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THE AVALOS – TRIGGIANI PROBLEM FOR THE LINEAR OSKOLKOV SYSTEM AND A SYSTEM OF WAVE EQUAIONS. II

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The Avalos – Triggiani problem for a system of wave equations and the linear Oskolkov system is investigated. The method proposed by G. Avalos and R. Triggiani is used to prove a theorem on the existence of a unique solution to the Avalos – Triggiani problem. The underlying mathematical model involves the linear Oskolkov system describing the flow of an incompressible viscoelastic Kelvin – Voigt fluid of zero order and a vector wave equation describing a structure immersed in the fluid.

Keywords: Avalos – Triggiani problem; incompressible viscoelastic fluid; linear Oskolkov system.

Dedicated to anniversary of Professor A.L. Shestakov

1. Introductory Part

Let Ω be a bounded domain in $\mathbb{R}^n, n = 2, 3$, with sufficiently smooth boundary $\partial\Omega$. Let $v = \mathrm{col}(v_1, v_2, ..., v_n)$ be a $n-$dimensional velocity vector $n = 2, 3$, the scalar function p be a pressure, and the vector $w = \mathrm{col}(w_1, w_2, ..., w_n)$ be a vector of displacement of a body, which occupies the domain Ω_s and is immersed in a fluid occupying the domain Ω_f. Therefore, $\Omega = \Omega_s \cup \Omega_f, \overline{\Omega}_s \cap \overline{\Omega}_f = \partial\Omega_s \equiv \Gamma_s$, is the common boundary of Ω_s and Ω_f. Let us denote the outer boundary of Ω_f by Γ_f (see Fig. 1). Our goal is to investigate the Avalos – Triggiani problem [1, 2] for the case when the fluid in Ω_f is an incompressible viscoelastic Kelvin – Voigt fluid of the zero-order [3,4]. The considered mathematical model is determined by the system

$$(1 - \kappa\nabla^2)v_t - \mu\nabla^2 v + \nabla p = 0 \quad \text{in } \mathbb{R} \times \Omega_f, \tag{1}$$

$$\nabla \cdot v = 0, \qquad \text{in } \mathbb{R} \times \Omega_f, \tag{2}$$

$$w_{tt} - \nabla^2 w + w = 0 \quad \text{in } \mathbb{R} \times \Omega_s, \tag{3}$$

with the boundary value conditions

$$v\big|_{\Gamma_f} \equiv 0, \qquad \text{on } \mathbb{R} \times \Gamma_f, \tag{4}$$

$$v \equiv w_t \qquad \text{on } \mathbb{R} \times \Gamma_s, \tag{5}$$

$$\frac{\partial v}{\partial \nu} - \frac{\partial w}{\partial \nu} = p\nu \qquad \text{on } \mathbb{R} \times \Gamma_s \tag{6}$$

and the initial value condition

$$(w(0,\cdot), w_t(0,\cdot), v(0,\cdot)) = (w_0, w_1, v_0) \in \mathbf{H}, \tag{7}$$

where $\mathbf{H} = (L^2(\Omega_s))^n \times (L^2(\Omega_s))^n \times \mathcal{H}_f$, where $\mathcal{H}_f$ is a closure in the norm of the space $(L^2(\Omega_s))^n$ that is the space of infinitely differentiable solenoidal functions and $\nu = \mathrm{col}(x_1, x_2, ..., x_n)$ is a normal to the boundary Γ_s.

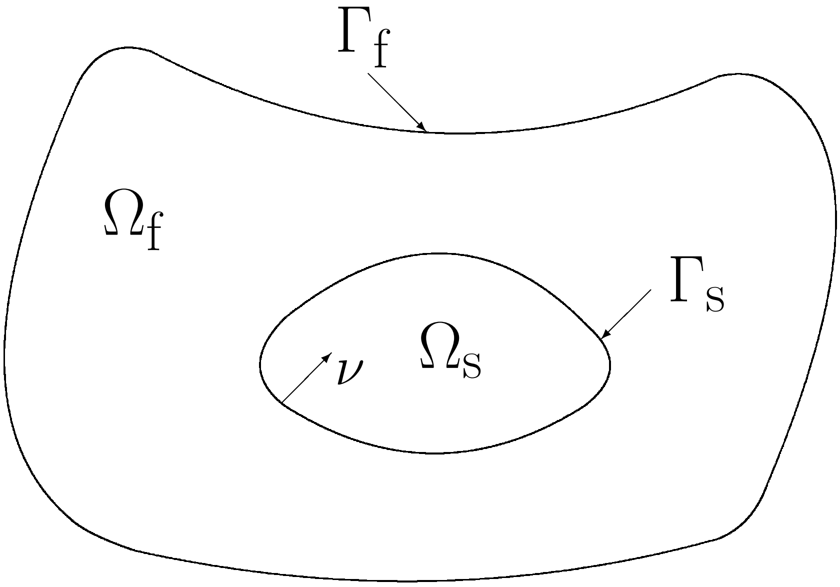

Fig. 1. Physical model

In system (1), the parameters κ and μ characterize the elastic and viscous properties of the fluid, respectively. Earlier, in the paper [5], we considered problem (1)–(7) and reduced this problem to a Sobolev-type linear operator equation. However, the question of the solvability of the initial problem was left without attention. This article is devoted to the elimination of this unfortunate misunderstanding.

