MSC 90X08 DOI: 10.14529/jcem150101

METHOD BASED ON THE APPLICATION OF BACKWARD DIFFERENTIATION FORMULAS (BDF) AND ITS MODIFICATIONS FOR CALCULATING OF DYNAMICS FOR THE TRIBOSYSTEM OF PISTON ENGINES

K. V. Gavrilov[1]***,*** gavrilovkv1@rambler.ru
A. A. Asaulyak[1]***,*** detroit_diesel@mail.ru
I. L. Kopyrkin[1]***,*** fypm_kins@mail.ru
[1] South Ural State University, Chelyabinsk, Russian Federation

Modifications of the BDF-method as applied to the calculation of trajectory of the moving elements of piston machines were analyzed. On the example of the calculation of the connecting-rod bearing of "Ruston & Hornsby 6VEB-X MK-III" engine as an international standard for testing we estimated the effectiveness of proposed algorithms for solving the equations of motion.

Keywords: tribosystem of piston machines, differential equations, order of BDF-method, hydromechanical characteristics.

Introduction

To solve the equations of motion of the movable elements of tribosystems well known such techniques as implicit BDF-method [1]. BDF-methods are based on the so-called backward differentiation formulas, when the derivative, for example, $du/d\tau$ in the point τ_{n+1} is calculated according value of u_{n+1} and values of $u(\tau)$ in previous k points $u_n, u_{n-1}, ..., u_{n+1-k}$, where k defines the order of method.

Exact solution $u(\tau)$ for the interval $(\tau_{n+1-k}; \tau_{n+1})$ is replaced by a polynomial for unequal intervals on the basis of which we forecast vector of unknown values.

BDF-method was studied in detail with reference to the specific case - for analyzing of the dynamics of autonomous supports. The traditional way is the reduction of the equations of motion of the second order to equations of the first order. Will consider this algorithm in a more general way for non-autonomous supports with misalignment of the shaft.

1. Differential equations of the first order

Represent the equations of motion of the movable elements of tribosystems, which described in papers [1,2,3], in the matrix form

$$\widehat{M}\,\dot{U} = \widehat{F}\,(U,\tau) \tag{1}$$

where

$$\hat{M} = \begin{bmatrix} \hat{E} & \hat{O} & \hat{O} & \hat{O} \\ \hat{O} & m\hat{E} & \hat{O} & \hat{O} \\ \hat{O} & \hat{O} & \hat{E} & \hat{O} \\ \hat{O} & \hat{O} & \hat{O} & J\hat{E} \end{bmatrix}; \quad U = \begin{bmatrix} u \\ v \\ \gamma \\ \upsilon \end{bmatrix}; \quad \hat{F}(U,\tau) = \begin{bmatrix} v \\ -P + f(\tau) \\ \upsilon \\ -M_a + \bar{M}(\tau) \end{bmatrix};$$

$$\hat{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}; \quad \hat{E} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

For discretization of system (1) we write the vector of displacement and derivatives in the next time step $\tau_{n+1} = \tau_n + \Delta\tau$ using functions $\dot{U}_{n+1} = \varphi\left(U_{n+1}\right); U_{n+1} = \xi\left(U_{n+1}\right)$. Then instead of continuous expression we obtain a next discrete task:

$$\widehat{M}\ \varphi\left(U_{n+1}\right) = \widehat{F}\ \left(\xi\left(U_{n+1}\right), \tau\right)\ . \tag{2}$$

For BDF-method of k-order the functions $\varphi,\ \xi$ for first approximation were determined by polynomials

$$U_{n+1}^{(1)} = L(\tau_{n+1}) = U_n + \sum_{i=0}^{k} \beta_i \Delta U_{n-i}, \tag{3}$$

$$\dot{U}_{n+1}^{(1)} = \dot{L}_1\left(\tau_{n+1}\right) = \sum_{i=-1}^{k-1} \alpha_i \Delta U_{n-i}\,, \tag{4}$$

where coefficients $\beta_i,\ \alpha_i$ were determined by polynomials $L,\ L_1$.

