MSC 34A55, 65M32, 65L09 DOI: 10.14529/jcem220106

PROCESSING OF INFORMATION ON RECOVERY OF THE EXTERNAL FORCE PARAMETER FOR THE MATHEMATICAL MODEL OF ION-ACOUSTIC WAVES IN PLASMA

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The article is devoted to the processing of input information for the mathematical model of ion-acoustic waves considered in plasma, taking into account the influence of a magnetic field, which is external to the system under consideration. The solution of the inverse problem for this model is the generalized potential of the electric field and the parameter of the external magnetic field. To find the solution, the method of successive approximations is used, and the model itself is reduced to a high-order Sobolev type equation. The first paragraph presents the results of an analytical study of this model. In the second paragraph, the developed algorithm for carrying out information processing and its scheme are presented. In the third, the results of information processing obtained on the basis of computational experiments of the developed program in the «Maple» environment are presented. All the results obtained can be applied in the field of mathematical modeling, for example, in calculating the generalized electric field potential for ion-acoustic waves in plasma.

Keywords: mathematical model of ion-acoustic waves; inverse problem; numerical study; plasma; Sobolev type equation; method of successive approximations.

Introduction

Let $\Omega = (0,a) \times (0,b) \times (0,c) \subset \mathbb{R}^3$. Consider equation of ion-acoustic waves in a plasma in an external magnetic field

$$(\Delta - \alpha)v_{tttt} + \beta(\Delta - \gamma)v_{tt} + \kappa\frac{\partial^2 v}{\partial x_3^2} + qf = 0, \quad (x,t) \in \Omega \times (0,T) \tag{1}$$

with the Cauchy condition

$$v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x), \quad v_{tt}(x,0) = v_2(x), \quad v_{ttt}(x,0) = v_3(x), \quad x \in \Omega, \tag{2}$$

the Dirichlet condition

$$v(x,t) = 0, \quad (x,t) \in \partial\Omega \times [0,T], \tag{3}$$

and overdetermination condition

$$\int\limits_\Omega v(x,t)K(x)dx = \Phi(t). \tag{4}$$

Let the functions $K(x)$, $v_0(x)$, $v_1(x)$, $v_2(x)$, $v_3(x)$, $f(x,t)$, $\Phi(t)$ be given, and

$$\int\limits_\Omega v(x,t)K(x)dx =< v(x,t), K(x) >$$

be the inner product in $L_2(\Omega)$. The coefficients α, β, γ, κ of (1) relate such quantities as the Debye radius, ion gyrofrequency, and Langmuir frequency. Conditions (2) and (3) define initial and boundary values, respectively. The overdetermination condition (4) defines some averaging of the function $v(x,t)$ over the entire domain Ω under consideration.

The inverse problem for ion-acoustic waves in a plasma in an external magnetic field is the problem of finding a pair of functions $v(x,t)$ and $q(t)$ from the relations (1)–(4), where $v(x,t)$ describes the generalized potential of the electric field, and $q(t)$ is the parameter of the external action of the magnetic field.

One of the first references to equation (1) was presented by A.G. Sveshnikov, A.B. Alshin, M.O. Korpusov, and Yu.D. Pletner [1]. Equation (1) belongs to the Sobolev type equations which are of great interest [2, 3, 4, 5, 6, 7]. For example, [2] considers the solvability and stability of a solution to the Showalter–Sidorov problem for a stochastic version of the linear Ginzburg–Landau equation in Hilbert spaces of smooth differential forms, reducing the original equation to an abstract Sobolev-type equation with a relatively radial operator. E.V. Bychkov's paper [3] is devoted to the consideration of the initial-boundary value problem for the modified Boussinesq equation, which describes the propagation of waves in shallow water under the condition of conservation of mass in the layer and taking into account capillary effects.

S.G. Pyatkov, together with M.V. Uvarova and T.V. Pronkina, in [8] considered the question of well-posedness of the inverse problem of determining the source function for a second-order quasilinear parabolic system. In the article by Ya.T. Megraliev and G.N. Iskenderova [9] one inverse boundary value problem for a second-order hyperbolic equation with an integral condition of the first kind was studied. This work is based on the theory of inverse problems [4, 5, 8, 9] and analytical results obtained in [6].

It should be noted that there are already works devoted to the study of ion-acoustic waves in different media and with different conditions [10, 11], but in this setting, the inverse problem (1)–(4) is studied for the first time. The work of A.E. Dubinov and I.N. Kitaev [10] is devoted to the study of oblique ion-acoustic waves in plasma, where ions move unidirectionally. M.O. Korpusov [11] proved the existence of a weak generalized solution and obtained sufficient conditions for the blow up of a weak generalized solution in a finite time for the equation of ion-acoustic waves in «non-magnetized» plasma, taking into account nonlinear sources localized at the boundary.

Among the «fresh» works on numerical research and information processing, it is worth highlighting [5, 7, 12]. E.A. Soldatova and A.V. Keller presented an algorithm for numerical research using the Galerkin method and information processing for a model of the pressure dynamics of a liquid filtering in a fractured-porous medium with a random external action [7]. In [12], the authors describe the mathematical model of optimal measurement in the presence of various types of noise, describe the approximations of the optimal measurement and prove their convergence to the precise one, and also describe the algorithm for finding approximations of the optimal measurement numerically.

