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MODELLING OF SHOCK WAVE EXPERIMENTS ON TWO-FOLD COMPRESSION OF POLYMETHYL METHACRYLATE

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This paper presents a mathematical model of one- and two-dimensional elastoplastic flows of a medium. The Prandtl – Reuss model is used to describe the plastic properties of a material. The presented model is implemented as a one-dimensional code in a plane geometry and a two-dimensional code in a cylindrical axisymmetric geometry. Also, these program codes allow to calculate the absorption of synchrotron radiation in the volume of the medium at different points in time, which makes it possible to interpret the results of shock-wave experiments using synchrotron diagnostics. In order to verify the numerical code that implements this model, we carried out mathematical modelling of the experiment on the impact of two plates of polymethyl methacrylate and the Taylor problem for a copper cylinder. The application of the Prandtl – Reuss plasticity model to the description of dynamic processes in polymethyl methacrylate showed that this model works quite well in a viscoplastic medium without using any fitting parameters. Also, in 1D and 2D formulations, we carried out mathematical modelling of an experiment with synchrotron diagnostics on shock-wave loading of a cylindrical sample of polymethyl methacrylate in counter-propagating shock waves. The calculated profiles of the relative absorption of synchrotron radiation are consistent with the experimental ones, which makes it possible to give an unambiguous interpretation of the results of experiments using synchrotron radiation. The study of the role of radial unloading showed that the stress profiles for 1D and 2D calculations at the stage of shock wave convergence to the center are in good agreement, however, as shown by two-dimensional calculations, a significant density inhomogeneity along the radius arises during unloading due to re-reflection of shock waves, which complicates interpretation of experimental results using synchrotron diagnostics.

Keywords: viscoplastic flow; elastoplastic flow; Prandtl – Reuss model; polymethyl methacrylate; synchrotron radiation.

Dedicated to anniversary of Professor A.L. Shestakov

Introduction

The appearance of a technique for recording phenomena using synchrotron radiation (SR) significantly expanded the information content about the processes occurring in shock-wave experiments. Measurement of the absorption of a direct SR beam makes it possible to obtain data on the dynamics of changes in the particle density of the studied object (various structural materials, explosives and detonation products), which makes it possible to determine the profiles of compression and rarefaction waves.

In order to demonstrate the possibilities of SR in diagnostics of shock-wave processes in solid inert bodies, the works [1, 2] choose a porous material, i.e. spheroplastic, and polymethyl methacrylate (PMMA) as the object of study, which has a complex of various physical and mechanical properties under such loading. In the experiments, we measure the intensity of the X-ray beam passing through the sample, and the intensity varies with the density of the material. Quantitative estimates of the observed processes are carried out on the basis of relations for the attenuation of the intensity of the transmitted beam. The applicability of SR to the study of this class of phenomena is shown.

As is well known, experiments with single and two-fold shock compression of substance are the basis for constructing the equations of state of materials at high pressures [3, 4]. Two-fold compression, compared with single compression, is accompanied by a smaller increase in internal energy, as a result of which the realized states are located below the shock adiabat of single compression, approaching the isotherm. This fact explains the importance of two-fold compression experiments for studying the state of substances and their equations of state at high pressures. The paper [5] presents the results of experiments on loading PMMA by incident and reflected shock waves, as well as during the collision of counterpropagating shock waves in the substance under study. Thus, the interpretation of the results of experiments using SR for the subsequent construction or refinement of the equations of state of substances is an urgent problem.

The purpose of this study is to mathematically model the results of experiments with synchrotron diagnostics on shock-wave loading of PMMA using mathematical models that describe one-dimensional and two-dimensional flows in condensed media in a plane and cylindrical axisymmetric geometry in Lagrange variables, as well as approbation of the technique for reconstructing the mass of substance along the SR beam. The results of mathematical modelling allow to make an unambiguous interpretation of the results of experiments using SR. In this paper, for the first time, the Prandtl – Reuss model of elastoplasticity is used to describe dynamic processes in a viscoplastic medium (PMMA), and application of the model is substantiated. Also, we show the limits of applicability of one-dimensional modelling of problems in the formulation corresponding to experiments using SR.