Put (3), (4) to (2) for $\tau = \tau_n + 1$ we obtain

$$\hat{M} \sum_{i=-1}^{k-1} \alpha_i \Delta U_{n-i} = \hat{F}\left(U_{n+1},\ \tau_{n+1}\right). \tag{5}$$

The system (5) is an implicit difference scheme, which can be efficiently solved by Newton's method. Will find a solution for U_{n+1}, writing the equation (5) in the form of Newton.

$$\left(\alpha_{-1}\hat{M} - \left(\frac{\partial \hat{F}}{\partial U}\right)_{n+1}^{(s)}\right)\left(U_{n+1}^{(s+1)} - U_n^{(s)}\right) =$$
$$\hat{F}\left(U_{n+1}^{(s)}, \tau_{n+1}\right) - \alpha_{-1}\hat{M}\left(U_{n+1}^{(s)} - U_n^{(s)}\right) - \sum_{i=0}^{k-1} \alpha_i \hat{M} \Delta U_{n-i} \tag{6}$$

We introduce the notation:

$$\left(\alpha_{-1}\hat{M} - \left(\frac{\partial \hat{F}}{\partial U}\right)_{n+1}^{(s)}\right) = A_{n+1}^{(s)}\,;$$

$$\hat{F}\left(U_{n+1}^{(s)},\ \tau_{n+1}\right) - \alpha_{-1}\hat{M}\left(U_{n+1}^{(s)} - U_n^{(s)}\right) - \sum_{i=0}^{k-1} \alpha_i \hat{M} \Delta U_{n-i} = G_{n+1}^{(s)},$$

then

$$A_{n+1}^{(s)}=\begin{vmatrix}
\alpha_{-1} & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & \alpha_{-1} & 0 & -1 & 0 & 0 & 0 & 0\\
-\frac{\partial\bar R_x}{\partial x} & -\frac{\partial\bar R_x}{\partial y} & m\alpha_{-1}-\frac{\partial\bar R_x}{\partial\dot x} & -\frac{\partial\bar R_x}{\partial\dot y} & -\frac{\partial\bar R_x}{\partial\alpha'} & -\frac{\partial\bar R_x}{\partial\beta'} & -\frac{\partial\bar R_x}{\partial\dot\alpha'} & -\frac{\partial\bar R_x}{\partial\dot\beta'}\\
-\frac{\partial\bar R_y}{\partial x} & -\frac{\partial\bar R_y}{\partial y} & -\frac{\partial\bar R_y}{\partial\dot x} & m\alpha_{-1}-\frac{\partial\bar R_y}{\partial\dot y} & -\frac{\partial\bar R_y}{\partial\alpha'} & -\frac{\partial\bar R_y}{\partial\beta'} & -\frac{\partial\bar R_y}{\partial\dot\alpha'} & -\frac{\partial\bar R_y}{\partial\dot\beta'}\\
0 & 0 & 0 & 0 & \alpha_{-1} & 0 & -1 & 0\\
0 & 0 & 0 & 0 & 0 & \alpha_{-1} & 0 & -1\\
-\frac{\partial\bar M_x}{\partial x} & -\frac{\partial\bar M_x}{\partial y} & -\frac{\partial\bar M_x}{\partial\dot x} & -\frac{\partial\bar M_x}{\partial\dot y} & -\frac{\partial\bar M_x}{\partial\alpha'} & -\frac{\partial\bar M_x}{\partial\beta'} & J\alpha_{-1}-\frac{\partial\bar M_x}{\partial\dot\alpha'} & -\frac{\partial\bar M_x}{\partial\dot\beta'}\\
-\frac{\partial\bar M_y}{\partial x} & -\frac{\partial\bar M_y}{\partial y} & -\frac{\partial\bar M_y}{\partial\dot x} & -\frac{\partial\bar M_y}{\partial\dot y} & -\frac{\partial\bar M_y}{\partial\alpha'} & -\frac{\partial\bar M_y}{\partial\beta'} & -\frac{\partial\bar M_y}{\partial\dot\alpha'} & J\alpha_{-1}-\frac{\partial\bar M_y}{\partial\dot\beta'}
\end{vmatrix}$$

$$G_{n+1}^{(s)}=\begin{vmatrix}
x_{n+1}-\alpha_{-1}\Delta x_{n+1}-\alpha_0\Delta x_n\\
y_{n+1}-\alpha_{-1}\Delta y_{n+1}-\alpha_0\Delta y_n\\
\left(f_x+\bar R_x\right)_{n+1}-m\left(\alpha_{-1}\Delta\dot x_{n+1}-\alpha_0\Delta\dot x_n\right)\\
\left(f_y+\bar R_y\right)_{n+1}-m\left(\alpha_{-1}\Delta\dot y_{n+1}-\alpha_0\Delta\dot y_n\right)\\
\alpha'_{n+1}-\alpha_{-1}\Delta\alpha'_{n+1}-\alpha_0\Delta\alpha'_n\\
\beta'_{n+1}-\alpha_{-1}\Delta\beta'_{n+1}-\alpha_0\Delta\beta'_n\\
\left(\bar M_x+\bar M_{ax}\right)_{n+1}-J\left(\alpha_{-1}\Delta\dot\alpha'_{n+1}-\alpha_0\Delta\dot\alpha'\right)\\
\left(\bar M_y+\bar M_{ay}\right)_{n+1}-J\left(\alpha_{-1}\Delta\dot\beta'_{n+1}-\alpha_0\Delta\dot\beta'\right)
\end{vmatrix}$$

