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PREDICTION OF MULTIDIMENSIONAL TIME SERIES BY METHOD OF INVERSE SPECTRAL PROBLEM

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The paper develops a new method for predicting time series by the inverse spectral problem. We show that it is possible to construct a differential operator such that its eigenvalues coincide with a given numerical sequence. The paper gives a theoretical justification of the proposed method. The algorithm for finding a solution and an example of constructing a differential operator with partial derivatives are given. In this paper, we present a generalization in the case of multidimensional time series.

Keywords: Laplace operator; inverse spectral problem; eigenvalues; time series.

Introduction

The problem of constructing time series models has different forms and represents different stochastic processes. We consider the following statement of the problem. Let a k-dimensional time series $\{t_1, t_2, \ldots, t_n\}$, $t_i = (t_{i1}, \ldots, t_{ik})$, $i = \overline{1, n}$ be known. This series is represented in the form of an observation matrix (t_{ij}). The problem is to construct a model that allows to predict the values of several terms of the time series based on several terms that precede the considered ones. The stated problem is widely known. Many different models were constructed to solve the problem with varying degrees of accuracy.

In the present work, we develop a method of constructing a differential operator such that the sequence of its eigenvalues allows to restore the time series. As a model operator, we choose the Laplace operator.

Previously, a similar problem was successfully solved in [?] for a one-dimensional time series. In this work, a generalization is presented in the case of a multidimensional time series. The author understands that the proposed method is much more complicated than those widely known in computational terms and is unlikely to find any practical application, however, an example from the work [?], as well as an example presented in the present paper, show that the method is quite justified and has the right to exist.

1. Problem Statement

Let Π be a rectangle with the sides a and b, where $\dfrac{a^2}{b^2}$ is irrational. In the space $H = L_2(\Pi)$, consider a self-adjoint non-negative operator T_0 generated by the Dirichlet problem:

$$-\Delta v = \lambda v, \quad v|_{\partial\Pi} = 0,$$

where Δ is the Laplacian.

Introduce the operator $T = \displaystyle\int_0^\infty \lambda^\beta dE(\lambda)$, where $E(\lambda)$ is a spectral decomposition of the operator unit T_0, $\beta > 3/2$, $\lambda^\beta > 0$ at $\lambda > 0$. The eigenvalues $\lambda_{kl} = \left(\dfrac{\pi^2 k^2}{a^2} + \dfrac{\pi^2 l^2}{b^2}\right)^\beta$

of the operator T correspond to the eigenfunctions

$$v_{kl}(x,y)=\frac{2}{\sqrt{ab}}\sin\left(\frac{\pi kx}{a}\right)\sin\left(\frac{\pi ly}{b}\right),\quad k,l=\overline{1,\infty},$$

which are orthonormal in $L_2(\Pi)$. Since a^2/b^2 is an irrational number, the spectrum $\sigma(T)$ of the operator T is single. For convenience, we enumerate the ascending natural numbers λ_n and the associated spectral objects with one natural index.

Let P be a bounded operator of multiplication by the function $p\in H$ acting in H. We look for the perturbed operator $T+P$, the eigenvalues of which allow to restore the time series. Therefore, the formulated problem of predicting the terms of the time series is solved as the inverse problem of spectral analysis.

2. Main Result

Denote by $R_0(\lambda)=(T-\lambda E)^{-1}$, $R(\lambda)=(T+P-\lambda E)^{-1}$ the resolvents of the operators T and $T+P$, respectively. It is known that if P is a bounded operator, then $T+P$ is discrete. Moreover, if R_0 is a kernel operator, then $R(\lambda)$ is a kernel operator as well. This allows to denote by μ_{kl} the eigenvalues of the operator $T+P$ and enumerate them in ascending order of the real parts taking into account algebraic multiplicity, and denote by u_{kl} the corresponding eigenfunctions orthonormal in H. Introduce an auxiliary rectangle $\Pi_4=\{(x,y)|\quad 0\le x\le\frac{a}{2},\quad 0\le y\le\frac{b}{2}\}$ and a total system of functions $\{\varphi_n\}_{n=1}^{\infty}$, $\varphi_n(x,y)=\varphi_{kl}(x,y)=h_{kl}\cos\dfrac{2\pi kx}{a}\cos\dfrac{2\pi ly}{b}$ orthonormal in $L_2(\Pi_4)$, where $\lambda_n=\left(\dfrac{\pi^2k^2}{a^2}+\dfrac{\pi^2l^2}{b^2}\right)^{\beta}$, $h_{kl}=\frac{2\sqrt{(1+\delta_{k0})(1+\delta_{0l})}}{\sqrt{ab}}$ $k,l=\overline{0,\infty}$.