1. Mathematical Model

The system of equations describing one-dimensional elastoplastic flows in Cartesian coordinates has the form

$$\dot{\rho} = -\rho \frac{\partial \upsilon}{\partial z}, \quad \rho \dot{\upsilon} = \frac{\partial}{\partial z}\left(S_{zz} - P\right), \quad \rho \dot{U} = \left(-P + S_{zz}\right)\frac{\partial \upsilon}{\partial z}, \quad \dot{S}_{zz} = \frac{4\mu}{3}\frac{\partial \upsilon}{\partial z}. \tag{1}$$

The system of equations describing two-dimensional elastoplastic flows in cylindrical coordinates has the form

$$\begin{gathered}
\dot{\rho} = -\rho \frac{\dot{V}}{V}, \quad \dot{V} = V\left(\upsilon_{rr} + \upsilon_{\phi\phi} + \upsilon_{zz}\right), \\
\rho \dot{\upsilon}_r = \frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial S_{rz}}{\partial z} + \frac{S_{rr} - S_{\phi\phi}}{r}, \quad \rho \dot{\upsilon}_z = \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial S_{zr}}{\partial r} + \frac{S_{zr}}{r}, \\
\rho \dot{U} = -P\frac{\dot{V}}{V} + \left(S_{rr}\upsilon_{rr} + S_{\phi\phi}\upsilon_{\phi\phi} + S_{zz}\upsilon_{zz} + 2S_{rz}\upsilon_{rz}\right) - \left(\nabla \vec{q}\right), \quad \vec{q} = -\aleph \nabla T, \\
\dot{S}^0_{rr} = 2\mu\left(\upsilon_{rr} - \frac{1}{3}\frac{\dot{V}}{V}\right), \quad \dot{S}^0_{zz} = 2\mu\left(\upsilon_{zz} - \frac{1}{3}\frac{\dot{V}}{V}\right), \quad \dot{S}^0_{\phi\phi} = -\dot{S}^0_{rr} - \dot{S}^0_{zz}, \quad \dot{S}^0_{rz} = 2\mu \upsilon_{rz},
\end{gathered} \tag{2}$$

where ρ, υ_r, υ_z are particle density and velocity components; P is a nonequilibrium pressure; T is a temperature; $q_k = -\aleph \partial T / \partial x_k$ is a heat flow; $\aleph$ is a coefficient of thermal conductivity; μ is a shear modulus, $\sigma^0_{ik} = -P^0\delta_{ik} + S^0_{ik}$ is an equilibrium part of the stress tensor, P^0, S^0_{ik} are equilibrium pressure and stress deviator tensor; $\sigma_{ik} = -P\delta_{ik} + S_{ik}$ is a nonequilibrium stress tensor, P, S_{ik} are non-equilibrium pressure and stress deviator tensor; U is a specific internal energy.

Systems (1), (2) are written taking into account the irreversibility of real physical processes. To this end, in the equations, we use nonequilibrium stresses, which take into account the finite time of system relaxation to an equilibrium state [6].

We represent the equilibrium stress tensor as the sum of stresses due to longitudinal and transverse deformations

$$\sigma^0_{ik} = \left(-P^0 + 4\mu\, u_{ll}/3\right)\delta_{ik} + T^0_{ik},$$

where $T^0_{ik} = 2\mu\,(u_{ik} - u_{ll}\delta_{ik})$ is a tensor that takes into account the addition to stresses due to transverse (with respect to the i-th axis) deformations.

In the case of adiabatic deformations, the rate of change of equilibrium stresses is as follows:

$$\dot{\sigma}^0_{ik} = \frac{\rho\, c_l^2}{V}\,\dot{V}\delta_{ik} + \dot{T}^0_{ik}, \tag{3}$$

where $c_l = \sqrt{(\partial P^0/\partial\rho)_S + 4c_t^2/3}$ is a longitudinal speed of sound, $c_t = \sqrt{\mu/\rho}$ is a transverse speed of sound, $\dot{T}^0_{ik} = 2\mu\,(\upsilon_{ik} - \upsilon_{ll}\delta_{ik})$, υ_{ik} is a deformation rate tensor.

