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A NEW TYPE OF THE MANIPULATOR DYNAMICS EQUATIONS FOR THE SYNTHESIS OF ADAPTIVE PID CONTROLLER FOR PROGRAM GRIPPING

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A new type of equations for the plane revolute-joints manipulator dynamics is derived according to the Appell's formalism in which the partial derivative of the acceleration energy is taken with respect to the coordinate acceleration, and on the left side of the equations, the parameters of the manipulator gripper position and their second time derivatives are received by identical transformations. This type of dynamics equations allows synthesizing an adaptive PID controller with a programmed gripping without solving the inverse kinematics problem. We consider an example of writing such equation and synthesizing, on its basis, an adaptive PID controller with a program gripping. In this example, the kinematic structure of the manipulator is a special case of the structure of a six-link manipulator with an angular coordinate system where certain joints are fixed, which corresponds to manipulations in the vertical plane. In conclusion, we describe how the obtained results can be extended to arbitrary manipulators.

Keywords: manipulator; rigid body system; equations of motion; Appell's equation of motion; adaptive PID controller.

Introduction

The solution of the first problem of the Robot Manipulation Systems (MS) dynamics is reduced to the derivation of formulas for calculating the driving forces and/or torques through the given movements of its bodies, i.e. to the derivation of the MS Dynamics Equations (DE). For MSs of three or more bodies, there is a problem of their cumbersome analytical form. For example, the DE of an angular six-link MS in a symbolic form takes up to forty typewritten pages [2]. At present, new methods were developed to derive the DE of six-link MS in an explicit analytical form, which takes less than one typewritten page depending on the joints of the bodies [2]. For MS on a plane, these methods make it possible to write the DE in several lines, the number of which is equal to the number of equations in the system. Unfortunately, these methods are not yet widely used. To present new results, articulated MSs on the plane are considered, and their DEs are derived according to the classical Appell's formalism with a transformation of desired DEs on the left side. In conclusion, it is described how to extend the obtained results to arbitrary MSs.

To control MS based on their DE, the Timofeev formula is often used [3], where program (desired) movements are specified for directly controlled coordinates, for example, angles relative to body rotation. The difference between the given (programmed) and the real (taken from sensors) relative state of bodies acts as an error in the MS state [3, 4, 5, 6]. In MS control problems, the aim is often the gripper control accuracy, therefore, to determine the necessary gripper movements, it is necessary to solve the inverse kinematics problem which increases the MS uncontrollability cycle and reduces the calculation accuracy which leads to increasing control errors.

In this regard, the article has the following aim: to derive the MS DEs on the basis of which, according to the Timofeev formula, an adaptive PID controller of the program gripping is synthesized.

1. Solution to First Problem of Dynamics

Introduce the following notation: N is a number of moving bodies (links); $\overline{Z}$ is a normal to the motion plane; OXY is stationary (base) coordinate system (SCS), m_i is the mass of the i-th body; O_i is the pole of the i-th body; C_i is the center of mass (CM) of the i-th body; q_i is the rotation angle of the i-th body relative to the previous body; $\alpha_j = \sum_{i=1}^{j} q_i$ is the absolute angle of the i-th body; $L_i = O_i O_{i+1}$ is the i-th body length; $a_i = O_i C_i$ is a distance from the pole to CM of the i-th body; J_k is the moment of inertia of the k-th body to the axis $C_k \overline{Z}$; Q_j is a sum of the Extended Driving Force (EDF) and Extended Gravity (EG) of the j-th body; x_i, y_i are the CM coordinates of the i-th body in SCS; $s_j = \sin(\alpha_j); c_j = \cos(\alpha_j)$.

Proposition 1. *The DE of the given MS can be represented as follows:*

$$\sum_{k=j}^{N} \left(H_{kj}^{y} \ddot{x}_k + H_{kj}^{x} \ddot{y}_k + J_k \ddot{\alpha}_k \right) = Q_j, \tag{1}$$

where $j = 1, 2..., N,\ \ H_{kj}^{x} = m_k \left(x_k - x_j + a_j s_j \right), \quad H_{kj}^{y} = m_k \left(y_j - y_k + a_j c_j \right).$