Introduce the following notations: $r_n=\frac{1}{2}\min\{\lambda_{n+1}-\lambda_n;\lambda_n-\lambda_{n-1}\}$, $\gamma_n=\{\lambda:|\lambda_n-\lambda|=r_n\}$, $\Gamma_n=\{\lambda:|\lambda|=\lambda_n\}$, $\Omega_n=\{\lambda:|\lambda_n-\lambda|\ge r_n\}$, $\Omega_N=\bigcap\limits_{n=N}^{\infty}\Omega_n$, $\|\cdot\|_1$, $\|\cdot\|_2$ which are the kernel norm and the Hilbert–Schmidt norm, respectively. Since the series $\sum\limits_n\frac{1}{\lambda_n}$ converges, the following statement is obvious.

Lemma 1. *There exists $N\in\mathbb{N}$ such that*
1) the same number of eigenvalues λ_n and μ_n take place within the contour Γ_N;
2) exactly one λ_n and one μ_n take place inside the contours γ_n for all $n>N$.

Lemma 2. [?, ?] *For $n\gg 1$, $\beta>3/2$, there exists an estimate:*

$$\|R_0(\lambda)\|_2^2\le\|R_0(\lambda)\|^2+\frac{1}{r_n^2}\left(2+\frac{\beta C}{2}+\frac{1}{C^{1/\beta}}\right),\quad \lambda\in\gamma_n.$$

Corollary 1. [?] *For $\beta>3/2$, the series $\sum\limits_n r_n^2\left(\max\limits_{\lambda\in\gamma_n}\|R_0(\lambda)\|_2\right)^4$ converges.*

Consider the identity operator

$$R(\lambda)=R_0(\lambda)-R_0(\lambda)PR_0(\lambda)+R(\lambda)(PR_0(\lambda))^2,\ \lambda\in\Omega_N.$$

Multiply the operator by $\frac{\lambda}{2\pi}$ and integrate along the contour γ_n. Therefore, we find the following:

$$\mu_n=\lambda_n+(Pv_n,v_n)+\alpha_n(p),\quad n>N,$$

where

$$\alpha_n(p) = -\frac{1}{2\pi i}\int_{\gamma_n} \lambda \mathrm{Sp}\,\left[R(\lambda)(PR_0(\lambda))^2\right] d\lambda.$$

Similarly, multiplying by $\frac{\lambda^q}{2\pi}$ and integrating along Γ_N, we get

$$\sum_{n=1}^{N} \mu_n^q = \sum_{n=1}^{N} \lambda_n^q + \sum_{n=1}^{N} q\lambda_n^{q-1}(Pv_n, v_n) + \alpha_q(p), \quad q \le N, \tag{1}$$

where

$$\alpha_q(p) = -\frac{1}{2\pi i}\int_{\Gamma_N} \lambda^q \mathrm{Sp}\,\left[R(\lambda)(PR_0(\lambda))^2\right] d\lambda.$$

It can be shown that for the operator $R(\lambda)$, the decomposition into a converging series is valid:

$$R(\lambda) = \sum_{k=0}^{\infty}(-1)^k R_0(\lambda)(PR_0(\lambda))^k,\ \lambda \in \Omega_N. \tag{2}$$

Substituting series (**??**) in α_n, we obtain $\alpha_n(p) = \sum\limits_{k=2}^{\infty} \alpha_n^{(k)}(p)$, where

$$\alpha_q^{(k)}(p) = -\frac{(-1)^k}{2\pi i}\int_{\Gamma_N} \lambda^q \mathrm{Sp}\,\left[R_0(\lambda)(PR_0(\lambda))^k\right] d\lambda, \quad q \le N,$$

$$\alpha_n^{(k)}(p) = -\frac{(-1)^k}{2\pi i}\int_{\gamma_n} \lambda \mathrm{Sp}\,\left[R_0(\lambda)(PR_0(\lambda))^k\right] d\lambda, \quad n > N.$$