The nonequilibrium spherical part of the stress tensor can be represented as $P = P^0 + \delta\sigma$, the nonequilibrium stress deviator tensor can be written as $S_{ik} = S^0_{ik} + \delta T_{ik}$, where $\delta\sigma$, δT_{ik} are non-equilibrium additions due to longitudinal and transverse deformations, respectively.

In the process of compression of a medium element during the time $\triangle t$, an inhomogeneous velocity field is formed in the element, which leads to the appearance of an excess particle density $\delta\rho$ in the disturbed domain whose size is $c_l\,\triangle t$. The nonequilibrium addition has the form [7] $\delta\rho = -\rho\,(\nabla\vec{\upsilon})\,\triangle t$ if $(\nabla\vec{\upsilon}) < 0$, and $\delta\rho = 0$ if $(\nabla\vec{\upsilon}) \geq 0$. Taking into account that longitudinal perturbations propagate with the speed c_l, the effective volume of the element of the medium, which is equal to the part V perturbed in the process of deformation, is equal to $V_{eff} = V\,c_l\triangle t/\xi$ under the compression $\left(\dot{V} < 0\right)$ and $V_{eff} = V$ under the tension $\left(\dot{V} \geq 0\right)$, where ξ is a characteristic linear size of the volume.

Since $(\nabla\vec{\upsilon}) = \dot{V}/V_{eff}$, then, taking into account (3), the expression for $\delta\sigma$ can be written as $\delta\sigma = \rho c_l^2 \tau_l \dot{V}/V$ for $\dot{V} < 0$, and $\delta\sigma = 0$ for $\dot{V} \geq 0$, where $\tau_l = \xi/c_l$ is a relaxation time for longitudinal deformations. Similarly, we can write an expression for δT_{ik}. For diagonal elements,

$$\delta T_{ii} = \begin{cases} \dot{T}^0_{ii}\,\tau_t, & \dot{T}^0_{ii} > 0 \\ 0, & \dot{T}^0_{ii} \leq 0 \end{cases}$$

where $\tau_t = \xi/c_t$ is the relaxation time for transverse deformations.

The non-diagonal components δT_{ik} describe the non-equilibrium addition to stresses due to shear. When estimating the relaxation time for a shear, the length of the contour enclosing the considered volume should be taken as a linear scale. Then the relaxation time for the shear has the form $\tau_t^{'} = \pi\xi/c_t$ and the expression for the nonequilibrium shear addition is

$$\delta T_{ik} = \dot{T}^0_{ik}\,\tau'_t, \quad i \neq k.$$

Various empirical models are used to describe plastic flows. The most commonly used model is the von Mises yield criterion, which limits shear stresses at the yield point

$$\dot{S}^0_{ik} = 2\mu\,\widehat{\upsilon}_{ik}, \quad \left(S^0_{ik}\right)^2 \leq 2Y_0^2/3, \tag{4}$$

where Y_0 is the yield limit for simple tension, $\widehat{\upsilon}_{ik} = \upsilon_{ik} - \upsilon_{ll}\delta_{ik}/3$. Realization (4) is carried out by multiplying each element of the stress deviator by the factor $\sqrt{2/3}\cdot Y_0\sqrt{\left(S^0_{ik}\right)^2}$ if condition (4) is not satisfied [8].

A more detailed description of plastic flows is implemented in the Prandtl – Reuss model [9, 10], in which the plastic deformation rate tensor appears explicitly. Taking into account plastic flows, in the general case, the equation for the components of the equilibrium stress deviator tensor has the form

$$\dot{S}^0_{ik} = 2\mu\left(\widehat{\upsilon}_{ik} - \dot{u}^p_{ik}\right), \tag{5}$$

where $\dot{u}^p_{ik}$ is the plastic deformation rate tensor, which is related to stresses by the von Mises equations $\dot{u}^p_{ik} = S^0_{ik}/\lambda$, λ is the modulus of plasticity. Expression (5) reflects the fact that only elastic deformations are responsible for elastic stresses.