Solving system (6) by Gauss method we obtain:

$$\Delta x_{n+1}^{(s+1)},\quad \Delta y_{n+1}^{(s+1)},\quad \Delta\alpha'^{(s+1)}_{n+1},\quad \Delta\beta'^{(s+1)}_{n+1}.$$

According to coordinate's increments in the polar coordinates we find:

$$\chi_{n+1}^{(s+1)},\ \delta_{n+1}^{(s+1)},\ \dot\chi_{n+1}^{(s+1)},\ \dot\delta_{n+1}^{(s+1)},\ \sigma_{n+1}^{(s+1)},\ \varepsilon_{n+1}^{(s+1)},\ \dot\sigma_{n+1}^{(s+1)},\ \dot\varepsilon_{n+1}^{(s+1)}.$$

The iterative process at the next step of the integration is carried out till next condition

$$\tilde E_{n+1}^{s+1}\le\varepsilon,$$

where

$$\tilde E_{n+1}^{s+1}=\max\begin{cases}\left|\dfrac{U_{n+1}^{s+1}-U_{n+1}^{s}}{U_{n+1}^{s}}\right|, & \text{if } \left|U_{n+1}^{s+1}\right|>1;\\ \left|U_{n+1}^{s+1}-U_{n+1}^{s}\right|, & \text{if } \left|U_{n+1}^{s+1}\right|\le 1;\end{cases}\tag{7}$$

The value of error ε was equal 0,0001.

The value of error we estimated for each components of vector and chosen the maximal value.

If we had $\tilde E_{n+1}^{s+1}>\varepsilon$ after i iterations the solving process was finished and we decreased the step of the integration. After that we calculated new values of U_{n+1}^{1}.

Since it is necessary to determine the forces $R(u,\dot u,\gamma,\dot\gamma)$at each time step to integrate the partial differential equation of second order to determine the pressure in the lubricating layer (Reynolds equation [2] or Elrod [4,5]), the time required are so large that in some cases it is impossible to perform parametric research. This is particularly evident in the tasks of nonlinear dynamics of journal bearings with supply of lubricating fluid through the holes and grooves, as in such cases the equation for the pressure distribution are integrated numerically.

On this basis, appear relevant research and development of methods and algorithms for solving of the equations of motion directly without reducing them to equations of the first order, which halves the dimension of the task.

2. Differential equations of the second order

Consider another method of solving of the equations of motion without reducing them to a system of equations of the first order. It allows to reduce the number of integrations of Reynolds equation [5]. This is especially important when we use numerical methods of integration, since the calculated time is essentially increasing. Consider it an example of solutions of equation of motion for autonomous tribosystem, written in the form

$$m\ddot{u} = f(\tau) - P(u, \dot{u}). \tag{8}$$

For discretization of the system (8) we replace the vector of derivatives in the next moment of time $\tau_{n+1} = \tau_n + \Delta\tau$ by the functions $\dot{u}_{n+1} = \varphi(u_{n+1})$, $\ddot{u} = \psi(u_{n+1})$. Then, instead of continuous writing obtain

$$m \cdot \psi(u_{n+1}) = f_{n+1} - P(u_{n+1}, \varphi(u_{n+1})). \tag{9}$$

The system of equations (9) is an implicit difference scheme, which is solved by Newton's method. At each iteration step is required to find the solution of algebraic equations in the form

$$J_{n+1}^{(s)}(u_{n+1}^{(s+1)} - u_{n+1}^{(s)}) = F_{n+1}^{(s)},$$

where

$$J_{n+1}^{(s)}(u_{n+1}^{(s)},\, \dot{u}_{n+1}^{(s)},\, \ddot{u}_{n+1}^{(s)}) = \left[m\frac{\partial \ddot{u}}{\partial u} + \frac{\partial P}{\partial \dot{u}} \cdot \frac{\partial \dot{u}}{\partial u} \right]_{n+1}^{(s)};$$