1. Analytical Investigation of the Mathematical Model

Let $\mathcal{U} = \{v \in W_2^{l+2}(\Omega) : v(x) = 0, x \in \partial\Omega\}$, $\mathcal{Y} = \mathcal{F} = W_2^l(\Omega)$ be Banach spaces, and $W_2^l(\Omega)$ be Sobolev spaces, where $l = 0, 1, \ldots$. According to [6], similarly to [4, 5], problem (1)–(4) can be reduced to the problem of finding functions $u \in C^4([0,T]; \mathcal{U}^1)$,

$w \in C^4([0,T];\mathcal{U}^0)$, $q \in C^1([0,T];\mathcal{Y})$ from the relations

$$u^{(4)}(t) = S_1 u''(t) + S_2 u(t) + (A^1)^{-1} Q q(t) f(t), \tag{5}$$

$$u^{(j)}(0) = u_j, \quad j = 0,1,2,3, \tag{6}$$

$$Cu(t) = \Psi(t) \equiv Cv(t), \tag{7}$$

$$H_1 w^{(4)}(t) = H_2 w''(t) + w(t) + (B_0^0)^{-1}(\mathbb{I} - Q) q(t) f(t), \tag{8}$$

$$w^{(j)}(0) = w_j, \quad j = 0,1,2,3, \tag{9}$$

where

$$S_1 = \sum_{\lambda_{kmn} \neq \alpha} \frac{\beta(\lambda_{kmn} - \gamma)}{\lambda_{kmn} - \alpha} < \cdot, \mathbb{X}_{kmn} > \mathbb{X}_{kmn},$$

$$S_2 = -\sum_{\lambda_{kmn} \neq \alpha} \frac{\kappa \pi^2 n^2}{(\lambda_{kmn} - \alpha) c^2} < \cdot, \mathbb{X}_{kmn} > \mathbb{X}_{kmn},$$

$$(B_0^0)^{-1} = -\sum_{\lambda_{kmn} = \alpha} \frac{c^2}{\kappa \pi^2 n^2} < \cdot, \mathbb{X}_{kmn} > \mathbb{X}_{kmn}, \quad (A^1)^{-1} = \sum_{\lambda_{kmn} \neq \alpha} \frac{< \cdot, \mathbb{X}_{kmn} >}{\lambda_{kmn} - \alpha} \mathbb{X}_{kmn},$$

$$\mathbb{I} - Q = \sum_{\lambda_{kmn} = \alpha} < \cdot, \mathbb{X}_{kmn} > \mathbb{X}_{kmn}, \quad Q = \sum_{\lambda_{kmn} \neq \alpha} < \cdot, \mathbb{X}_{kmn} > \mathbb{X}_{kmn},$$

$$w_j = \sum_{\lambda_{kmn} = \alpha} < v_j, \mathbb{X}_{kmn} > \mathbb{X}_{kmn}, \quad u_j = \sum_{\lambda_{kmn} \neq \alpha} < v_j, \mathbb{X}_{kmn} > \mathbb{X}_{kmn}, \quad j = 0,1,2,3,$$

$$H_1 = -\sum_{\lambda_{kmn} = \alpha} \frac{(\lambda_{kmn} - \alpha) c^2}{\kappa \pi^2 n^2} < \cdot, \mathbb{X}_{kmn} > \mathbb{X}_{kmn},$$

$$H_2 = -\sum_{\lambda_{kmn} = \alpha} \frac{\beta(\lambda_{kmn} - \gamma) c^2}{\kappa \pi^2 n^2} < \cdot, \mathbb{X}_{kmn} > \mathbb{X}_{kmn},$$

$$C = \sum_{\lambda_{kmn} \neq \alpha} < \cdot, K(x) > .$$

Here $\lambda_{kmn} = -\pi^2 \left(\left(\frac{k}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2 \right)$ are the eigenvalues of the Laplace operator, and $\mathbb{X}_{kmn} = \left\{ \sin\left(\frac{\pi k x_1}{a}\right) \sin\left(\frac{\pi m x_2}{b}\right) \sin\left(\frac{\pi n x_3}{c}\right) \right\}$ are the corresponding eigenfunctions from $L_2(\Omega)$, where $k, m, n \in \mathbb{N}$.

Theorem 1. *Let K, u_0, u_1, u_2, $u_3 \in \mathcal{U}^1$, $\Phi \in C^6([0,T];\mathcal{Y})$, $f \in C^4([0,T];\mathcal{F})$, one from the conditions $\alpha \notin \sigma(\Delta)$ or $(\alpha \in \sigma(\Delta)) \wedge (\alpha = \gamma)$ be fulfilled. Let also the conditions*

$$\sum_{\alpha = \lambda_{kmn}} < v_j, \mathbb{X}_{kmn} >= 0, \quad j = 0,1,2,3,$$

$$\sum_{\lambda_{kmn} \neq \alpha} \frac{< f(\cdot, t), K(x) >}{\lambda_{kmn} - \alpha} \neq 0,$$

$$< v_3, K(x) >= \Phi'''(0)$$

be satisfied for initial value $v_3 \in \mathcal{U}^1$*, and the initial values* $w_j = (\mathbb{I} - P)v_j \in \mathcal{U}^0$*, for* $k : \lambda_{kmn} = \alpha$ *satisfy*

$$< v_j - \Big(\frac{c^2}{\kappa\pi^2 n^2}\Big)\frac{\partial^j}{\partial t^j}\Big(q(t)f(\cdot,t)\Big)\Big|_{t=0}, \mathbb{X}_{kmn} >= 0 \text{ for } j = 0,1,2,3. \tag{10}$$

