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THE OSKOLKOV SYSTEM WITH A MULTIPOINT INITIAL-FINAL VALUE CONDITION

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The article considers a deterministic Oskolkov model with a multipoint initial-final value condition. The article, in addition to the introduction and the list of references, contains two parts. The first part provides theoretical information about a deterministic Sobolev type equation with a multipoint initial-final value condition. In the second part, the solvability of the Oskolkov model with a multipoint initial-final value condition is investigated.

Keywords: Sobolev type equations; Oskolkov model; multipoint initial-final value condition.

Introduction

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with the boundary $\partial\Omega$ of the class C^∞. In the cylinder $\Omega \times \mathbb{R}$ consider the Dirichlet problem

$$v(x,t) = 0, \quad (x,t) \in \partial\Omega \times \mathbb{R}, \tag{1}$$

for a linear system of Oskolkov equations [1], [2]

$$(\lambda - \nabla^2)v_t = \nu\nabla^2 v - \nabla p + g, \quad \nabla \cdot v = 0, \tag{2}$$

modeling the dynamics of a viscoelastic incompressible Kelvin–Voigt fluid. Here the desired functions $v = v(x,t)$, $p = p(x,t)$ correspond to the velocity and pressure of the liquid, the specified function $g = g(x,t)$ corresponds to an external effect on the liquid; the parameters $\lambda \in \mathbb{R}$, $\nu \in \mathbb{R}_+$ characterize the elastic and viscous properties of the liquid, respectively, and negative values do not contradict the physical meaning.

Let $\mathfrak{U}$ and $\mathfrak{F}$ be Banach spaces; the operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ (i.e. linear and continuous), $M \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$ (i.e. linear, closed, and densely defined). Let the operator M (L,p) be bounded [2], moreover L-spectrum of the operator M is satisfied the condition [3]

$$\left.\begin{array}{c} \sigma^L(M) = \bigcup\limits_{j=0}^{m} \sigma_j^L(M),\ m \in \mathbb{N},\ \text{where } \sigma_j^L(M) \neq \emptyset,\ \text{there exists} \\ \text{a cloused contour } \gamma_j \subset \mathbb{C} \text{ that bounds the domain } D_j \supset \sigma_j^L(M), \\ \text{and is such that } \overline{D_j} \cap \sigma_0^L(M) = \emptyset,\ \overline{D_k} \cap \overline{D_l} = \emptyset \text{ for all } j,k,l = \overline{1,m}, k \neq l. \end{array}\right\} \tag{3}$$

Then there are projectors

$$P_j = \frac{1}{2\pi i}\int_{\gamma_j} R_\mu^L(M)d\mu \in \mathcal{L}(\mathfrak{U}); \quad Q_j = \frac{1}{2\pi i}\int_{\gamma_j} L_\mu^L(M)d\mu \in \mathcal{L}(\mathfrak{F}), j = \overline{0,m} \tag{4}$$

where closed contours γ_r are defined (3), $R_\mu^L = (\mu L - M)^{-1}L$ and $L_\mu^L = L(\mu L - M)^{-1}$ ane right and left L-operator resolvents M, respectively. Let the operator M (L,p) be bounded, then there are projectors $P \in \mathcal{L}(\mathfrak{U})$, $Q \in \mathcal{L}(\mathfrak{F})$ such that $PP_j = P_jP = P_j$, $QQ_j = Q_jQ = Q_j$, $j = \overline{0,m}$, и $P_kP_l = P_lP_k = \mathbb{O}$, $Q_kQ_l = Q_lQ_k = \mathbb{O}$, k, $l = \overline{0,m}$, $k \neq l$.

This article is devoted to the study of a deterministic model (1), (2), which is considered together with a multipoint initial-final value condition.

$$P_j(u(\tau_j) - u_j) = 0, \quad j = \overline{0,m}, \tag{5}$$

Here $\tau_j \in \mathbb{R}$, $j = \overline{0,m}$, such that $\tau_{j+1} > \tau_j$, $\tau_0 \geq 0$.

The article, in addition to the introduction and the list of references, contains two parts. The first part provides theoretical information on the solvability of Sobolev-type equations with a multipoint initial-final value condition. In the second part, the solvability of the Oskolkov model with a multipoint initial-final value condition is investigated.