By integrating in parts, it is easy to obtain the following formula:

$$\mathrm{Sp}\int_{\gamma} g(\lambda)R_0(\lambda)(PR_0(\lambda))^k d\lambda = \frac{-1}{k}\mathrm{Sp}\int_{\gamma} g'(\lambda)(PR_0(\lambda))^k d\lambda.$$

From here we get

$$\alpha_q^{(k)}(p) = \frac{(-1)^k q}{2\pi i k}\int_{\Gamma_N} \lambda^{q-1} \mathrm{Sp}\,\left[(PR_0(\lambda))^k\right] d\lambda, \quad q \le N,$$

$$\alpha_n^{(k)}(p) = \frac{(-1)^k}{2\pi i k}\int_{\gamma_n} \mathrm{Sp}\,\left[(PR_0(\lambda))^k\right] d\lambda, \quad n > N.$$

Write (**??**) in the matrix form $WV = M$, where

$$W = \begin{pmatrix} 1 & 1 & \dots & 1 \\ \lambda_1 & \lambda_2 & \dots & \lambda_N \\ \dots & \dots & \dots & \dots \\ \lambda_1^{N-1} & \lambda_2^{N-1} & \dots & \lambda_N^{N-1} \end{pmatrix}, \quad V = \begin{pmatrix} (Pv_1, v_1) \\ (Pv_2, v_2) \\ \dots \\ (Pv_N, v_N) \end{pmatrix}, \quad M = \begin{pmatrix} m_1 \\ m_2 \\ \dots \\ m_N \end{pmatrix},$$

$m_q = \frac{1}{q}\Big[\sum\limits_{n=1}^{N}(\mu_n^q - \lambda_n^q) - \alpha_N^q(p)\Big]$. The Vandermond determinant $|W| \ne 0$, therefore the matrix W is invertible and $V = W^{-1}M$. Denote by w_{nq}^- the elements of the inverse matrix W^{-1}. Therefore, we get $(Pv_n, v_n) = \sum\limits_{q=1}^{N} w_{nq}^- m_q$, $n \le N$.

Let $r = \min\{\|P_1\|, \|P_2\|\}$. We estimate the difference $|\alpha_n(p_1) - \alpha_n(p_2)|$ for $n > N$.

$$|\alpha_n^{(k)}(p_1) - \alpha_n^{(k)}(p_2)| = \frac{1}{2\pi k}\left|\int_{\gamma_n} \mathrm{Sp}\ \left[(P_1R_0(\lambda))^k - (P_2R_0(\lambda))^k\right] d\lambda\right| \leq$$

$$\leq \frac{r_n}{k}\max_{\lambda\in\gamma_n}\left\|(P_1R_0(\lambda))^k - (P_2R_0(\lambda))^k\right\|_1 \leq$$

$$\leq \frac{r_n}{k}\max_{\lambda\in\gamma_n}\left\|\sum_{s=0}^{k-1}(P_2R_0(\lambda))^s(P_1-P_2)R_0(\lambda)(P_1R_0(\lambda))^{k-s-1}\right\|_1 =$$

$$= \frac{r_n}{k}\max_{\lambda\in\gamma_n}\left(\sum_{s=0}^{k-1}\|P_1-P_2\|r^{k-1}\|R_0(\lambda)\|_2^2\,\|R_0(\lambda)\|^{k-2}\right) =$$

$$= r_n\|P_1-P_2\|r^{k-1}\max_{\lambda\in\gamma_n}\left(\|R_0(\lambda)\|_2^2\,\|R_0(\lambda)\|^{k-2}\right).$$

Next, we estimate the absolute value of the difference:

$$|\alpha_n(p_1) - \alpha_n(p_2)| \leq r_n r\|P_1-P_2\|\max_{\lambda\in\gamma_n}\|R_0(\lambda)\|_2^2\sum_{k=0}^{\infty}r^k\max_{\lambda\in\gamma_n}\|R_0(\lambda)\|^k \leq$$

$$\leq \|P_1-P_2\|\max_{\lambda\in\gamma_n}\|R_0(\lambda)\|_2^2\frac{rr_n}{1-r/r_n}.$$

