2). In system of equations (2), m is the number of years for which retrospective data were collected ($m > 4$).

$$\begin{cases} \sum\limits_{i=1}^{m} \ln CP_i = m \cdot \ln A + \alpha \cdot \sum\limits_{i=1}^{m} \ln FOT_i + \\ \qquad + \beta \cdot \sum\limits_{i=1}^{m} FA_i + \gamma \cdot \sum\limits_{i=1}^{m} \ln CA_i + \lambda \cdot \sum\limits_{i=1}^{m} t_i, \\ \sum\limits_{i=1}^{m} (\ln CP_i \cdot \ln FOT_i) = \ln A \cdot \sum\limits_{i=1}^{m} \ln FOT_i + \alpha \cdot \sum\limits_{i=1}^{m} (\ln FOT_i)^2 + \\ \qquad + \beta \cdot \sum\limits_{i=1}^{m} (\ln FA_i \cdot \ln FOT_i) + \gamma \cdot \sum\limits_{i=1}^{m} (\ln CA_i \cdot \ln FOT_i) + \lambda \cdot \sum\limits_{i=1}^{m} (t_i \cdot \ln FOT_i), \\ \sum\limits_{i=1}^{m} (\ln CP_i \cdot \ln FA_i) = \ln A \cdot \sum\limits_{i=1}^{m} \ln FA_i + \alpha \cdot \sum\limits_{i=1}^{m} (\ln FA_i \cdot \ln FOT_i) + \\ \qquad + \beta \cdot \sum\limits_{i=1}^{m} (\ln FA_i)^2 + \gamma \cdot \sum\limits_{i=1}^{m} (\ln FA_i \cdot \ln CA_i) + \lambda \cdot \sum\limits_{i=1}^{m} (t_i \cdot \ln FA_i), \\ \sum\limits_{i=1}^{m} (\ln CP_i \cdot \ln CA_i) = \ln A \cdot \sum\limits_{i=1}^{m} \ln CA_i + \alpha \cdot \sum\limits_{i=1}^{m} (\ln CA_i \cdot \ln FOT_i) + \\ \qquad + \beta \cdot \sum\limits_{i=1}^{m} (\ln CA_i \cdot \ln FA_i) + \gamma \cdot \sum\limits_{i=1}^{m} (\ln CA_i)^2 + \lambda \cdot \sum\limits_{i=1}^{m} (t_i \cdot \ln CA_i), \\ \sum\limits_{i=1}^{m} (\ln CP_i \cdot t_i) = \ln A \cdot \sum\limits_{i=1}^{m} t_i + \alpha \cdot \sum\limits_{i=1}^{m} (t_i \cdot \ln FOT_i) + \\ \qquad + \beta \cdot \sum\limits_{i=1}^{m} (t_i \cdot \ln FA_i) + \gamma \cdot \sum\limits_{i=1}^{m} (t_i \cdot \ln CA_i) + \lambda \cdot \sum\limits_{i=1}^{m} (t_i)^2. \end{cases} \tag{2}$$

The use of the method of power-law production functions for the analysis of the economics of production is often difficult because system of equations (2) may not have a solution. This is explained by the fact that there may be a dependence between statistical data, due not so much to their functional connection, but to the proximity in time of sets of exogenous variables, when all quantities change proportionally. In this case, there exists a phenomenon, which Mendershausen called the effect of multicollinearity between independent variables. In order to overcome this barrier, the following transformations are necessary. Let us divide the total differential of function (1) by the function itself. We get the equation

$$dCP/CP = \alpha \cdot dN/N + \beta \cdot dFA/FA + \gamma \cdot dCA/CA + \lambda \cdot dt. \tag{3}$$

Let us introduce the notation

$$\frac{dCP}{CP} = 2 \cdot \frac{CP_{i+1} - CP_i}{CP_{i+1} + CP_i} = z, \qquad \frac{dN}{N} = 2 \cdot \frac{N_{i+1} - N_i}{N_{i+1} + N_i} = x, \qquad dt = t_{i+1} - t_i = 1,$$

$$\frac{dFA}{FA} = 2 \cdot \frac{FA_{i+1} - FA_i}{FA_{i+1} + FA_i} = y, \qquad \frac{dCA}{CA} = 2 \cdot \frac{CA_{i+1} - CA_i}{CA_{i+1} + CA_i} = w.$$