Then there exists a unique solution (v, q) *of the inverse problem* (1)–(4)*, where* $q \in C^2([0,T];\mathcal{Y})$*,* $v = u + w$*, whence* $u \in C^4([0,T];\mathcal{U}^1)$ *is a solution of* (5)–(7) *and the function* $w \in C^4([0,T];\mathcal{U}^0)$ *is a solution of* (8)*,* (9) *given by*

$$w(t) = \sum_{\lambda_{kmn}=\alpha}\Big(\frac{c^2}{\kappa\pi^2 n^2}\Big) < q(t)f(\cdot,t), \mathbb{X}_{kmn} > \mathbb{X}_{kmn}. \tag{11}$$

Proof. The conditions of [6, Lemma 1 and Theorem 1] are satisfied. Since $K \in \mathcal{U}^1$, then $\mathcal{U}^0 \subset ker\ C$. For $y \in \mathcal{Y}$ due to the orthonormality of the system of eigenfunctions in $L_2(\Omega)$

$$C(A^1)^{-1}Qy = \Big(\sum_{\lambda\neq\lambda_{kmn}}\frac{< f(\cdot,t), \mathbb{X}_{kmn} >< \mathbb{X}_{kmn}, K >}{\lambda_{kmn}-\alpha}\Big)y = \Big(\sum_{\lambda_{kmn}\neq\alpha}\frac{< f(\cdot,t), K(x) >}{\lambda_{kmn}-\alpha}\Big)y.$$

This operator is reversible in $\mathcal{Y}$ when

$$\sum_{\lambda_{kmn}\neq\alpha}\frac{< f(\cdot,t), K(x) >}{\lambda_{kmn}-\alpha} \neq 0,$$

and the inverse operator is continuously differentiable by t due to the conditions on the function $f(\cdot, t)$.

Thus, all the conditions of [6, Theorem 4] are satisfied, therefore there exists a unique solution (v, q) of inverse problem (1)–(4), where $q \in C^2([0,T];\mathcal{Y})$, $v = u + w$, whence $u \in C^4([0,T];\mathcal{U}^1)$ is the solution of (5)–(7) and the function $w \in C^4([0,T];\mathcal{U}^0)$ is a solution of (8), (9) given by (11).

□

2. Algorithm of Numerical Method

Let us describe the developed algorithm for processing information for restoring the parameter of the influence of an external magnetic field and the generalized potential of the electric field in steps corresponding to the blocks shown in Figure 1.

Start of the program.

Step 1. Input initial data: the values a, b, c of the upper boundaries of the study domain; ion-acoustic wave equation parameters α, β, γ, κ; time limit value T; the value ε of the allowable deviation between adjacent successive approximations of the function $q(t)$; the numbers K, M, N of terms in the Galerkin approximations; a function $f(x,t)$ of a known external load; initial values $v_0(x)$, $v_1(x)$, $v_2(x)$, $v_3(x)$ for the generalized electric field potential function; kernel $K(x)$ in the overdetermination condition (4); the right side $\Phi(t)$ of the overdetermination condition (4).

Step 2. Solving of the Sturm-Liouville problem. Hence, getting eigenvalues λ_{kmn} and eigenfunctions $\mathbb{X}_{kmn}$.