Similarly, for $q \leq N$ we get

$$|\alpha_q(p_1) - \alpha_q(p_2)| \leq \|P_1-P_2\|\max_{\lambda\in\Gamma_N}\|R_0(\lambda)\|_2^2\frac{qrr_N^q}{1-r/r_N}.$$

Lemma 3. [?] *If the function p satisfies the following conditions:*
(i) $p(x, b-y) = p(x,y) = p(a-x,y)$, for almost all $(x,y)\in\Pi$;
(ii) $(p,\varphi_{0k})_{L_2(\Pi_4)} = (p,\varphi_{k0})_{L_2(\Pi_4)} = 0$, $k = \overline{0,\infty}$, then

$$(Pv_n, v_n)_{L_2(\Pi)} = \frac{1}{\sqrt{ab}}(p,\varphi_n)_{L_2(\Pi_4)}.$$

Theorem 1. *If there exist $N\in\mathbb{N}$ and $r>0$ for the sequence $\{\xi_n\}$ such that the following inequalities hold:*

$$\omega^2 := \frac{4r^2}{ab}\Big\{\Big[\max_{\lambda\in\Gamma_N}\|R_0(\lambda)\|_2^2\frac{1}{1-r/r_N}\Big]^2\sum_{n=1}^{N}\Big[\sum_{q=1}^{N}|w_{nq}^-|r_N^q\Big]^2 +$$

$$+\sum_{n=N+1}^{\infty}\max_{\lambda\in\gamma_n}\|R_0(\lambda)\|_2^2\frac{r_n}{1-r/r_n}\Big]^2\Big\} < 1,$$

$$\sum_{n=1}^{N}\left|\sum_{q=1}^{N}\frac{w_{nq}^-}{q}\sum_{s=1}^{N}(\xi_s^q-\lambda_s^q)\right|^2 + \sum_{n=N+1}^{\infty}|\xi_n-\lambda_n|^2 \leq r_N^2\omega^2 ab,$$

then there exists the operator P such that the spectrum $\sigma(T+P)$ coincides with the sequence $\{\xi_n\}$.

Proof. In the space $H_1 = L_2(\Pi_4)$, consider the equation with respect to p:

$$p = \alpha_0 - \alpha(p),$$

where

$$\alpha_0 = \sum_{n=1}^{N}\sum_{q=1}^{N}\frac{w_{nq}^{-}}{q}\sum_{s=1}^{N}(\xi_s^q - \lambda_s^q)\varphi_n + \sum_{n=N+1}^{\infty}(\xi_n - \lambda_n)\varphi_n,$$

$$\alpha(p) = \sum_{n=1}^{N}\sum_{q=1}^{N}\frac{w_{nq}^{-}}{q}\alpha_q(p)\varphi_n + \sum_{n=N+1}^{\infty}\alpha_n(p)\varphi_n.$$

Introduce the operator $A : H_1 \to H_1$:

$$Ap = \alpha_0 - \alpha(p).$$

Let us find

$$\|\alpha(p_1) - \alpha(p_2)\|_{H_1}^2 \le \sum_{n=1}^{N}\Big[\sum_{q=1}^{N}|w_{nq}^{-}|\|P_1 - P_2\| \max_{\lambda\in\Gamma_N}\|R_0(\lambda)\|_2^2\frac{rr_N^q}{1 - r/r_N}\Big]^2 +$$

$$+ \sum_{n=N+1}^{\infty}\Big[\|P_1 - P_2\| \max_{\lambda\in\gamma_n}\|R_0(\lambda)\|_2^2\frac{rr_n}{1 - r/r_n}\Big]^2 =$$

$$r^2\|P_1 - P_2\|^2\Big\{\Big[\max_{\lambda\in\Gamma_N}\|R_0(\lambda)\|_2^2\frac{1}{1 - r/r_N}\Big]^2\sum_{n=1}^{N}\Big[\sum_{q=1}^{N}|w_{nq}^{-}|r_N^q\Big]^2 +$$

$$+ \sum_{n=N+1}^{\infty}\max_{\lambda\in\gamma_n}\|R_0(\lambda)\|_2^2\frac{r_n}{1 - r/r_n}\Big]^2\Big\} = \|p_1 - p_2\|_{H_1}^2\omega^2.$$

Therefore, α is the contraction operator.