Then expression (3) is converted into the equation

$$z = \alpha \cdot x + \beta \cdot y + \gamma \cdot w + \lambda$$

and system (2) is converted into the system

$$\begin{cases} \sum\limits_{i=1}^{m} z_i = \lambda \cdot m + \alpha \cdot \sum\limits_{i=1}^{m} x_i + \beta \cdot \sum\limits_{i=1}^{m} y_i + \gamma \cdot \sum\limits_{i=1}^{m} w_i, \\ \sum\limits_{i=1}^{m} (x_i \cdot z_i) = \lambda \cdot \sum\limits_{i=1}^{m} x_i + \alpha \cdot \sum\limits_{i=1}^{m} (x_i)^2 + \beta \cdot \sum\limits_{i=1}^{m} (x_i \cdot y_i) + \\ \qquad + \gamma \cdot \sum\limits_{i=1}^{m} (x_i \cdot w_i), \\ \sum\limits_{i=1}^{m} (y_i \cdot z_i) = \lambda \cdot \sum\limits_{i=1}^{m} (t_i \cdot y_i) + \alpha \cdot \sum\limits_{i=1}^{m} (x_i \cdot y_i) + \beta \cdot \sum\limits_{i=1}^{m} (y_i)^2 + \\ \qquad + \gamma \cdot \sum\limits_{i=1}^{m} (y_i \cdot w_i), \\ \sum\limits_{i=1}^{m} (w_i \cdot z_i) = \lambda \cdot \sum\limits_{i=1}^{m} (w_i) + \alpha \cdot \sum\limits_{i=1}^{m} (w_i \cdot x_i) + \beta \cdot \sum\limits_{i=1}^{m} (y_i \cdot w_i) + \\ \qquad + \gamma \cdot \sum\limits_{i=1}^{m} (w_i)^2. \end{cases} \tag{4}$$

Based on the transformed initial data from system of equations (4), we find the elasticity coefficients $\alpha, \beta, \gamma, \lambda$. According to the found numerical values of the elasticity coefficients, the coefficient A is found as follows:

$$A = \frac{\sum\limits_{i=1}^{m} z_i \cdot x_i^{\alpha} \cdot y_i^{\beta} \cdot w_i^{\gamma} \cdot e^{\lambda t}}{\sum\limits_{i=1}^{m} (x_i^{\alpha} \cdot y_i^{\beta} \cdot w_i^{\gamma} \cdot e^{\lambda t})^2}$$

2. Construction of Mathematical Model of JSC «Chelyabinskgorgaz»

The described modelling algorithm was formalized in the form of a computer program written in the Java language and registered by the Federal Intellectual Property Service in the State Register of Computer Programs [2]. The initial modelling base was the data of a sample of accounting statements of JSC «Chelyabinskgorgaz» for 2011–2020 (see Table).

Table

Accounting data of JSC «Chelyabinskgorgaz», thousand rubles

Year	CP	FA	CA	FOT
2011	523 808	780 587	1 156 748	142 225
2012	563 215	844 313	1 259 080	185 120
2013	595 509	881 020	1 342 182	195 734
2014	681 484	1 647 590	2 206 085	230 864
2015	1 695 606	1 824 475	2 481 095	252 143
2016	756 558	1 881 142	2 647 293	334 348
2017	772 416	1 906 703	2 705 338	358 314
2018	860 217	1 973 022	2 763 786	381 936
2019	866 102	2 099 599	2 900 870	390 760
2020	838 392	2 208 774	2 993 194	409 447

As a result of modelling, a mathematical model of the operational activity of JSC «Chelyabinskgorgaz» was obtained as

$$CP = 9.22715 \cdot FOT^{-3.26669} \cdot FA^{-7.26697} \cdot CA^{10.63808} \cdot e^{0.12876 \cdot t}. \tag{5}$$

The reliability of the constructed model is estimated by the coefficient of determination

$$R^2 = 1 - \frac{\sum e_i^2}{\sum (y_i - \bar{y})^2} = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2},$$

where $y_i, \hat{y}_i$ are the actual and calculated values of the explained variable.

The coefficient of determination takes values in the range from 0 to 1, and the closer the value of the coefficient to 1, the better the constructed model describes the initial data. For the constructed model (5), the value of the determination coefficient is close to 1, which corresponds to the high accuracy of the model.