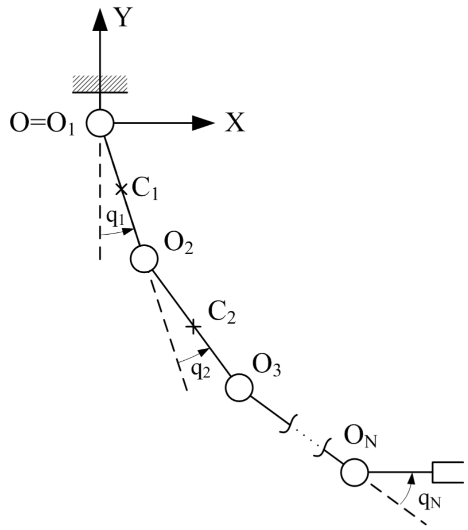

Fig. 1. N-link manipulation system with revolute joints

Proof. From the Kinematic Structure (KS) shown in Fig. 1, it follows that the CM coordinates of the 1-st and 2-nd bodies (links) are calculated by the formulas

$$\begin{cases} x_1 = a_1 \sin(q_1) = a_1 s_1, \\ y_1 = -a_1 \cos(q_1) = -a_1 c_1, \end{cases}$$

$$\begin{cases} x_2 = L_1 s_1 + a_2 \sin(q_1 + q_2) = L_1 s_1 + a_2 s_2, \\ y_2 = -L_1 c_1 - a_2 \cos(q_1 + q_2) = -L_1 c_1 - a_2 c_2. \end{cases}$$

By induction, for the coordinates of the k-th body, we obtain

$$x_k = \sum_{i=1}^{k-1} L_i s_i + a_k s_k, \qquad y_k = -\sum_{i=1}^{k-1} L_i c_i - a_k c_k. \tag{2}$$

From this, for the projections of the CM velocity vector of the k-th body on the SCS axis, we obtain the formulas

$$\dot{x}_k = \sum_{i=1}^{k-1} L_i \dot{s}_i + a_k \dot{s}_k, \qquad \dot{y}_k = -\sum_{i=1}^{k-1} L_i \dot{c}_i - a_k \dot{c}_k,$$

where $\dot{s}_k = c_k \dot{\alpha}_k, \dot{c}_k = -s_k \dot{\alpha}_k$. Now, for the projections of the CM acceleration vector of the k-th body on the SCS axis, we have the formulas

$$\ddot{x}_k = \sum_{i=1}^{k-1} L_i \ddot{s}_i + a_k \ddot{s}_k, \qquad \ddot{y}_k = -\sum_{i=1}^{k-1} L_i \ddot{c}_i - a_k \ddot{c}_k,$$

where $\ddot{c}_k = \left(-s_k \dot{\alpha}_k\right)_t' = -s_k \ddot{\alpha}_k - c_k \dot{\alpha}_k^2, \ddot{s}_k = \left(c_k \dot{\alpha}_k\right)_t' = c_k \ddot{\alpha}_k - s_k \dot{\alpha}_k^2$. It follows that:

$$\ddot{x}_k = \sum_{i=1}^{k-1} L_i c_i \ddot{\alpha}_i + a_k c_k \ddot{\alpha}_k + ..., \qquad \ddot{y}_k = \sum_{i=1}^{k-1} L_i s_i \ddot{\alpha}_i + a_k s_k \ddot{\alpha}_k + ... \tag{3}$$

In (3), dots denote terms that do not contain accelerations and can be ignored, since these terms are equal to zero in the partial derivatives of $\ddot{x}_k$,$\ddot{y}_k$ with respect to the acceleration $\ddot{q}_i$.

According to the Appell's formalism [7], the DE of the j-th MS body is derived from

$$\frac{\partial S}{\partial \ddot{q}_j} = Q_j, \tag{4}$$

where S is the MS acceleration energy calculated by

$$S = \frac{1}{2} \sum_{k=1}^{N} \left[m_k \left(\ddot{x}_k^2 + \ddot{y}_k^2 \right) + J_k \ddot{\alpha}_k^2 \right]. \tag{5}$$