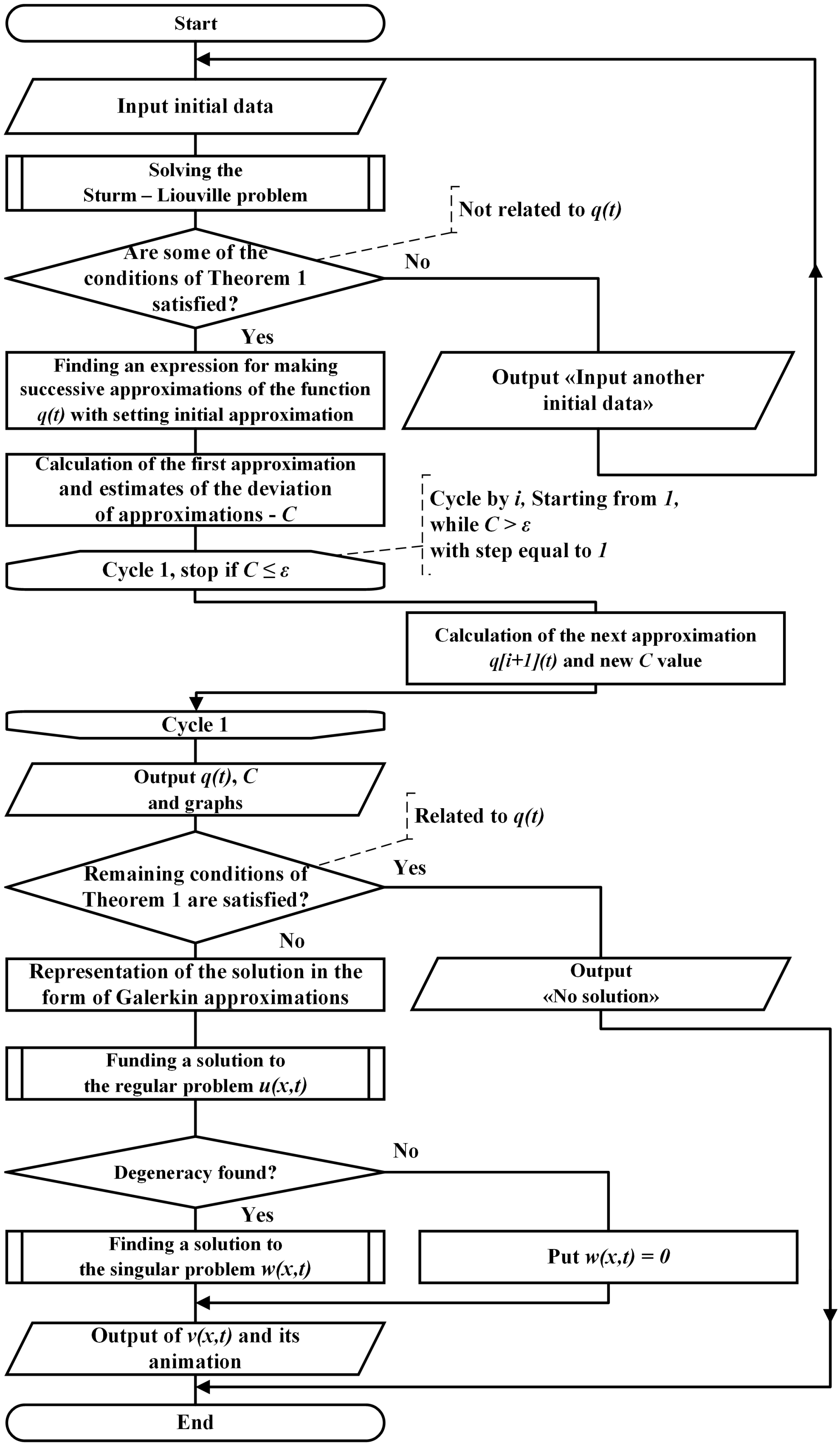


Fig. 1. Scheme of the algorithm

Step 3. Checking the fulfillment of conditions of Theorem 1:

i) one of the conditions: $\alpha \notin \sigma(\Delta)$ or $(\alpha \in \sigma(\Delta)) \wedge (\alpha = \gamma)$ is fulfilled;

ii) conditions $\sum\limits_{\alpha=\lambda_{kmn}} < v_j, \mathbb{X}_{kmn} >= 0$ for $j = 0, 1, 2, 3$ are satisfied;

iii) condition $\sum\limits_{\lambda_{kmn}\neq\alpha} \frac{<f(\cdot,t),K(x)>}{\lambda_{kmn}-\alpha} \neq 0$ is true;

iv) for initial value $v_3 \in \mathcal{U}^1$, condition $< v_3, K(x) >= \Phi'''(0)$ is fulfilled.

If all conditions are met, then go to step 5, otherwise go to step 4.

Step 4. Print «Input another initial data» go to step 1.

Step 5. Compiling an expression for restoring the function $q(t)$ using the method of successive approximations, using the formula from [6]:

$$q[i+1](t) = q_0(t) + \frac{1}{\sum\limits_{\lambda_{kmn}\neq\alpha} \frac{<f(x,t),K(x)>}{\lambda_{kmn}-\alpha}} \times$$

$$\times\left(\sum_{\lambda_{kmn}\neq\alpha} \frac{\kappa\pi^2 n^2}{c^2(\lambda_{kmn}-\alpha)^2} \int\limits_0^t \left(\sum_{j=1}^4 \frac{e^{\mu^j_{kmn}(t-s)}}{M_j} \right) < f(x,s)q[i](s), K(x) > ds - \right. \tag{12}$$

$$\left. - \sum_{\lambda_{kmn}\neq\alpha} \frac{(\lambda_{kmn}-\gamma)\beta}{(\lambda_{kmn}-\alpha)^2} \int\limits_0^t \left(\sum_{j=1}^4 \frac{(\mu^j_{kmn})^2 e^{\mu^j_{kmn}(t-s)}}{M_j} \right) < f(x,s)q[i](s), K(x) > ds \right).$$

Step 6. Calculate the first approximation $q[1](t)$ from the given initial approximation $q[0](t) = 0$, using formula (12). Calculate the estimation error C, which is equal to the norm (in space $L_2(\Omega)$) of the difference between the 1st approximation and the initial one.

Step 7. Cycle over i starting from 1 while $C > \varepsilon$. If the cycle condition is satisfied go to step 8, otherwise go to step 10.

Step 8. Calculate the next approximation $q[i+1](t)$ from the previous approximation $q[i](t)$ by formula (12). Calculate a new estimation error C, which is equal to the norm of the difference between the $(i+1)$-th approximation and the i-th.

Step 9. Increse index i by one, go to step 7.

Step 10. Print of the restored approximate value of the parameter $q(t)$, as well as the resulting estimation error C for the found function. Plot the found function $q(t)$ and the functions of all obtained successive approximations.