From the Mises equations and (4), (5) we can obtain the following relations:

$$\frac{1}{\lambda} = \frac{3}{2}\frac{S^0_{ik}\,\widehat{\upsilon}_{ik}}{Y_0^2}, \tag{6}$$

$$\dot{S}^0_{ik} + S^0_{ik}/\tau^p = 2\mu\,\widehat{\upsilon}_{ik}, \tag{7}$$

where $\tau^p = \lambda/2\mu$ is the time of elastic stress relaxation due to plastic flows (Maxwell time of relaxation). It follows from (6) and (7) that, at $\tau^p \to 0$, the stresses reach the limiting value, which is determined by the von Mises yield criterion (4).

Thus, when describing plasticity within the Prandtl – Reuss model, the equations for $\dot{S}^0_{ik}$ in (1), (2) should be replaced by equations (6) and (7). In this case, the limitation of elastic stresses at the yield point is performed automatically. In the one-dimensional case, equation (5) has the form $\dot{S}_{zz} + S_{zz}/\tau^p = 4\mu\upsilon_{zz}/3$, and the modulus of plasticity is given by $1/\lambda = 3S_{zz}\upsilon_{zz}/2Y_0^2$ for $S_{zz}\upsilon_{zz} > 0$, that is, for deformations that lead to stress growth.

Note that equations (5) have the same form as the equations for stresses in the model of a viscoelastic medium (Maxwell model) [8], in which the role of elastic and viscous processes is determined by the relaxation time. Since the relaxation time is determined in the Prandtl – Reuss model, there is no need to introduce an additional fitting parameter, i.e. the viscosity [11].

Systems of equations (1), (2) must be endowed with the corresponding initial and boundary value conditions and equations of state of substances [12, 13].

For the numerical solution of the proposed mathematical model, the semi-analytical method [6] was used, the distinctive feature of which is that only spatial derivatives are replaced by finite differences. In this case, the system of equations of continuum mechanics is reduced to a system of differential equations that admit an approximate analytical solution at a certain time step.

2. Reconstruction of Substance Mass along SR Beam

In this work, for numerical modelling, we use the results of a series of experiments on shock-wave loading of polymethyl methacrylate in counterpropagating shock waves using synchrotron radiation.

In the process of shock-wave loading of the material under study, the density of the substance changes significantly along the SR beam, and, consequently, the absorption spectrum changes. When conducting experiments for the material under study, the DIMEX detector is calibrated by measuring the absorption depending on the mass along the SR beam, the results of which are used to construct a logarithmic dependence of the relative absorption [14]. The intensity recorded by the detector before the experiment has the form

$$J_{before} = J_{air} \exp\left(-\alpha_1 m_0 + \alpha_2 m_0^2\right),$$

where J_{air} is the SR flux intensity for the unperturbed sample, m_0 is the unperturbed mass of substance along the SR beam ($m_0 = \rho_0 d_0$, where ρ_0 is the unperturbed sample density and d_0 is the sample diameter), α_1 and α_2 are the interpolated absorption coefficients.

The intensity recorded by the detector during the experiment can be represented as

$$J_{\exp} = J_{air} \exp\left(-\alpha_1 m_x + \alpha_2 m_x^2\right),$$

where m_x is the perturbed substance mass along the beam.

Following [14], the logarithm of the ratio of $J_{\exp}$ and J_{before} is

$$\ln\left(\frac{J_{\exp}}{J_{before}}\right) = g = -\alpha_1 \left(m_x - m_0\right) + \alpha_2 \left(m_x^2 - m_0^2\right). \tag{8}$$

In the case when the regime of one-dimensional flows is realized (along the z axis), the perturbed mass along the SR beam has the form $m_x = \rho(z,t)\, d_0$ and expression (8) takes the form

$$g(z,t) = -\alpha_1 \rho_0 d_0 \left(\frac{\rho(z,t)}{\rho_0} - 1\right) + \alpha_2 \left(\rho_0 d_0\right)^2 \left(\left(\frac{\rho(z,t)}{\rho_0}\right)^2 - 1\right). \tag{9}$$

Thus, the measurement of the intensity of the SR passed through the sample under study makes it possible to trace the dynamics of the particle density distribution.