1. Sobolev Type Deterministic Equation with Multipoint Initial-Final Value Condition

Let $\mathfrak{U}$ and $\mathfrak{F}$ be Banach spaces, the operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ (i.e. linear and continuous) and $M \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$ (i.e. linear, closed, and densely defined). In addition, suppose that the operator M (L,p)-bounded [3], $p \in \mathbb{N}_0$ (here and below $\mathbb{N}_0 \equiv \{0\} \cup \mathbb{N}$), then there exist degenerate analytic groups of resolving operators

$$U^t = \frac{1}{2\pi i}\int_\gamma R_\mu^L(M)e^{\mu t}d\mu \quad \text{и} \quad F^t = \frac{1}{2\pi i}\int_\gamma L_\mu^L(M)e^{\mu t}d\mu,$$

defined on the spaces $\mathfrak{U}$ и $\mathfrak{F}$ respectively, and $U^0 \equiv P$, $F^0 \equiv Q$ are projectors. Here γ – is a contour bounding the domain D, containing the L-spectrum $\sigma^L(M)$ of the operator M. For a degenerate analytic group, we define a *kernel* $\ker U^\cdot = \ker P = \ker U^t$ ($\ker F^\cdot = \ker Q = \ker F^t$) for any $t \in \mathbb{R}$ and an *image* $\operatorname{im} U^\cdot = \operatorname{im} P = \operatorname{im} U^t$ ($\operatorname{im} F^\cdot = \operatorname{im} Q = \operatorname{im} F^t$) for any $t \in \mathbb{R}$. Denote $\mathfrak{U}^0 = \ker U^\cdot$, $\mathfrak{U}^1 = \operatorname{im} U^\cdot$, and $\mathfrak{F}^0 = \ker F^\cdot$, $\mathfrak{F}^1 = \operatorname{im} F^\cdot$, then $\mathfrak{U}^0 \oplus \mathfrak{U}^1 = \mathfrak{U}$ and $\mathfrak{F}^0 \oplus \mathfrak{F}^1 = \mathfrak{F}$. Also, denote by L_k (M_k) the restriction of the operator L (M) to $\mathfrak{U}^k$ ($\operatorname{dom} M \cap \mathfrak{U}^k$), $k = 0, 1$.

Theorem 1. [4] *Let the operator M be (L,p)-bounded, $p \in \mathbb{N}_0$, and condition* (3) *be satisfied. Then*
(i) there exist degenerate analytic groups

$$U_j^t = \frac{1}{2\pi i}\int\limits_{\gamma_j} R_\mu^L(M)e^{\mu t}d\mu, \quad F_j^t = \frac{1}{2\pi i}\int\limits_{\gamma_j} L_\mu^L(M)e^{\mu t}d\mu, j = \overline{1,m};$$

(ii) $U^tU_j^s = U_j^sU^t = U_j^{s+t}$, $F^tF_j^s = F_j^sF^t = F_j^{s+t}$ *for all* s, $t \in \mathbb{R}$, $j = \overline{1,m}$;
(iii) $U_k^tU_l^s = U_l^sU_k^t = \mathbb{O}$, $F_k^tF_l^s = F_l^sF_k^t = \mathbb{O}$ *for all* s, $t \in \mathbb{R}$, k, $l = \overline{1,m}$, $k \neq l$;
(iv) $U_0^t = U^t - \sum\limits_{k=1}^m U_k^t$, $F_0^t = F^t - \sum\limits_{k=1}^m F_k^t$, *for* $t \in \mathbb{R}$.

Remark 1. The units $P_j \equiv U_j^0$, $Q_j \equiv F_j^0$, $j = \overline{0,m}$, built by virtue of the condition (3), of degenerate analytic groups $\{U_j^t : t \in \mathbb{R}\}$, $\{F_j^t : t \in \mathbb{R}\}$, $j = \overline{0,m}$. Note that the projectors P_j и Q_j were built earlier (4).

Let us introduce the subspaces $\mathfrak{U}^{1j} = \operatorname{im} P_j$, $\mathfrak{F}^{1j} = \operatorname{im} Q_j$, $j = \overline{0, m}$. By construction,

$$\mathfrak{U}^1 = \bigoplus_{j=0}^{m} \mathfrak{U}^{1j} \text{ and } \mathfrak{F}^1 = \bigoplus_{j=0}^{m} \mathfrak{F}^{1j}.$$

Denote by L_{1j} the restriction of the operator L to $\mathfrak{U}^{1j}$, $j = \overline{0, m}$, and denote by M_{1j} the restriction of the operator M to $\operatorname{dom} M \cap \mathfrak{U}^{1j}$, $j = \overline{0, m}$. Since, as it is easy to show, $P_j\varphi \in \operatorname{dom} M$, if $\varphi \in \operatorname{dom} M$, then the domain $\operatorname{dom} M_{1j} = \operatorname{dom} M \cap \mathfrak{U}^{1j}$ is dense in $\mathfrak{U}^{1j}$, $j = \overline{0, m}$.