Step 11. Check the remaining conditions of Theorem 1:

$$< v_j + \left(\frac{c^2}{\kappa\pi^2 n^2}\right) \frac{\partial^j}{\partial t^j} \Big(q(t) f(\cdot, t) \Big)\Big|_{t=0}, \mathbb{X}_{kmn} >= 0, \quad j = 0, 1, 2, 3 \text{ for } k : \lambda_{kmn} = \alpha.$$

If all conditions are met, then go to step 13, otherwise go to step 12.

Step 12. Print «No solution». Stop the program.

Step 13. Represent the solution $v(x,t)$ as a Galerkin approximation.

Step 14. Obtain an approximate solution of the ion-acoustic waves equation (1) with the obtained function $q(t)$. The cycles in k, m, n, starting from 1, while $k \leq K$, $m \leq M$,

$n \leq N$ with the step equal to 1. Multiplying equation (1), as well as the initial conditions, scalarly by the eigenfunction $\mathbb{X}_{kmn}$. Solution of obtained second order ordinary differential equation with initial conditions. Go to the next iteration. Getting a solution to the regular problem $u(x,t)$ from (5)–(7).

Step 15. Check the degeneracy. If no degeneracy is found, then go to step 16, otherwise go to step 17.

Step 16. The solution $w(x,t)$ to the singular problem (8), (9) is equal to zero. Go to step 18.

Step 17. Find the solution $w(x,t)$ to the singular problem (8), (9) given by (11).

Step 18. Calculate the required function $v(x,t)$ as the sum of two previously obtained functions $u(x,t)$ and $w(x,t)$. Print the resulting function $v(x,t)$. Plot an animated graph of the function $v(x,t)$ by variable t.

End of program.

3. Results of Information Processing

Let us present the results of information processing according to the developed algorithm, which was implemented in the Maple environment. Information processing was carried out on the basis of computational experiments.

Example 1. Let the following input information be given:

$$\alpha = 1,\ \beta = 2,\ \gamma = -4,\ \kappa = -4,\ \varepsilon = 0.01,\ T = 2,\ k = 2,\ m = 1,\ n = 1,\ a = b = c = \pi,$$

$$v_0(x) = \sin(x_1)\sin(x_2)\sin(x_3),\ v_1(x) = 2\sin(x_1)\sin(x_2)\sin(x_3) - \sin(2x_1)\sin(2x_2)\sin(2x_3),$$

$$v_2(x) = \sin(2x_1)\sin(x_2)\sin(x_3) - \sin(x_1)\sin(2x_2)\sin(x_3),\ v_3(x) = \pi\sin(x_1)\sin(x_2)\sin(x_3),$$

$$f(x) = \sin(x_1),\quad K(x) = \sin(\pi x_1),\quad F(t) = \frac{-4\pi}{\pi^2 - 1}\cos(t + \pi^2).$$

Consequently, the mathematical model of ion-acoustic waves in a plasma in an external magnetic field (1)–(4) takes the form

$$(\Delta - 1)v_{tttt} + 2(\Delta + 4)v_{tt} - 4\frac{\partial^2 v}{\partial x_3^2} + q(t)\sin(x_1) = 0,$$

$$v(0, x_2, x_3, t) = v(\pi, x_2, x_3, t) = v(x_1, 0, x_3, t) = 0,$$

$$v(x_1, \pi, x_3, t) = v(x_1, x_2, 0, t) = v(x_1, x_2, \pi, t) = 0,$$

$$v(x, 0) = \sin(x_1)\sin(x_2)\sin(x_3),$$

$$v_t(x, 0) = 2\sin(x_1)\sin(x_2)\sin(x_3) - \sin(2x_1)\sin(2x_2)\sin(2x_3),$$

$$v_{tt}(x, 0) = \sin(2x_1)\sin(x_2)\sin(x_3) - \sin(x_1)\sin(2x_2)\sin(x_3),$$

$$v_{ttt}(x, 0) = \pi\sin(x_1)\sin(x_2)\sin(x_3),$$

$$\int_0^\pi\int_0^\pi\int_0^\pi v(x,t)\sin(\pi x_1)dx = \frac{-4\pi}{\pi^2 - 1}\cos(t + \pi^2).$$

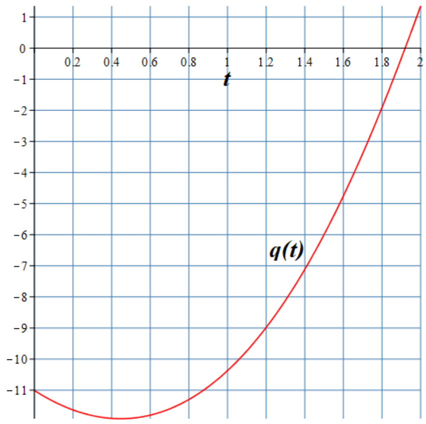
For this input information, all the conditions of Theorem 1 are satisfied. Using the developed algorithm, the information was processed and the parameter of the equation was restored

$$q(t) = \Big(9.6429 - 2.6403\cos(t) + 0.61029\cos^2(t) - 0.081055\cos^3(t)\Big)\cos(t + 9.8696) +$$

$$+0.045293\cos^4(t) + \Big(-0.34103 + 0.077974\sin(t)\Big)\cos^3(t) + \Big(-0.5871\sin(t) + 1.4754\Big)\cos^2(t) +$$

$$+\Big(2.54\sin(t) - 5.3884\Big)\cos(t) - 9.2764\sin(t)$$

reaching admissible error $0.0009094859448 < \varepsilon$ between neighboring approximations of the function $q(t)$, at the 4th step of successive approximations. Figure 2 shows the graph of the function $q(t)$ and the graph of its successive approximations.