3. Test Problems

For the one-dimensional case, verification of the mathematical model was carried out using experimental data [15]. The experiments were carried out using PMMA as the material of the projectile plate, the specimen, and the «window» supporting the specimen, through which the particle velocity was recorded using a laser interferometer. Thus, the specimen and the «window» formed a continuous layer in which the particle velocity was measured in a certain plane.

Below we present the results of one-dimensional calculations in the geometry corresponding to the experiment [15]. The impact of two PMMA plates is considered. Geometry is as follows: initial thickness of impactor and sample is $h = 6.35$ mm, initial thickness of ≪window≫ is $h = 25$ mm. At the initial moment of time, the impactor speed was set equal to $u_0 = 450$ m/s and $u_0 = 640$ m/s.

The results of numerical modelling are shown in Fig. 1. Note that the work [15] presents data averaged over a series of experiments with impactor velocities equal to 450.1 m/s and 639.1–641.2 m/s.

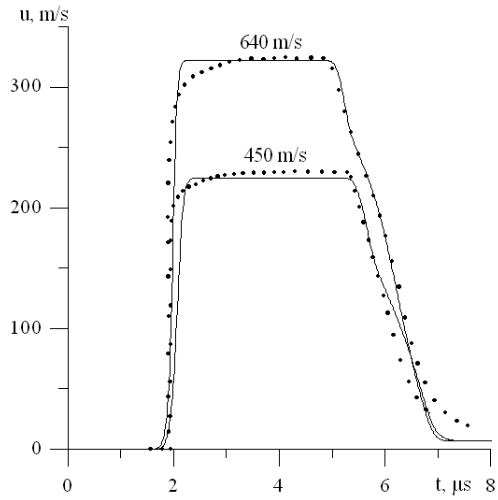

Fig. 1. Time dependence of the particle velocity considered on the plane given by sample and window: points denote experiment, lines denote numerical calculation.

The calculation results presented in Fig. 1 are in good agreement with the experiment in the domain of both the shock wave and rarefaction wave. Thus, the Prandtl – Reuss plasticity model describes with high accuracy the viscoplastic character of PMMA deformation. For the two-dimensional case, verification of the mathematical model was carried out on the Taylor problem (impact of a rod on a rigid barrier) presented in [16].

Let us present the statement of the problem. Consider a normal impact of a solid copper cylinder on a rigid wall. The problem has axial symmetry (z is the axis of symmetry). Geometry is as follows: the initial length L_0 is 100 mm and the rod radius r is 10 mm. The control (final) moment of time is determined by the time when the deformation of the material stops. As a reference solution, we use experimental data on the finite length of the rod L_0 depending on the impact velocity u_0. The results of numerical modelling are presented in Table 1.

Table 1

Comparison table of experimental and numerical modelling results

u_0, m/s	L_0, mm (experiment [16])	L_0, mm (calculation)
110	90	90.03
158	82	81.75
205	72	72.51
250	62	63.33

Thus, a comparison of the calculation results with experimental data showed that the constructed mathematical model gives a correct description of shock-wave processes in solid deformable bodies that have both viscoplastic and elastic-plastic nature of deformation.

4. Numerical Experiment Results

This paper presents the results of numerical modelling of an experiment on shock-wave loading of a cylindrical PMMA specimen in counterpropagating shock waves [5]. Let us present the statement of the problem. Consider a symmetrical impact of two solid copper cylinders by the end surfaces of a cylindrical PMMA specimen. The problem has axial symmetry (z is the axis of symmetry). Geometry is as follows: the initial thickness of each copper impactor is 0.5 mm, the radius is 10 mm, the thickness of the PMMA specimen is 20 mm, the radius is 10 mm. At the initial moment of time, the velocities of cylindrical impactors made of copper are 400 m/s. The unperturbed density of PMMA is 1.18 g/cm^3.