Theorem 2. [4]. (Generalised spectral theorem). *Suppose that the operators $L \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ and $M \in \mathcal{C}l(\mathfrak{U}; \mathfrak{F})$, while the operator M is (L, p)-bounded, $p \in \mathbb{N}_0$, and condition* (3) *is satisfied. Then*
(i) the operators $L_0 \in \mathcal{L}(\mathfrak{U}^0; \mathfrak{F}^0)$, $M_0 \in \mathcal{C}l(\mathfrak{U}^0; \mathfrak{F}^0)$, $M_0^{-1} \in \mathcal{L}(\mathfrak{F}^0; \mathfrak{U}^0)$;
(ii) the operators $L_1 \in \mathcal{L}(\mathfrak{U}^1; \mathfrak{F}^1)$, $L_{1j} \in \mathcal{L}(\mathfrak{U}^{1j}; \mathfrak{F}^{1j})$;
(iii) the operators $M_1 \in \mathcal{L}(\mathfrak{U}^1; \mathfrak{F}^1)$, $M_{1j} \in \mathcal{L}(\mathfrak{U}^{1j}; \mathfrak{F}^{1j})$, $j = \overline{0, m}$;
(iv) there exist the operators $L_1^{-1} \in \mathcal{L}(\mathfrak{F}^1; \mathfrak{U}^1)$, $L_{1j}^{-1} \in \mathcal{L}(\mathfrak{F}^{1j}; \mathfrak{U}^{1j})$, $j = \overline{0, m}$.

Let $H = M_0^{-1} L_0 \in \mathcal{L}(\mathfrak{U}^0)$, $S = L_1^{-1} M_1 \in \mathcal{L}(\mathfrak{U}^1)$, $S_j = L_{1j}^{-1} M_{1j} \in \mathcal{L}(\mathfrak{U}^{1j})$, $j = \overline{0, m}$.

So, let condition (3) be satisfied. Fix $\tau_j \in \mathbb{R}$, $(\tau_j < \tau_{j+1})$, the vectors $u_j \in \mathfrak{U}$, $j = \overline{0, m}$, the vector function $f \in C^\infty(\mathbb{R}; \mathfrak{F})$ and consider the linear inhomogeneous equation of Sobolev type

$$L\dot{u} = Mu + f. \tag{6}$$

The vector function $u \in C^\infty(\mathbb{R}; \mathfrak{U})$ satisfying equation (6) is called a *solution to equation* (6). Solution $u = u(t)$ to equation (6), $t \in \mathbb{R}$, satisfying the conditions

$$P_j(u(\tau_j) - u_j) = 0, \quad j = \overline{0, m}, \tag{7}$$

is said to be a *solution to the multipoint initial-final value problem for equation* (6).

Based on Theorem 2, we reduce equation (6) to the system

$$\begin{cases} H\dot{u}^0 = u^0 + M_0^{-1}(\mathbb{I} - Q)f, \\ \dot{u}^{1j} = S_j u^{1j} + L_{1j}^{-1} Q_j f, \quad j = \overline{0, m}, \end{cases} \tag{8}$$

where $u^0 = (\mathbb{I} - P)u$, $u^{1j} = P_j u$, $j = \overline{0, m}$, and each equation is defined on «its own» subspace. From the first equation of (8), by differentiation of the equation and multiplication from the left by H, due to the nilpotency of the operator H we obtain

$$u^0(t) = -\sum_{k=0}^{p} H^K M_0^{-1}(\mathbb{I} - Q) f^{(k)}(t). \tag{9}$$

For the remaining equations of (8), conditions (7) become the Cauchy conditions

$$u^{1j}(\tau_j) = P_j u_j, \quad j = \overline{0, m}. \tag{10}$$

Solving these problems step by step, we obtain

$$u^{1j}(t) = U_j^{t-\tau_j} u_j + \int_{\tau_j}^{t} U_j^{t-s} L_{1j}^{-1} Q_j f(s) ds, \ j = \overline{0, m}. \tag{11}$$