The projections $\ddot{x}_k$,$\ddot{y}_k$ and the absolute angular acceleration $\ddot{\alpha}_k$ contain the relative angular acceleration $\ddot{q}_j$ for $k \geq j$. Therefore, according to formalism (4), using (5), we obtain

$$\frac{\partial S}{\partial \ddot{q}_j} = \frac{1}{2} \frac{\partial}{\partial \ddot{q}_j} \sum_{k=j}^{N} \left[m_k \left(\ddot{x}_k^2 + \ddot{y}_k^2 \right) + J_k \ddot{\alpha}_k^2 \right] =$$

$$= \sum_{k=j}^{N} \left[m_k \left(\ddot{x}_k \frac{\partial \ddot{x}_k}{\partial \ddot{q}_j} + \ddot{y}_k \frac{\partial \ddot{y}_k}{\partial \ddot{q}_j} \right) + J_k \ddot{\alpha}_k \frac{\partial \ddot{\alpha}_k}{\partial \ddot{q}_j} \right] = Q_j. \tag{6}$$

From the equation $\ddot{\alpha}_k = \sum_{i=j}^{k} \ddot{q}_i$, we obtain $\frac{\partial \ddot{\alpha}_k}{\partial \ddot{q}_j} = \frac{\partial}{\partial \ddot{q}_j} \sum_{i=j}^{k} \ddot{q}_i = \frac{\partial \ddot{q}_j}{\partial \ddot{q}_j} = 1$.

Hence, from (3), we obtain

$$\frac{\partial \ddot{x}_k}{\partial \ddot{q}_j} = \sum_{i=j}^{k-1} L_i c_i + a_k c_k, \qquad \frac{\partial \ddot{y}_k}{\partial \ddot{q}_j} = \sum_{i=j}^{k-1} L_i s_i + a_k s_k. \tag{7}$$

Substituting (7) into equation (6), we obtain the MS DE in the following form:

$$\sum_{k=j}^{N} \left\{ m_k \left[\ddot{x}_k \left(\sum_{i=j}^{k-1} L_i c_i + a_k c_k \right) + \ddot{y}_k \left(\sum_{i=j}^{k-1} L_i s_i + a_k s_k \right) \right] + J_k \ddot{\alpha}_k \right\} = Q_j. \tag{8}$$

Now convert DE (8) to the desired form, so that the expressions in parentheses can be represented in terms of the absolute coordinates of the CM bodies. To do this, we split the sums in (2) into two parts, and also add and subtract the terms $a_j s_j$,$-a_j s_j$ in the expression x_k and the terms $a_j c_j$,$-a_j c_j$ in the expression y_k. Then equalities (2) take the form:

$$x_k = \sum_{i=1}^{j-1} L_i s_i + \sum_{i=j}^{k-1} L_i s_i + a_k s_k + a_j s_j - a_j s_j = x_j - a_j s_j + \sum_{i=j}^{k-1} L_i s_i + a_k s_k,$$

$$y_k = -\sum_{i=1}^{j-1} L_i c_i - \sum_{i=j}^{k-1} L_i c_i - a_k c_k + a_j c_j - a_j c_j = y_j + a_j c_j - \sum_{i=j}^{k-1} L_i c_i - a_k c_k.$$

So, we obtain $\sum\limits_{i=j}^{k-1} L_i s_i + a_k s_k = x_k - x_j + a_j s_j$, $\quad \sum\limits_{i=j}^{k-1} L_i c_i + a_k c_k = y_j - y_k + a_j c_j$. Thus, DE (8) take the desired form (1). This completes proof of Proposition 1.

□

Now consider an example of using MS DE in the form (1) for the synthesis of an adaptive PID controller by program gripping.

2. Modelling Example

Let us consider the 3-link MS (Fig.2). From the general form of DE (1) of the given MS class for $N = 3, j = 1$, we write

$$\sum_{k=1}^{3} (H_{k1}^{y} \ddot{x}_k + H_{k1}^{x} \ddot{y}_k + J_k \ddot{\alpha}_k) = H_{31}^{y} \ddot{x}_3 + H_{31}^{x} \ddot{y}_3 + J_3 \ddot{\alpha}_3 + h_1 = Q_1,$$

where $h_1 = \sum\limits_{k=1}^{2} (H_{k1}^{y} \ddot{x}_k + H_{k1}^{x} \ddot{y}_k + J_k \ddot{\alpha}_k)$.

For $j = 2$, from (1), we write

$$\sum_{k=2}^{3} (H_{k2}^{y} \ddot{x}_k + H_{k2}^{x} \ddot{y}_k + J_k \ddot{\alpha}_k) = H_{32}^{y} \ddot{x}_3 + H_{32}^{x} \ddot{y}_3 + J_3 \ddot{\alpha}_3 + h_2 = Q_2,$$

where $h_2 = H_{22}^{y} \ddot{x}_2 + H_{22}^{x} \ddot{y}_2 + J_2 \ddot{\alpha}_2$.