3. Study of Mathematical Model of JSC «Chelyabinskgorgaz»

In order to analyze the constructed model, in terms of determining the influence of individual production factors on the economic result of an enterprise, the following algorithm can be recommended. Based on equation (1), we find partial derivatives of production resources. We obtain the following equations:

$$\begin{cases} \dfrac{\partial CP}{\partial N} = A \cdot \alpha \cdot FOT^{\alpha-1} \cdot FA^{\beta} \cdot CA^{\gamma} \cdot e^{\lambda t}, \\ \dfrac{\partial CP}{\partial FA} = A \cdot \beta \cdot FOT^{\alpha} \cdot FA^{\beta-1} \cdot CA^{\gamma} \cdot e^{\lambda t}, \\ \dfrac{\partial CP}{\partial CA} = A \cdot \gamma \cdot FOT^{\alpha} \cdot FA^{\beta} \cdot CA^{\gamma-1} \cdot e^{\lambda t}. \end{cases} \tag{6}$$

Divide equations (6) by equation (1) and get the system

$$\begin{cases} \dfrac{\partial CP}{CP} = \alpha \cdot \dfrac{\partial FOT}{FOT}, \\ \dfrac{\partial CP}{CP} = \beta \cdot \dfrac{\partial FA}{FA}, \\ \dfrac{\partial CP}{CP} = \gamma \cdot \dfrac{\partial CA}{CA}. \end{cases}$$

From this system it follows that the total increment of the final product of the enterprise is the sum of the increments due to individual factors of production :

$$\sum \frac{\partial CP}{CP} = \alpha \cdot \frac{\partial FOT}{FOT} + \beta \cdot \frac{\partial FA}{FA} + \gamma \cdot \frac{\partial CA}{CA}.$$

Based on the constructed mathematical model, we find the indicator $h = \alpha + \beta + \gamma$ called the elasticity of production. Its value shows how the scale of production affects output. If $h = 1$, then function (5) assumes a constant growth of scale of the production. If $h > 1$, the accruing effect of the growth of scale of production prevails, i.e. in this production process, the return on production resources increases. If $h < 1$, there exists a diminishing effect of the growth of scale of production, that is, the return on resources involved in production at the enterprise decreases, i.e. the final result of production decreases: revenue or marketable output.

In the resulting model (5), $h = 0.104$, which indicates a decreasing effect of growth in the scale of production, i.e. additional involvement of resources (employees and non-current assets) reduces the final result of production (output).

The dynamics of the indicator h can be used to assess the economic stability of the enterprise. If $\partial h/\partial t > 0$, at $t = \overline{1, T}$, then there exists an economic stability of the enterprise, otherwise we arrive at a decrease in the economic stability of the enterprise.

Conclusions

Since 2016, the dynamics of the indicator h was negative, which means that the company is not economically stable and bankruptcy is inevitable. The absence of an economic analysis of the company's operating activities may result in its liquidation.

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ПОСТРОЕНИЕ МАТЕМАТИЧЕСКОЙ МОДЕЛИ ДЕЯТЕЛЬНОСТИ АО «ЧЕЛЯБИНСКГОРГАЗ»

В. Г. Мохов

В статье рассмотрена математическая модель производственной деятельности основного поставщика газа жителям г. Челябинска, построенная на основе модифицированной производственной функции Кобба – Дугласа за 2011-2020 годы. Приведен авторский алгоритм построения модели, по которому на языке Java написана и зарегистрирована в государственном Реестре программ для ЭВМ компьютерная программа моделирования операционной деятельности предприятия. Адекватность построенной модели проверена по коэффициенту детерминации, значение которого показало ее высокую достоверность. Анализ построенной математической модели позволил сделать следующие выводы: эффективность операционной деятельности не отвечает современным требованиям – в течение всего анализируемого периода имеет место убывающий эффект масштаба; начиная с 2016 г. на предприятии отсутствует экономическая устойчивость и процедура банкротства становится неизбежной.

Результаты исследования могут быть рекомендованы научному сообществу при проведении аналогичных работ, направленных на моделирование операционной деятельности, что возможно, так как авторская компьютерная программа моделирования зарегистрирована в государственном реестре программ для ЭВМ и находится в открытом доступе.

Ключевые слова: математическая модель; анализ; поставщик газа; программное обеспечение; экономическая устойчивость.

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