$$F_{n+1}^{(s)} = -m\ddot{u}_{n+1} - P(u_{n+1},\, \dot{u}_{n+1}) + f_{n+1}.$$

Having constructed the same as in the previous method, interpolation polynomials according to (3) and (4) we can to determine the value of the function and its first derivative $\dot{u}_{n+1}^{(1)}$ at the point τ_{n+1}. The value of the second derivative of the function $L_1(\tau)$ we obtain from the expression

$$\ddot{u}_{n+1} = \ddot{L}_1(\tau_{n+1}) = \sum_{i=-1}^{k-1} \beta_i \Delta u_{n-i}, \tag{10}$$

where coefficients β_i determines from $L_1(\tau)$

Substituting (3), (4), (10) in the equation of motion (8) for moment of time τ_{n+1} we obtain

$$m \sum_{i=-1}^{k-1} \beta_i \Delta u_{n-i} + P(u_{n+1},\, \sum_{i=-1}^{k-1} \alpha_i \Delta u_{n-i}) = f_{n+1}. \tag{11}$$

The solution of equation (11) for the unknown value u_{n+1} we find by Newton's iterative method

$$\left[m\beta_{-1}\hat{E} + \left(\frac{\partial P}{\partial U}\right)_{n+1}^{(s)} + \alpha_{-1}\left(\frac{\partial P}{\partial U}\right)_{n+1}^{(s)} \right] \left(u_{n+1}^{(s+1)} - u_{n+1}^{(s)}\right) = F_{n+1}^{(s)}, \tag{12}$$

where $F^{(s)}_{n+1} = f_{n+1} - P\left(u^{(s)}_{n+1}\sum\limits_{i=-1}^{k-1}\alpha_i\Delta u^{(s)}_{n-i}\right) - m\sum\limits_{i=-1}^{k-1}\beta_i\Delta u^{(s)}_{n-i}$; $\hat{E} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $u^{(1)}_{n+1}$, $\dot{u}^{(s)}_{n+1}$, $\ddot{u}^{(s)}_{n+1}$ – are calculated by formulas (3), (4), (10).

In general, for the coordinates x, y we can be written solution of the system (11) as follows:

$$x^{(s+1)}_{n+1} = x^{(s)}_{n+1} + \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}; \quad y^{(s+1)}_{n+1} = y^{(s)}_{n+1} + \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}, \tag{13}$$

where

$$a_1 = m\beta_{-1} - \frac{\partial R_x}{\partial x} - \alpha_{-1}\frac{\partial R_x}{\partial \dot{x}}; \quad a_2 = -\frac{\partial R_y}{\partial x} - \alpha_{-1}\frac{\partial R_y}{\partial \dot{x}};$$

$$b_1 = -\frac{\partial R_x}{\partial y} - \alpha_{-1}\frac{\partial R_x}{\partial \dot{y}}; \quad b_2 = m\beta_{-1} - \frac{\partial R_y}{\partial y} - \alpha_{-1}\frac{\partial R_y}{\partial \dot{y}};$$

$$c_1 = f_{n+1,x} + R^{(s)}_x - m\sum_{i=-1}^{k-1}\beta_i\Delta x^{(s)}_{n-i}; \quad c_2 = f_{n+1,x} + R^{(s)}_y - m\sum_{i=-1}^{k-1}\beta_i\Delta y^{(s)}_{n-i};$$

We named this approach – **algorithm 1**.

Numerical experiments have shown that a significant increase in effectiveness of BDF-method can be achieved if instead of Newton's method in the form of (12) to apply it's modification

$$\left[m\beta_{-1}\hat{E} + \left(\frac{dP}{du}\right)^{(s)}_{n+1}\right]\cdot\left[u^{(s+1)}_{n+1} - u^{(s)}_{n+1}\right] = F^{(s)}_{n+1}, \tag{14}$$

where $F^{(s)}_{n+1} = f_{n+1} - m\sum\limits_{i=-1}^{k-1}\beta_i\cdot\Delta u^{(s)}_{n-i} - P\left(u^{(s)}_{n+1}, \sum\limits_{i=-1}^{k-1}\alpha_i\Delta u^{(s)}_{n-i}\right)$; $\Delta u^{(s)}_{n+1} = u^{(s)}_{n+1} - u_n$; $\Delta u^{(s)}_{n-i} = \Delta u_{n-i}$, $(i = 0, 1, ..., k-1)$.