Put $R = \min\{r, r_N\sqrt{ab}(1-\omega)\}$. Since the operator α_0 is completely continuous, then, according to the combination principle, there exists a solution to the equation in the ball $U(\alpha_0, R) \subset H_1$. Note that the combination principle does not guarantee the uniqueness of the solution.

Let p be a solution to the equation, P be the operator of multiplication by the function p, $\sigma(T + P) = \{\mu_n\}$ be the spectrum of the founded operator. From the construction of the equation, it is obvious that the sequences $\{\mu_n\}$ and $\{\xi_n\}$ are the same.

□

3. Algorithm

Let us give the algorithm arising from the Banach principle.

1. Specify the accuracy δ.

2. Choose m. The number m is chosen arbitrarily, the larger m is, the more accurate approximate solution is found. If the amount of ξ_n is finite, then m is determined naturally.

3. Put $p_0 \equiv 0$.

4. $p_1 = \alpha_0 - \alpha(p_0) = \alpha_0$,

$$\alpha_0 = \sum_{n=1}^{m} (\xi_n - \lambda_n)\varphi_n, \quad \varphi_{kl}(x) = \frac{4}{\sqrt{ab}} \cos\frac{2\pi kx}{a} \cos\frac{2\pi lx}{b}.$$

5. Calculate $p_{j+1} = \alpha_0 - \alpha(p_j)$, $j = 1, 2, \dots$ by the formulas:

$\alpha(p) = \sum_{n=1}^{m} \Big(\alpha_n^2(p) + \alpha_n^3(p) + \alpha_n^4(p)\Big)\varphi_n,$

$$\alpha_n^2(p) = -\sum_{i \neq n} \frac{V_{in}^2}{\lambda_i - \lambda_n}, \quad \text{where} \quad V_{ni} = (Pv_n, v_i),$$

$$\alpha_n^3(p) = \sum_{i,j \neq n} \frac{V_{ni}V_{ij}V_{jn}}{(\lambda_i - \lambda_n)(\lambda_j - \lambda_n)} - \sum_{i \neq n} \frac{V_{ni}^2 V_{nn}}{(\lambda_i - \lambda_n)^2},$$

$$\alpha_n^4(p) = -\sum_{i,j,k \neq n} \frac{V_{ni}V_{ij}V_{jk}V_{kn}}{(\lambda_i - \lambda_n)(\lambda_j - \lambda_n)(\lambda_k - \lambda_n)} + \sum_{i,j \neq n} \frac{V_{nn}V_{ni}V_{ij}V_{jn}}{(\lambda_i - \lambda_n)(\lambda_j - \lambda_n)^2} +$$

$$+ \sum_{i,j \neq n} \frac{V_{nn}V_{ni}V_{ij}V_{jn}}{(\lambda_i - \lambda_n)^2(\lambda_j - \lambda_n)} + \sum_{i,j \neq n} \frac{V_{ni}^2 V_{jn}^2}{(\lambda_i - \lambda_n)^2(\lambda_j - \lambda_n)} - \sum_{i \neq n} \frac{V_{nn}^2 V_{ni}^2}{(\lambda_i - \lambda_n)^3}.$$

6. Calculate μ_n, $n = \overline{1, m}$:

$$\mu_n = \lambda_n + V_{nn} + \alpha_n^2(p) + \alpha_n^3(p) + \alpha_n^4(p).$$

7. Compare μ_n obtained at Step 5 with ξ_n according to any criterion, for example, the least squares:

$$MNK = \sum_{n=1}^{m} |\xi_n - \mu_n|^2 < \delta^2.$$

7. If the value of the criterion decreases compared to the previous one, then we proceed to the next iteration, i.e. to Step 4. If the value increases and the required accuracy was achieved at the previous iteration, then the approximate solution $\tilde{p} = p_{j+1}$ is found. If the value increases, but the required accuracy was not achieved at the previous iteration, then go to Step 1, increasing m.