For $j = 3$, from (1), we write

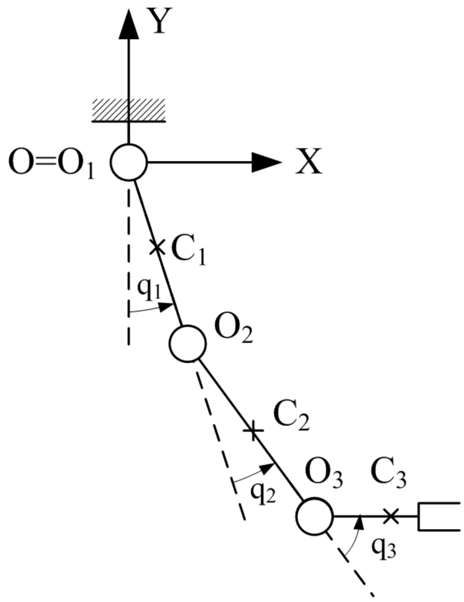

Fig. 2. Three-link manipulator with revolute joints

$$H_{33}^{y}\ddot{x}_3 + H_{33}^{x}\ddot{y}_3 + J_3\ddot{\alpha}_3 = Q_3$$

Represent the written DE in the form

$$\mathbf{H}\ddot{\mathbf{q}} + \mathbf{h} = \mathbf{Q}, \tag{9}$$

where $\mathbf{H} = \begin{pmatrix} H_{31}^{y} & H_{31}^{x} & J_3 \\ H_{32}^{y} & H_{32}^{x} & J_3 \\ H_{33}^{y} & H_{33}^{x} & J_3 \end{pmatrix}$, $\quad \ddot{\mathbf{q}} = \begin{pmatrix} \ddot{x}_3 \\ \ddot{y}_3 \\ \ddot{\alpha}_3 \end{pmatrix}$, $\quad \mathbf{h} = \begin{pmatrix} h_1 \\ h_2 \\ 0 \end{pmatrix}$, $\quad \mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$.
Here, $\mathbf{q} = (x_3, y_3, \alpha_3)$ is a vector of parameters of the position and orientation of the MS gripper at the current moment of time t. Let x_3^p, y_3^p, α_3^p be the given (desired) program parameters of the position and orientation of the gripper at the current time t, i.e. Program Gripping (PG). Then, $x_3^e = x_3^p - x_3$, $y_3^e = y_3^p - y_3$, $\alpha_3^e = \alpha_3^p - \alpha_3$ are PG control errors and $\dot{x}_3^e = \dot{x}_3^p - \dot{x}_3$, $\dot{y}_3^e = \dot{y}_3^p - \dot{y}_3$, $\dot{\alpha}_3^e = \dot{\alpha}_3^p - \dot{\alpha}_3$ are PG speed control errors.

We calculate Q by the formula

$$Q = H\left(\ddot{p} + D\dot{x} + Px\right) + h, \tag{10}$$

where $\ddot{p} = \left(\ddot{x}_3^p, \ddot{y}_3^p, \ddot{\alpha}_3^p\right)^T$ is a column vector of PG accelerations, $D = diag\left(d_x, d_y, d_\alpha\right)$, $P = diag\left(p_x, p_y, p_\alpha\right)$ are diagonal matrices of constant coefficients.

Let us combine DE (9) with control (10):

$$H\ddot{q} + h = H\left(\ddot{p} + D\ddot{x} + Px\right) + h. \tag{11}$$

We can cancel the values h in equation (11), and multiply the result by the inverse matrix H^{-1}. Then, taking into account the equalities $H^{-1}H = E$, $Eb = b$, where E is an identity matrix, b is a column vector, we obtain $q = \ddot{p} + D\dot{x} + Px$.