2. Main Part

Following [1, 2], we assume that p satisfies the following elliptic problem:

$$\begin{aligned} &\Delta p = 0 \quad \text{in } \mathbb{R} \times \Omega_f, \\ &p = \frac{\partial v}{\partial \nu} \cdot \nu - \frac{\partial w}{\partial \nu} \cdot \nu \quad \text{on } \mathbb{R} \times \Gamma_s, \\ &\frac{\partial p}{\partial \nu} = \Delta v \cdot \nu \quad \text{on } \mathbb{R} \times \Gamma_f. \end{aligned} \tag{8}$$

Then the pressure p can be represented as follows:

$$p = D_s\{(\frac{\partial v}{\partial \nu} \cdot \nu - \frac{\partial w}{\partial \nu} \cdot \nu)_{\Gamma_s}\} + N_f((\Delta v \cdot \nu)_{\Gamma_f}) \quad \text{in } \mathbb{R} \times \Omega_f;$$

where the Dirichlet map D_s is defined by the relations

$$h = D_s(g) \Leftrightarrow \begin{cases} \Delta h = 0 \quad \text{in} \quad \Omega_f, \\ h = g \quad \text{on} \quad \Gamma_s, \\ \dfrac{\partial h}{\partial \nu} = 0 \quad \text{on} \quad \Gamma_f, \end{cases}$$

and the Neumann map N_f is defined by the relations

$$h = N_f(g) \Leftrightarrow \begin{cases} \Delta h = 0 \quad \text{in} \quad \Omega_f, \\ h = 0 \quad \text{on} \quad \Gamma_s, \\ \dfrac{\partial h}{\partial \nu} = g \quad \text{on} \quad \Gamma_f. \end{cases}$$

Then original system (1)–(3), which describes the interaction of the fluid and the body immersed in the fluid, takes the form

$$(1 - \kappa\nabla^2)v_t - \mu\nabla^2 v + G_1 w + G_2 w = 0 \quad \text{in } \mathbb{R} \times \Omega_f, \tag{9}$$

$$\nabla \cdot v = 0, \qquad \text{in } \mathbb{R} \times \Omega_f, \tag{10}$$

$$w_{tt} - \nabla^2 w + w = 0 \quad \text{in } \mathbb{R} \times \Omega_s \tag{11}$$

with the boundary value conditions

$$v|_{\Gamma_f} \equiv 0, \qquad \text{on } \mathbb{R} \times \Gamma_f, \tag{12}$$

$$v \equiv w_t, \qquad \text{on } \mathbb{R} \times \Gamma_s, \tag{13}$$

where

$$G_1 w \equiv \nabla\{D_s\{(\frac{\partial w}{\partial \nu} \cdot \nu)_{\Gamma_s}\}\},$$

$$G_2 w \equiv -\nabla\{D_s\{(\frac{\partial v}{\partial \nu} \cdot \nu)_{\Gamma_s}\} + N_f((\Delta v \cdot \nu)_{\Gamma_f})\}.$$

Let us rewrite problem (9)–(13), in which pressure is excluded, in the form of an abstract Cauchy problem:

$$L\dot{u} = Mu, \quad u(0) = u_0, \tag{14}$$

where the operators L and M are defined by the matrices

$$L := \begin{pmatrix} I & O & O \\ O & I & O \\ O & O & A_\kappa \end{pmatrix}, \quad M := \begin{pmatrix} O & I & O \\ \Delta - I & O & O \\ G_1 & O & \mu\Delta + G_2 \end{pmatrix},$$

and $u = \text{col}(w, w_t, v)$, $\quad A_\kappa = 1 - \kappa\nabla^2$, $\quad I$ is a unit operator whose domain is clear out of context. We study problem (14) based on the results obtained in [6–8].

Lemma 1. *Let $\kappa \in \mathbb{R}_+$ and $\nu \in \mathbb{R}$, then the operators $L, M \in \mathcal{L}(\mathbf{G}, \mathbf{H})$ (linear continuous operators from $\mathbf{G}$ to $\mathbf{H}$), and there exists an operator $L^{-1} \in \mathcal{L}(\mathbf{H})$. Here the space $\mathbf{G} =$*

$(H^2(\Omega_s))^n \times (H^2(\Omega_s))^n \times \mathcal{G}_f$, *where* $\mathcal{G}_f$ *is a closure in the norm of the space* $(H^2(\Omega_s))^n$ *that is the space of infinitely differentiable solenoidal functions such that (12) and (13) hold.*

Theorem 1. *For any* $\kappa \in \mathbb{R}_+$, $\mu \in \mathbb{R}$ *and* $u_0 \in \mathbf{G}$, *there exists a unique solution to problem (14)* $u \in C^\infty(\mathbb{R}, \mathbf{G})$.

In conclusion, note that in the future we intend to develop our research in the direction indicated in [9, 10].

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ЗАДАЧА АВАЛОС – ТРИГГИАНИ ДЛЯ ЛИНЕЙНОЙ СИСТЕМЫ ОСКОЛКОВА И СИСТЕМЫ ВОЛНОВЫХ УРАВНЕНИЙ. II

Т. Г. Сукачева, Г. А. Свиридюк

Исследуется задача Авалос–Триггиани для системы волновых уравнений и линейной системы Осколкова. На основе метода, предложенного авторами указанной задачи доказана теорема существования единственного решения задачи Авалос–Триггиани. Математическая модель содержит линейную систему Осколкова, описывающую течение несжимаемой вязкоупругой жидкости Кельвина – Фойгта нулевого порядка, и волновое векторное уравнение, соответствующее некоторой структуре, погруженной в указанную жидкость.

Ключевые слова: задача Авалос–Триггиани; несжимаемая вязкоупругая жидкость; линейная система Осколкова.

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