The results of numerical modelling are shown in Fig. 2.

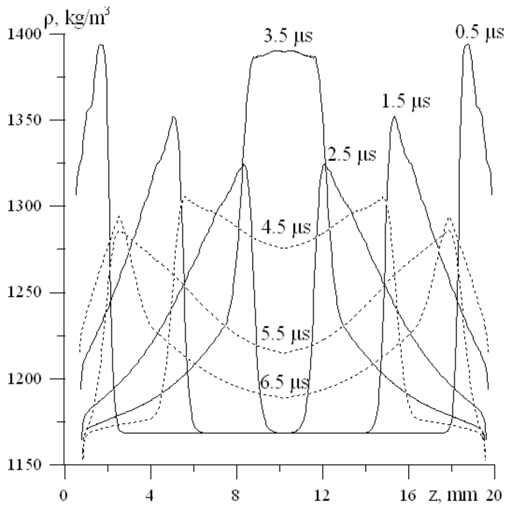

Fig. 2. Distribution of PMMA particle density at different time points.

Only the PMMA domain is highlighted in Fig. 2. The time is counted from the moment of impact of the impactors and the specimen. Time points 0.5 μs, 1.5 μs, 2.5 μs, 3.5 μs correspond to the convergence of shock waves to the center of the specimen, and time points 4.5 μs, 5.5 μs, 6.5 μs correspond to unloading.

The results shown in Fig. 2 were used to interpret the results of synchrotron diagnostics in experiments on two-fold compression [5]. Expression (8) and the results shown in Fig. 2 make it possible to reproduce the results of synchrotron diagnostics and to compare the results thus obtained with experiment.

To make such a comparison, it is necessary to reduce the calculated data of «relative absorption» to the same scale that was used in the experiments [5], and also to find the coefficients α_1 and α_2 used in (8).

It follows from (8) and (9) that the quantity $g(z,t)$ is an alternating quantity, which is positive when $J_{\exp}/J_{before} > 1$, that is, for $\rho(z,t) < \rho_0$, and negative for $J_{\exp}/J_{before} < 1$, i.e. for $\rho(z,t) > \rho_0$. In [5], the value of «relative absorption» is only positive. For the time point equal to 0.5 μs, the value of «relative absorption» at the center of the specimen, where the perturbation does not yet arrived, is equal to unity. Therefore, we assume that, in the experiment [5], the «relative absorption» is understood as the quantity $g(z,t) + 1$.

From a comparison of the calculated density distribution and the experiment for the time point equal to 0.5 μs, the values α_1=0.5 cm^2/g and $\alpha_2 = 10^{-4}$ (cm^2/g)^2 are found.

The application of the obtained parameters to find the «relative absorption» at the time point equal to 4.5 μs leads to agreement with the experiment (Fig. 3).

Thus, the above comparison shows that the modelling of medium flows in shock-wave experiments allows to give a correct interpretation of the results of synchrotron diagnostics.

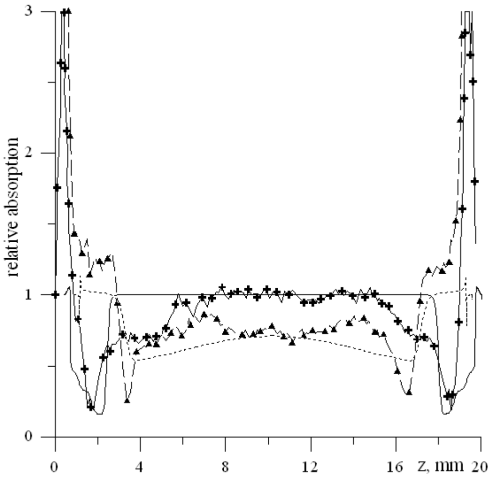

Fig. 3. Comparison of the calculated profiles of the relative absorption of SR with the experimental ones. Lines without markers denote calculated profiles, lines with markers denote experimental ones

To assess the effect of lateral unloading in a cylindrical PMMA specimen and the limits of applicability of the one-dimensional complex, the results of numerical modelling in one- and two-dimensional cases are compared. Figs. 4–7 show the density profiles at the time points equal to 2.5 μs and 3.7 μs, corresponding to the convergence of shock waves to the center of the specimen, and the time points equal to 5 μs and 7.5 μs, corresponding to unloading. In Fig.8, the stress fields are presented at the same time points for the one- and two-dimensional cases.