The expression $\dfrac{dP}{du}$ is a matrix of full derivatives of function

$$P(u) = P\left(u_{n+1}, \sum_{i=-1}^{k-1}\alpha_i\Delta u_{n-i}\right).$$

Thus, in the solution is not necessary to compute partial derivatives of functions P by $\dot{u}$, so the calculation time is essentially reduced. We named this approach - fast BDF-method or **algorithm 2**. In the available literature this modification of the method could not be found.

To determine the partial derivatives we used difference relations:

$$\frac{\partial R_x}{\partial x} = \frac{R_x\left(\tilde{u}^{(s)}_{n+1}, \sum\limits_{i=-1}^{k-1}\alpha_i\Delta\tilde{u}_{n-i}\right) - R_x\left(u^{(s)}_{n+1}, \sum\limits_{i=-1}^{k-1}\alpha_i\Delta u^{(s)}_{n-i}\right)}{\Delta x}, \tag{15}$$

where

$$\tilde{u}^{(s)}_{n+1} = \begin{pmatrix} x^{(s)}_{n+1} + \Delta x \\ y^{(s)}_{n+1} \end{pmatrix} \quad \Delta\tilde{u}^{(s)}_{n+1} = \begin{pmatrix} x^{(s)}_{n+1} + \Delta x - x_n \\ y^{(s)}_{n+1} - y_n \end{pmatrix},$$

$$\Delta\tilde{u}^{(s)}_{n+1} = \Delta u_{n-1}, \ (i = 0,\ 1,\ \ldots, k-1).$$

Solution of system (14) we found by formulas (13), where

$$a_1 = m\beta_{-1} + \frac{\partial R_x}{\partial x}; \quad a_2 = \frac{\partial R_y}{\partial x}; \quad b_1 = \frac{\partial R_x}{\partial y}; \quad b_2 = m\beta_{-1} + \frac{\partial R_y}{\partial y};$$

$$c_1 = (F_{n+1}^{(s)})_x; \quad c_2 = (F_{n+1}^{(s)})_y.$$

It is known that Newton's method converges for the right choice of the initial approximation. To select the initial positions and velocities of the shaft we use a polynomial of the second degree, and for accelerations - third-degree polynomial.

We introduce a new coordinate axis T. Positive direction which is opposite to the direction of the axis τ, the beginning corresponding to the current point in time τ_{n+1}. Discrete values T_i are determined by the values τ_{n-i} according to the expression

$$T_i = \tau_{n+1} - \tau_{n-i}, (i = -1, 0, 1, ..., k-1, k)\,.$$

In the coordinate system T we can write the expression (3), (4) for polynomials of second degree ($k = 2$)

$$u_{n+1} = u_n + \gamma_0 \Delta u_n + \gamma_1 \Delta u_{n-1}, \tag{16}$$

$$\dot{u}_{n+1} = \alpha_{-1} \Delta u_{n+1} + \alpha_0 \Delta u_n, \tag{17}$$

where

$$\gamma_0 = \frac{T_0}{T_1 - T_0}(1 + \frac{T_1}{T_2 - T_0}); \quad \gamma_1 = -\frac{T_0 T_1}{(T_2 - T_0)(T_2 - T_1)};$$

$$\alpha_{-1} = \frac{1}{T_0} + \frac{1}{T_1}; \quad \alpha_0 = -\frac{T_0}{T_1(T_1 - T_0)}.$$

For determination $\ddot{u}_{n+1}$ (10) we use polynomials of third degree ($k = 3$)

$$\ddot{u}_{n+1} = \beta_{-1} \Delta u_{n+1} + \beta_0 \Delta u_n + \beta_1 \Delta u_{n-1}, \tag{18}$$

where

$$\beta_{-1} = \frac{2(T_0 + T_1 + T_2)}{T_0 T_1 T_2};$$

$$\beta_0 = -2\frac{(T_2 - T_0)(T_2 + T_0) + T_1(T_2 + T_1)}{T_1 T_2 (T_1 - T_0)(T_2 - T_0)};$$

$$\beta_1 = \frac{2(T_0 + T_1)}{T_2(T_2 - T_1)(T_2 - T_0)}.$$

If we use the interpolation polynomial of the second degree $L_1(\tau)$, then the approximation of the derivative of the second order becomes a constant, which reduces the accuracy and stability of the method.

The value of the integration step was limited by the range of variation $T_{0\,\min} < T_0 < T_{0\,\max}$, where $T_{0\,\min}$, $T_{0\,\max}$ respectively the minimum and maximum size of discretization of τ, which selected experimentally in solving test tasks. To correct time step in the algorithm we used Fowler procedure [5].