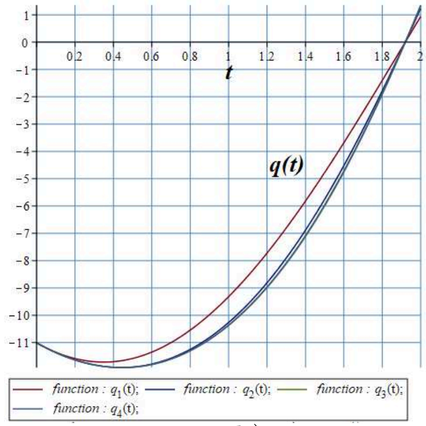


Fig. 2. Graph: a) of function $q(t)$; b) of functions of all approximations

Further in the program, the required function

$$v(x,t) = 0.047409\sin(x_3)\Bigg(-2.9295\sin(x_1)\Bigg(24.808e^{-1.1317t} -$$

$$-2\Big(-14.254\sin(0.88362t) + 57.697\cos(0.88362t)\Big) - 28.901e^{1.1317t}\Bigg) +$$

$$+3.4606\sin(2x_1)\Big(7.3082e^{-0.72278t} + 7.3082e^{0.72278t} - 14.6164\cos(1.0459t)\Big) +$$

$$+\sin(x_1)(\cos(2t) + 0.35839\sin(2t) - 0.0082070\sin(3t) - 0.022559\cos(3t) +$$

$$+0.000481313\cos(4t) - 89.047\sin(t) + 0.0001751\sin(4t) - 252.39\cos(t) - 16.873)\Bigg)\sin(x_2)$$

representing the generalized potential of the electric field was obtained. The last step of the program was to plot the time-animated graph of the found function $v(x,t)$. Figure 3 shows the graph of the function $v(x,t)$ at different time points t in the section $x_3 = \frac{1}{2}$.

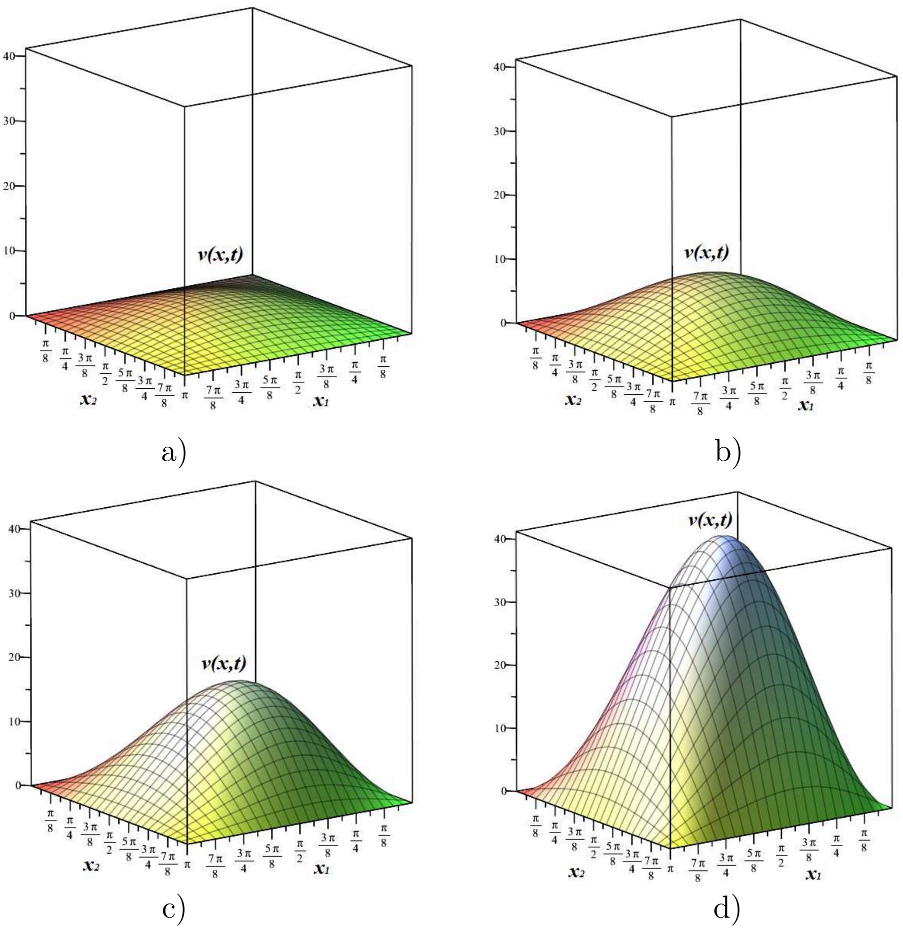


Fig. 3. Function $v(x,t)$ graph at $x_3 = \frac{1}{2}$ and: a) $t = 0$; b) $t = 0.52$ c) $t = 1.16$; d) $t = 2$