Therefore, we arrive at

Theorem 3. [4] *Let the operator* M *be* (L,p)*-bounded,* $p \in \mathbb{N}_0$*, and condition* (3) *be satisfied. Then for any* $f \in C^\infty(\mathbb{R};\mathfrak{F})$*,* $u_j \in \mathfrak{U}$*,* $j=\overline{0,m}$*, there exists a unique solution to problem* (6)*,* (5)*, of the form*

$$u(t) = -\sum_{k=0}^{p} H^q M_0^{-1}(\mathbb{I}-Q)f^{(k)}(t) + \sum_{j=0}^{m}\left(U_j^{t-\tau_j}u_j + \int_{\tau_j}^{t} U_j^{t-s}L_{1j}^{-1}Q_j f(s)ds\right). \qquad (12)$$

2. The Oskolkov Model with Multipoint Initial-Final Value Condition

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with the boundary $\partial\Omega$ of the class C^∞. In the cylinder $\Omega \times \mathbb{R}$ consider the Dirichlet problem

$$v(x,t) = 0, \quad (x,t) \in \partial\Omega \times \mathbb{R},$$

for a linear system of Oskolkov equations

$$(\lambda - \nabla^2)v_t = \nu\nabla^2 v - \nabla p + g, \quad \nabla \cdot v = 0.$$

We will reduce the problem (1), (2) to an equation (6), based on the results [2]. First of all, let 's redefine ∇p through p, motivating this by the fact that in many cases the consideration of the pressure gradient is preferable to the consideration of the pressure gradient. Then the system of equations (2) takes the form

$$(\lambda - \nabla^2)v_t = \nu\nabla^2 v - p + g, \quad \nabla(\nabla \cdot v) = 0. \qquad (13)$$

Note that by virtue of the Gauss–Ostrogradsky theorem, the sets of solutions to problems (1), (2) and (1), (13) coincide. Denote by $\mathbb{H}^2 = (W_2^2)^n$, $\overset{\circ}{\mathbb{H}}{}^1 = (\overset{\circ}{W}{}_2^1)^n$, $\mathbb{L}^2 = (L_2)^n$ the spaces of vector functions $v = (v_1, v_2, \ldots, v_n)$, defined on Ω. Consider the linear $\mathfrak{L} = \{v \in (C_n^\infty)^n : \nabla \cdot v = 0\}$ vector functions, solenoidal and finite in the domain Ω. The closure $\mathfrak{L}$ according to the norm of space $\mathbb{L}^2$ is denoted by $\mathbb{H}_\sigma$. A Hilbert space $\mathbb{H}_\sigma$ with an inner product $\langle\cdot,\cdot\rangle$, inherited from $\mathbb{L}^2$, in addition, there is splitting $\mathbb{L}^2 = \mathbb{H}_\sigma \oplus \mathbb{H}_\pi$, where $\mathbb{H}_\pi$ is an orthogonal complement to $\mathbb{H}_\sigma$. Denote by $\Sigma : \mathbb{L}^2 \to \mathbb{H}_\sigma$ the corresponding orthoprojector. The narrowing of the projector Σ на $\mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1$ is a continuous operator, $\Sigma : \mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1 \to \mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1$. Therefore, we represent the space $\mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1$ as a direct sum $\mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1 = \mathbb{H}_\sigma^2 \oplus \mathbb{H}_\pi^2$, где $\mathbb{H}_\sigma^2 = \operatorname{im}\Sigma$, $\mathbb{H}_\pi^2 = \ker\Sigma$. Continuous and dense embeddings take place $\mathbb{H}_\sigma^2 \hookrightarrow \mathbb{H}_\sigma$ and $\mathbb{H}_\pi^2 \hookrightarrow \mathbb{H}_\pi$. The space $\mathbb{H}_\pi^2$ consists of vector functions equal to zero on $\partial\Omega$ and which are gradients of functions from $W_2^3(\Omega)$.