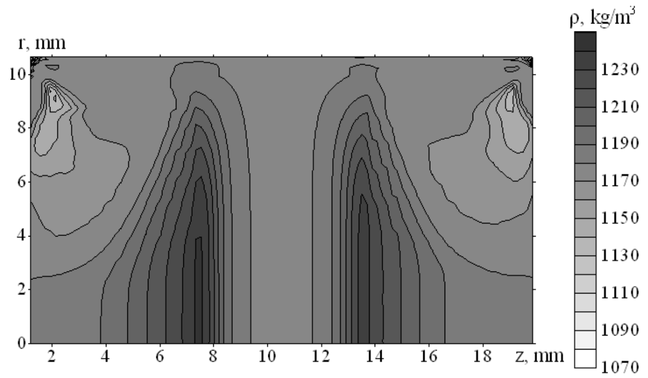

Fig. 4. Density distribution at the time point equal to 2.5 μs

Conclusion

In this work, we implement a mathematical model in the form of an one- and two-dimensional code, which makes it possible to calculate viscous-elastic-plastic flows of a solid deformable body in Lagrangian variables. To describe dynamic processes in

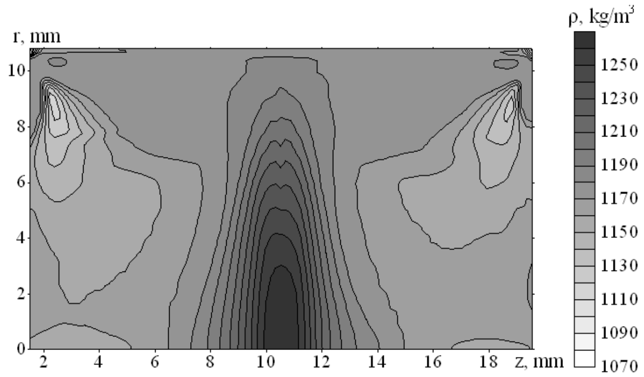

Fig. 5. Density distribution at the time point equal to 3.7 μs

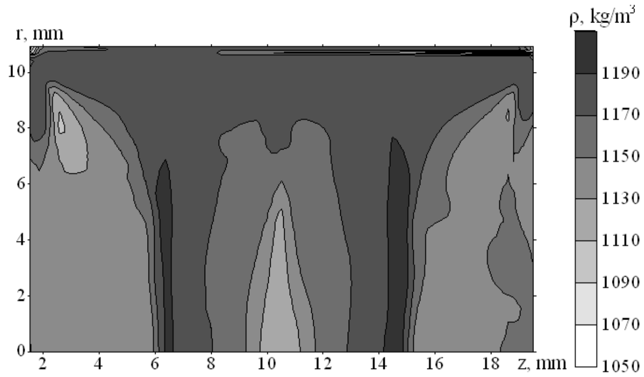

Fig. 6. Density distribution at the time point equal to 5 μs

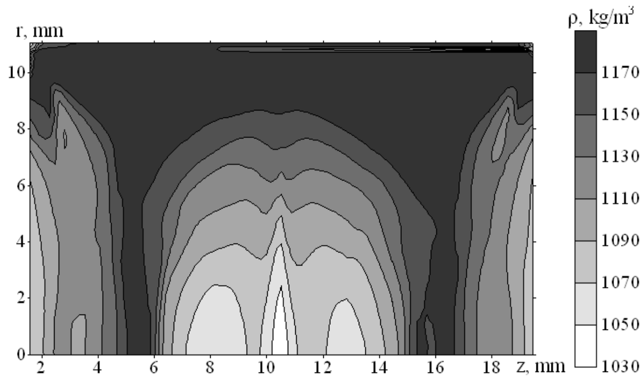

Fig. 7. Density distribution at the time point equal to 7.5 μs

PMMA, the Prandtl – Reuss model of elastoplasticity is applied, and the use of the model is proved. To verify the numerical code, we carry out mathematical modelling of the experiment on the impact of two PMMA plates and the Taylor problem. Mathematical modelling of an experiment on shock-wave loading of a cylindrical PMMA specimen in counterpropagating shock waves was also carried out in 1D and 2D formulations. The calculated profiles of the relative absorption of synchrotron radiation are in good agreement with the experimental ones. The study of the role of radial unloading shows that the stress

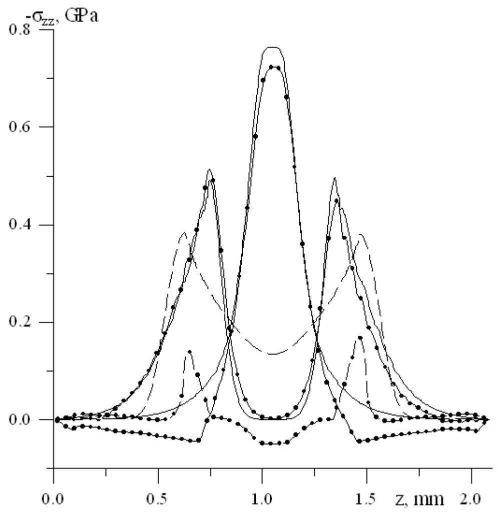

Fig. 8. Stress fields: lines without markers denote the calculated profiles for the one-dimensional case, lines with markers denote the calculated profiles for the two-dimensional case

profiles for 1D and 2D calculations at the stage of shock wave convergence to the center are in good agreement, however, as shown by two-dimensional calculations, a significant density inhomogeneity along the radius arises during unloading due to re-reflection of shock waves, which complicates interpretation of experimental results using synchrotron diagnostics.

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МОДЕЛИРОВАНИЕ УДАРНО-ВОЛНОВЫХ ЭКСПЕРИМЕНТОВ ПО ДВУКРАТНОМУ СЖАТИЮ ПОЛИМЕТИЛМЕТАКРИЛАТА