Example 2. Let the following input information be given:

$$\alpha = 1,\ \beta = 2,\ \gamma = -4,\ \kappa = -4,\ \varepsilon = 1,\ T = 3,\ k = 2,\ m = 1,\ n = 1,\ a = b = c = \pi,$$

$$v_0(x) = x_1x_2x_3(x_1 - \pi)(x_2 - \pi)(x_3 - \pi),\quad v_1(x) = 2\sin(x_1)\sin(x_2)\sin(x_3),$$

$$v_2(x) = \sin(2x_1)\sin(x_2)\sin(x_3),\quad v_3(x) = \pi\sin(x_1)\sin(x_2)\sin(x_3),$$

$$f(x) = \sin(x_1),\quad K(x) = \sin(\pi x_1),\quad F(t) = \frac{-4\pi}{\pi^2 - 1}\cos(t + \pi^2).$$

Consequently, the mathematical model of ion-acoustic waves in a plasma in an external magnetic field (1)–(4) takes the form

$$(\Delta - 1)v_{tttt} + 2(\Delta + 4)v_{tt} - 4\frac{\partial^2 v}{\partial x_3^2} + q(t)\sin(x_1) = 0,$$

$$v(0, x_2, x_3, t) = v(\pi, x_2, x_3, t) = v(x_1, 0, x_3, t) = 0,$$

$$v(x_1, \pi, x_3, t) = v(x_1, x_2, 0, t) = v(x_1, x_2, \pi, t) = 0,$$

$$v(x, 0) = x_1x_2x_3(x_1 - \pi)(x_2 - \pi)(x_3 - \pi),\quad v_t(x, 0) = 2\sin(x_1)\sin(x_2)\sin(x_3),$$

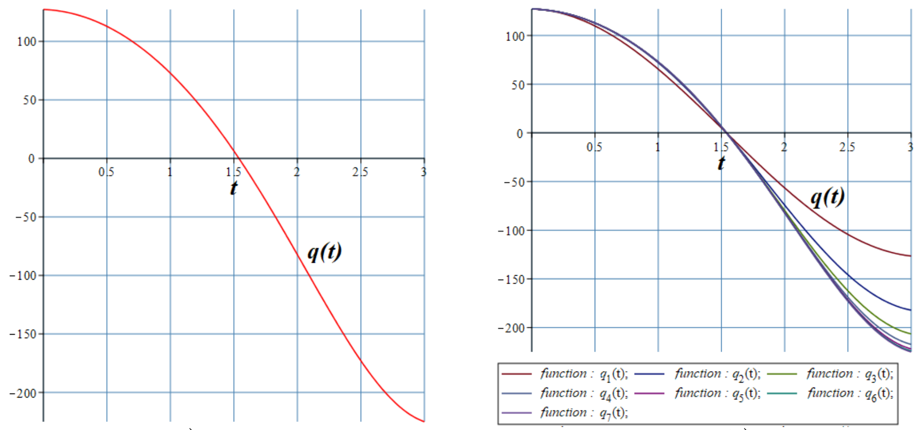
$$v_{tt}(x, 0) = \sin(2x_1)\sin(x_2)\sin(x_3),\quad v_{ttt}(x, 0) = \pi\sin(x_1)\sin(x_2)\sin(x_3),$$

$$\int_0^\pi \int_0^\pi \int_0^\pi v(x,t)\sin(\pi x_1)dx = \frac{-4\pi}{\pi^2-1}\cos(t+\pi^2).$$

For this input information, all the conditions of Theorem 1 are satisfied. Using the developed algorithm, the information was processed and the parameter of the equation was restored

$$q(t) = \Big(0.050734\cos^4(t) - 0.0091845\cos^5(t) + 0.00087227\cos^6(t) - 2.7369\cos(t) +$$
$$+0.77026\cos^2(t) - 0.20959\cos^3(t) + 9.6657\Big)\cos(t+9.8696) + 0.015525\cos^7(t) + \Big(-0.16347-$$
$$-0.00083912\sin(t)\Big)\cos^6(t) + \Big(0.0088354\sin(t) + 0.90295\Big)\cos^5(t) + \Big(-3.7303-$$
$$-0.048805\sin(t)\Big)\cos^4(t) + \Big(13.709 + 0.20162\sin(t)\Big)\cos^3(t) + \Big(-48.711-$$
$$-0.74098\sin(t)\Big)\cos^2(t) + \Big(2.6329\sin(t) + 172.03\Big)\cos(t) - 9.2983\sin(t)$$

reaching admissible error $0.2728150334 < \varepsilon$ between neighboring approximations of the function $q(t)$, at the 7th step of successive approximations. Figure 4 shows the graph of the function $q(t)$ and the graph of its successive approximations.



a) b)

Fig. 4. Graph: a) of function $q(t)$; b) of functions of all approximations

Further in the program, the required function

$$v(x,t) = -0.59513\sin(x_3)\Bigg(9.2729\sin(x_1)\Big(1.9353e^{-1.1317t}+$$
$$+2\Big(-0.36171\sin(0.88362t) + 18.136\cos(0.88362t)\Big) + 0.58378e^{1.1317t}\Big)-$$
$$-2.7567\sin(2x_1)\Big(-1.46164\cos(1.0459t) + 0.73082e^{-0.72278t} + 0.73082e^{0.72278t}\Big)+$$
$$+\sin(x_1)\Big(-0.028418\cos(3t) + \cos(2t) + 10^{-6}2.5151\cos(6t) - 0.000059042\cos(5t)+$$

$$+0.0012215\cos(4t) + 0.00086642\sin(3t) - 0.030293\sin(2t) - 10^{-8}6.4679\cos(7t) -$$
$$-10^{-8}7.9150\sin(6t) + 10^{-6}1.8426\sin(5t) - 0.000037649\sin(4t) + 7.1436\sin(t) - 16.682 +$$
$$+10^{-9}2.0354\sin(7t) - 236.45\cos(t)\Big)\Big)\sin(x_2)$$

representing the generalized potential of the electric field was obtained. The last step of the program was to plot the time-animated graph of the found function $v(x,t)$. Figure 5 shows the graph of the function $v(x,t)$ at different time points t in the section $x_2 = \frac{1}{2}$.