4. Example

As an example, consider the two-dimensional time series dollar/ruble and euro/ruble rates from 06.08.2019 to 10.08.2019 on MOEX. For shortness, we consider the series containing only five terms:

$$\{t_n\}_{n=1}^{5} = \left\{ \begin{array}{ccccc} 65.0546 & 65.2030 & 65.0932 & 65.1299 & 65.2543 \\ 72.3732 & 73.0730 & 72.8914 & 73.0432 & 73.0196 \end{array} \right\}.$$

Select the parameters $\beta = 2$, $a = \pi$, $b = 20$, then the first eigenvalues of the Dirichlet problem discussed above are as follows:

$$\{\lambda_{ij}\} = \left\{ \begin{array}{ccccc} 1.0500 & 1.2071 & 1.4934 & 1.9454 & 2.1420 \\ 16.1980 & 16.7993 & 17.8258 & 19.3141 & 21.3153 \end{array} \right\}.$$

Construct a two-dimensional series $\{\xi_{ij}\}$, $\xi_{ij} = 0.0001t_{ij} + \lambda_{ij}$. Arrange in increasing order the numbers $\{\xi_{ij}\}$, the eigenvalues λ_{ij}, as well as the eigenfunction v_{ij} corresponding to these eigenvalues, enumerating each obtained sequence with the same index. We look for the function p in the form of a series on the system of functions orthonormed in $L_2(\Pi)$. Select $\varphi_{ij} = \frac{4}{\sqrt{ab}}\cos(\frac{2\pi ix}{a})\cos(\frac{2\pi jy}{b})$ as the basis functions, also numbered by the same index accordingly. According to the above algorithm, as a result of two iterations, we find the function

$$p = 0.0260218423\cos(2x)\cos(0.3141y) + 0.0260812002\cos(2x)\cos(0.6283y)+$$

$$0.0260372764\cos(2x)\cos(0.9425y) + 0.0260519598\cos(2x)\cos(1.2566y)+$$

$$0.0261017189\cos(2x)\cos(1,5707y) + 0.0289492922\cos(4x)\cos(0.3141y)+$$

$$0.0292292000\cos(4x)\cos(0.6283y) + 0.0291565600\cos(4x)\cos(0.9425y)+$$

$$0.0292172800\cos(4x)\cos(1.2566y) + 0.0292078400\cos(4x)\cos(1.5707y).$$

Let us assume that the following terms of the decomposition have the same Fourier coefficients as the found higher coefficients, and add to this function the term

$$0.0261017189\cos(2x)\cos(1.8849y) + 0.0292078400\cos(4x)\cos(1.8849y).$$

Find the eigenvalues for the obtained function p by the formula of Step 5 of the algorithm:

$$\mu_n = \{1.0564; 1.2136; 1.4999; 1.9519; 2.6207; 35720;$$
$$16.2052; 16.8066; 17.8331; 19.3214; 21.3226; 23.9024\}$$

We enumerate μ_n with two indexes μ_{ij} according to the numbering of the numbers λ_n and λ_{ij}. Then return to the original variables $\eta_{ij} = \frac{\mu_{ij} - \lambda_{ij}}{0.0001} =$

$$\left\{ \begin{array}{cccccc} 65.0546000 & 65.2029999 & 65.0931955 & 65.1298973 & 65.2542987 & 65.2543001 \\ 72.3732000 & 73.0729893 & 72.8914172 & 73.0432015 & 73.0196135 & 73.0196091 \end{array} \right\}.$$

The least squares criterion of the sets $\{\eta_n\}_{n=1}^{5}$ and $\{t_n\}_{n=1}^{5}$ equals $6 \cdot 10^{-10}$.

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ПРОГНОЗИРОВАНИЕ МНОГОМЕРНОГО ВРЕМЕННОГО РЯДА МЕТОДОМ ОБРАТНОЙ ЗАДАЧИ СПЕКТРАЛЬНОГО АНАЛИЗА

А. И. Седов

В работе развивается новый метод прогнозирования временных рядов методом обратной задачи спектрального анализа. Показано, что можно построить такой дифференциальный оператор, что его собственные числа совпадут с данной числовой последовательностью. В работе дано теоретическое обоснование предложенного метода. Приводится алгоритм нахождения решения и пример построения дифференциального оператора с частными производными. В представленной работе сделано обобщение на многомерные временные ряды.

Ключевые слова: оператор Лапласа; обратная задача спектрального анализа; собственные числа; временной ряд.

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