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В данной работе представлена математическая модель одно- и двумерных упругопластических течений среды. Для описания пластических свойств материала применяется модель Прандтля – Рейса. Представленная модель реализована в виде одномерного кода в плоской геометрии и двумерного кода в цилиндрической осесимметричной геометрии. Данные программные коды позволяют вычислять также поглощение синхротронного излучения в объеме среды в различные моменты времени, что дает возможность интерпретировать результаты ударно-волновых экспериментов с применением синхротронной диагностики. Для верификации численного кода, реализующего данную модель, проведено математическое моделирование эксперимента по соударению двух пластин из полиметилметакрилата и задачи Тейлора для медного цилиндра. Применение модели пластичности Прандтля – Рейса к описанию динамических процессов в полиметилметакрилате показало, что данная модель достаточно хорошо работает и в вязкопластической среде без использования каких-либо подгоночных параметров. Также проведено в 1D- и 2D-постановке математическое моделирование эксперимента с синхротронной диагностикой по ударно-волновому нагружению цилиндрического образца полиметилметакрилата во встречных ударных волнах. Расчетные профили относительного поглощения синхротронного излучения согласуются с экспериментальными, что позволяет дать однозначную интерпретацию результатов экспериментов с применением синхротронного излучения. Исследование роли радиальной разгрузки показало, что профили напряжений для 1D- и 2D-расчетов на стадии схождения ударных волн к центру хорошо согласуются, однако, как показали двумерные расчеты, при разгрузке возникает существенная неоднородность плотности по радиусу из-за переотражения ударных волн, что усложняет интерпретацию результатов экспериментов с использованием синхротронной диагностики.

Ключевые слова: вязкопластическое течение; упругопластическое течение; модель Прандтля – Рейса; полиметилметакрилат; синхротронное излучение.

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