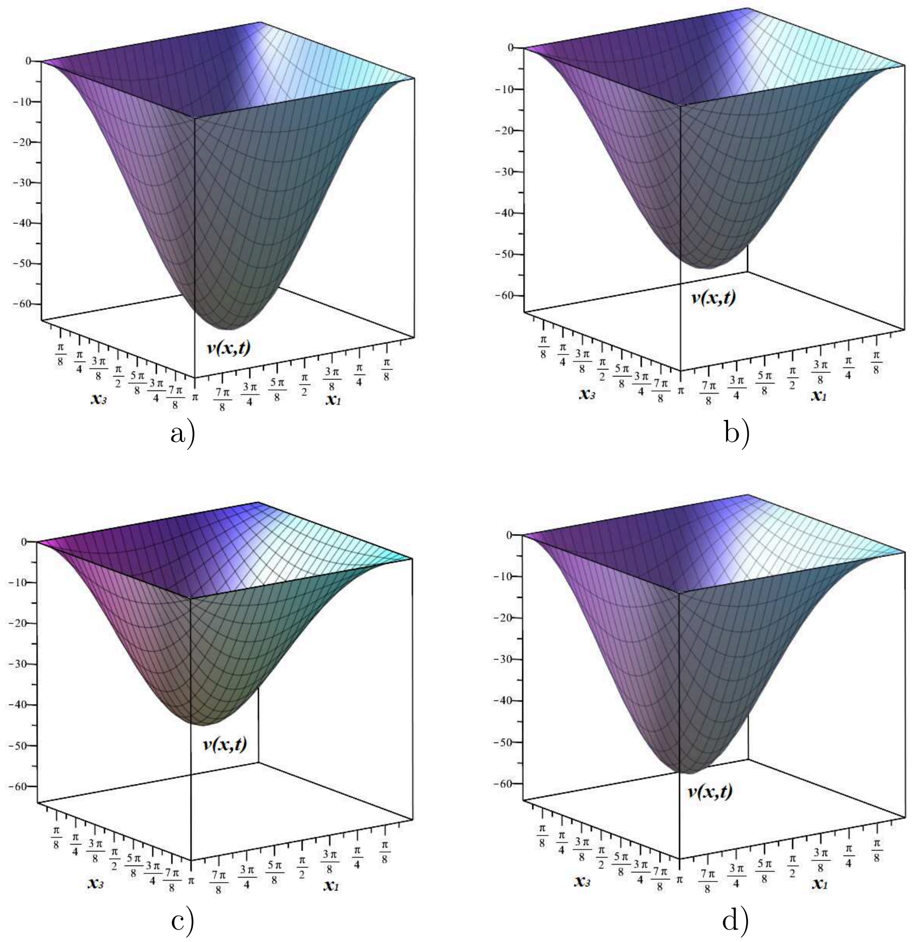


Fig. 5. Function $v(x,t)$ graph at $x_2 = \frac{1}{2}$ and: a) $t = 0$; b) $t = 1.03$ c) $t = 2.28$; d) $t = 3$

The reported study was funded by RFBR, project number 19-31-90137.

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Received Febrary 20, 2022

УДК 519.63, 517.9 **DOI: 10.14529/jcem220106**

ОБРАБОТКА ИНФОРМАЦИИ ПО ВОССТАНОВЛЕНИЮ ПАРАМЕТРА ВНЕШНЕГО ВОЗДЕЙСТВИЯ ДЛЯ МАТЕМАТИЧЕСКОЙ МОДЕЛИ ИОННО-ЗВУКОВЫХ ВОЛН В ПЛАЗМЕ

А. А. Замышляева, А. В. Лут

Статья посвящена проведению обрабоки входной информации для математической модели ионно-звуковых волн рассматриваемых в плазме при учете воздействия магнитного поля, которое является внешним по отношению к рассматриваемой системе. Решение обратной задачи для данной модели представляет обобщенный потенциал электрического поля и параметр воздействия магнитного поля. Для нахождения решения используется метод последовательных приближений, а сама модель сводится к полному уравнению соболевского типа высокого порядка. В первом параграфе приведены результаты аналитического исследования данной модели. Во втором параграфе приведен разработанный алгоритм проведения обработки информации и его схема. В третьем, приведены результаты обработки информации, полученные на основе вычислительных экспериментов разработанной программы в среде «Maple». Все полученные результаты могут быть применены в области математического моделирования, например, при вычислении обобщенного потенциала электрического поля для ионно-звуковых волн в плазме.

Ключевые слова: математическая модель ионно-звуковых волн; обратная задача; численное исследование; плазма; уравнение соболевского типа; метод последовательных приближений.

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Поступила в редакцию 20 февраля 2022 г.