The **algorithm 1** uses an iterative Newton's scheme. At each iteration is required to make nine integrations of Reynolds equation. The **algorithm 2**, due to the approximate calculation of the Jacobian at each iteration of Newton's method, requires only five integrations of Reynolds equation. It significantly decreases time of calculation of hydromechanical characteristics of tribosystems.

3. Results and calculations

Studies, which performed for calculation of trajectories of movement parts of connecting rod bearings for engines Ch 12/12 and ChN 21/21 by rapid BDF-method shown that the computation time compared with the BDF method for equations of the 1st order twice decreased, while the number of integrations of Reynolds equation was respectively: 1531 and 5931, (engine Ch 12/12); 1497 and 5621 (engine ChN 21/21).

Results of calculation of the connecting-rod bearing of "Ruston & Hornsby 6VEB-X MK-III"engine as an international standard for testing [5] are presented in Table. We analyzed main output hydromechanical characteristics of journal bearings for piston engines, which include: the minimal ($\inf h_{\min}$) and average ($h^*_{\min}$) film thicknesses of the loading cycle; the maximum ($\sup p_{\max}$) and average ($p^*_{\max}$) hydrodynamic pressures in a lubricant film; the integrated average friction losses N^* for a loading cycle. These parameters indirectly characterize wear resistance, durability and reliability of tribosystems of piston engines.

Table 1

Results of calculation

Algorithm	Hydromechanical parameters					The average number of integrations of Reynolds equation for calculation step
	$\inf h_{\min}$	$\sup p_{\max}$	$h^*_{\min}$	$p^*_{\max}$	N^*	
1	3.60	34.32	9.92	19.27	1359	13.81
2	3.60	34.42	9.92	19.22	1359	7.06

As can be seen from Table, **algorithm 2** is the most efficient. Compared with the classical **algorithm 1** solution time is reduced more than two time, and the number of integrations of Reynolds equation for calculation step – by 2.5 times.

This work was supported by a grant of the Ministry of Education and Science of the Russian Federation for applied research, code 2014-14-579-0109. The unique identifier for Applied Scientific Research (project) is RFMEFI57714X0102. Agreement no. 14.577.21.0102.

References

1. Goryacheva I.G., Morozov A.V. Rozhdestvenskiy Y.V., Gavrilov K.V. Doykin A.A. Development of Method for Calculating and Experimentally Evaluating Tribological Parameters of "Piston-Cylinder" Tribosystem of Diesel Engine. *Journal of Friction and Wear*, 2013, vol. 34, no. 5, pp. 339–348. doi: 10.3103/S106836661305005X

2. Zadorozhnaya E.A., Levanov I.G., Pyryev S.A. [Calculation of Efficiency Tribo-Unit "Gudgeon Pin - the Little End" of Tractor Diesel]. *Journal of Mechanical Engineering*, 2013, no. 12, pp. 42–47. (in Russian)

3. Surkin V.I., Zadorozhnaya E.A., Niyazov H.M. [Calculation Heavy-Loaded Tribo-Units of High Performance Diesel Engine]. *Bulletin of the South Ural State University. Series "Engineering"*, 2012, no. 12(271), pp. 97–102. (in Russian)

4. Prokop'ev V.N, Boyarshinova A.K., Gavrilov K.V. [Hydromechanical Characteristics of Heavy-Loaded Bearings with the Non-Circular Journal and Bearing]. *Problems of Mechanical Engineering and Reliability of the Machines*, 2009, no. 4, pp. 98–104. (in Russian)

5. Gavrilov K.V. [*Algorithm of Mass Conservation of Calculation of Hydromechanical Characteristics and Optimization of Parameters of Heavy-Loaded Bearings*]. Dis. … kand. tekhn. nauk, Chelyabinsk, 2006. (in Russian)

Konstantin V. Gavrilov, Candidate of Technical Sciences, Associate Professor, Department of Motor Transports and Service of Automobiles, South Ural State University, Chelyabinsk, Russian Federation, gavrilovkv1@rambler.ru.

Arseny A. Asaulyak, Postgraduate student, Department of Motor Transports and Service of Automobiles, South Ural State University, Chelyabinsk, Russian Federation, detroit_diesel@mail.ru.

Ilya L. Kopyrkin, Student, Department of Motor Transports and Service of Automobiles, South Ural State University, Chelyabinsk, Russian Federation, fypm_kins@mail.ru.

Received February 20, 2015