Lemma 1. *[3] (i) The formula* $A = (-\nabla^2)^n : \mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1 \to \mathbb{L}^2$ *defines a linear continuous operator with a positive discrete finite–fold spectrum* $\sigma(A)$*, which condenses only to a point* $+\infty$*, moreover , the mapping* $A : \mathbb{H}_{\sigma(\pi)}^2 \to \mathbb{H}_{\sigma(\pi)}$ *is bijective.*
(ii) The formula $B : v \to -\nabla(\nabla \cdot v)$ *defines a linear continuous surjective operator* $B : \mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1 \to \mathbb{H}_\pi$*, moreover* $\ker B = \mathbb{H}_\sigma^2$*.*

Put $\mathfrak{U} = \mathbb{H}_\sigma^2 \times \mathbb{H}_\pi^2 \times \mathbb{H}_p$, $\mathfrak{F} = \mathbb{H}_\sigma \times \mathbb{H}_\pi \times \mathbb{H}_p$, $\mathbb{H}_p = \mathbb{H}_\pi$, $A_\lambda = \lambda\mathbb{I} + A$ and build operators

$$L = \begin{pmatrix} \Sigma A_\lambda & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \Pi A_\lambda & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix}, \quad M = \begin{pmatrix} -\nu \sum A & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & -\nu \Pi A & -\Pi \\ \mathbb{O} & \Pi B & \mathbb{O} \end{pmatrix}.$$

Obviously, $L, M \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$, with $\operatorname{im} L = \mathbb{H}_\sigma \times \mathbb{H}_\pi \times \{0\}$, $\ker L = \{0\} \times \{0\} \times \mathbb{H}_p$.

Lemma 2. *[3] Under any conditions $\lambda \in \mathbb{R} \setminus \sigma(A)$, $\nu \in \mathbb{R}_+$ the operator M $(L, 1)$ is limited.*

If we put $f = \operatorname{col}(\Sigma g, \Pi g, 0)$, then the reduction of the problem (1), (12) to the equation (6) is finished. Find L-the spectrum $\sigma^L(M)$ of the operator M. As it is easy to see, the operator $\mu L - M$ is invertible exactly when we reverse the operator $\Sigma(\mu\lambda\mathbb{I} - (\mu - \nu)A) : \mathbb{H}_\sigma^2 \to \mathbb{H}_\sigma$.Denote by $\tilde{A}$ narrowing the operator A to $\mathbb{H}_\sigma^2$. The spectrum of the operator $\tilde{A} \in \mathcal{L}(\mathbb{H}_\sigma^2; \mathbb{H}_\sigma)$ is positive, discrete, finite-fold, and condenses only to $+\infty$ (the Solonnikov–Vorovich–Yudovich theorem). Denote by $\{\lambda_k\}$ the set of eigenvalues of the operator $\tilde{A}$, numbered by non-decreasing taking into account multiplicity. Then

$$\sigma^L(M) = \left\{ \mu_k = \frac{\nu \lambda_k}{\lambda_k - \lambda} : \lambda_k \in \sigma(\tilde{A}) \setminus \{\lambda\} \right\}.$$

It is clear that it is possible to choose contours for such a set $\gamma_j \subset \mathbb{C}$. We will build

$$U_j^t = \begin{pmatrix} \sum\limits_{\lambda_k \in \sigma_j^L(M)} e^{\lambda_k t} \langle \cdot, \varphi_k \rangle_\sigma \varphi_k & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix}, j = \overline{0, m}.$$

The theorem is valid

Theorem 4. *For any $\lambda \in \mathbb{R} \setminus \sigma(A)$, $\nu \in \mathbb{R}_+$, $\tau_j \in \mathbb{R}$, $u_j \in \mathfrak{U}$, $j = \overline{0, m}$ vector function $g : [\tau_0, \tau_m] \to L_2$ such that $\Sigma g \in C([\tau_0, \tau_m]; \mathbb{H}_\sigma)$, $\Pi g \in C^1((\tau_0, \tau_m), \mathbb{H}_\pi) \cap C([\tau_0, \tau_m]; \mathbb{H}_\pi)$ there is a single solution to the problem (1), (2), (5).*

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СИСТЕМА ОСКОЛКОВА С МНОГОТОЧЕЧНЫМ НАЧАЛЬНО-КОНЕЧНЫМ УСЛОВИЕМ

М. М. Якупов, А. С. Конкина

В статье рассматривается детерминированная модель Осколкова с многоточечным начально-конечным условием. Статья, кроме введения и списка литературы, содержит две части. В первой частях приводятся теоретические сведения о детерминированном уравнение соболевского типа с многоточечным начально-конечным условием. Во второй части исследуется разрешимость модели Осколкова с многоточечным начально-конечным условием.

Ключевые слова: уравнения соболевского типа; модель Осколкова; многоточечное начально-конечное условие.

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