From here, taking into account the equality $\ddot{p} - \ddot{q} = \ddot{x}$, we obtain the equation for the vector of control errors

$$\ddot{x} + D\dot{x} + Px = 0.$$

Hence, for the elements of the column vector x, we obtain three scalar equations:

$$\ddot{x}_3^e + d_x\dot{x}_3^e + p_x x_3^e = 0, \quad \ddot{y}_3^e + d_y\dot{y}_3^e + p_y y_3^e = 0, \quad \ddot{\alpha}_3^e + d_\alpha\dot{\alpha}_3^e + p_\alpha \alpha_3^e = 0. \tag{12}$$

It follows from the theory of second-order homogeneous differential equations with constant coefficients that for the known initial control errors $x_3^e(0)$, $y_3^e(0)$, $\alpha_3^e(0)$ it is always possible to choose constant coefficients d_x, p_x, d_y, p_y, d_α, p_α, under which the control errors $x_3^e(t)$, $y_3^e(t)$, $\alpha_3^e(t)$ tend to zero with a given quality, for example, fall into the ε–neighborhood of zero for the time T according to the aperiodic law (Fig. 3).

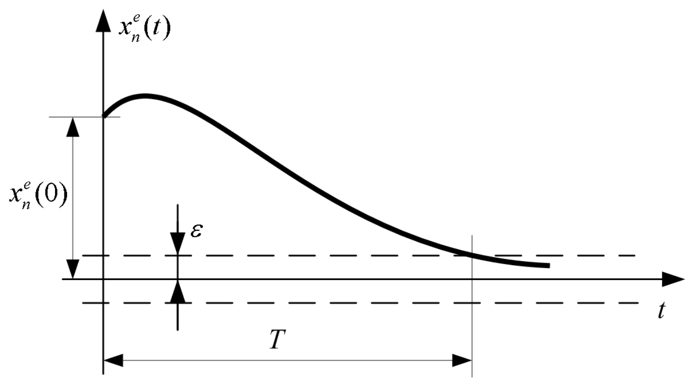

Fig. 3. Graph of change control error along the coordinate x for the point O_N of the gripper

Control (10) belongs to the class of adaptive PID regulators (controllers). Indeed, (10) can be represented as

$$Q = px + d\dot{x} + I\left(t, q, \dot{q}\right), \tag{13}$$

where $p = H(q)P$ is a proportional coefficient of control depending on q; $d = H(q)D$ is a differential coefficient of control depending on q; $I = H(q)\ddot{p}(t) + h$ is an integral part of the control depending on q.

Thus, the coefficients of PID controller (13) depend on the parameters x_3, y_3, α_3 of the actual position of the gripper.

Conclusion

To extend the above results to an arbitrary MS, it is recommended to use the article formalism [2]. This article solves the problem of the cumbersomeness of the MS DE consisting of prismatic and revolute joints, and proposes a new formalism for writing the MS DE in direction cosines and their time derivatives from which DE in other parameters, including quasi-coordinates and quasi-velocities, are easily obtained. For quasi-velocities, to synthesize an adaptive PID controller by a program MS gripper movement, it is sufficient to choose, for example, the projections of the absolute velocity of the gripper suspension point on the axes of the coordinate system of the MS base, as well as the projections of the absolute angular velocity of the gripper on its associated axes. It is also possible to choose other parameters of the gripper position depending on the requirements for its program movements in the given MS.

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НОВЫЙ ВИД УРАВНЕНИЙ ДИНАМИКИ МАНИПУЛЯТОРОВ ДЛЯ СИНТЕЗА АДАПТИВНОГО ПИД-РЕГУЛЯТОРА ПРОГРАММНЫМ ДВИЖЕНИЕМ СХВАТА

М. В. Носиков, А. И. Телегин

Новый вид уравнений динамики шарнирных манипуляторов на плоскости выведен по формализму Аппеля, в котором частная производная от энергии ускорения взята по ускорению управляемой координаты, а в левой части уравнений путем тождественных преобразований выделены параметры положения схвата манипулятора и их вторые производные по времени. Такой вид уравнений динамики позволяет синтезировать адаптивный ПИД-регулятор программным движением схвата без решения обратной задачи кинематики. Рассмотрен пример выписывания такого уравнения и синтеза на его основе адаптивного ПИД-регулятора программным движением схвата. Кинематическая схема манипулятора в этом примере является частным случаем схемы шестизвенного манипулятора с ангулярной системой координат в пространстве, в которой неподвижны определенные сочленения, что соответствует манипуляциям в вертикальной плоскости. В заключении описано, как можно распространить полученные результаты на произвольные манипуляторы.

Ключевые слова: манипулятор; система жесткого тела; уравнения движения; уравнение движения Аппеля; адаптивный ПИД-регулятор.

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