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STABILIZATION OF THE STOCHASTIC BARENBLATT – ZHELTOV – KOCHINA EQUATION

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The article is devoted to the stabilization of solutions to the stochastic Barenblatt – Zheltov – Kochina equation. The Barenblatt – Zheltov – Kochina equation is a model of filtration of a viscous liquid in a porous medium. This equation also models the processes of moisture transfer in the soil. We consider the problem for the Barenblatt – Zheltov – Kochina equation with random initial data. The equation is considered as a system of equations given on stable and unstable invariant spaces. The problem of stabilization is as follows. It is required to find a controlling effect on the system so that its solutions become asymptotically stable. For the stochastic Barenblatt – Zheltov – Kochina equation, we find feedback such that the closed system is asymptotically stable. Numerical solutions to the stochastic Barenblatt – Zheltov – Kochina equation and the stabilized equation are found. Graphs of solutions are constructed.

Keywords: stochastic Sobolev type equations; stable and unstable invariant spaces; stabilization of solutions.

Introduction

Consider the stochastic analogue of the Barenblatt – Zheltov – Kochina equation

$$(\lambda - \Delta)u_t = \alpha\Delta u. \tag{1}$$

For this, we reduce the equation to the stochastic linear Sobolev type equation

$$L \overset{\circ}{\eta} = M\eta \tag{2}$$

where the operators $L,\ M$ are linear and continuous, η is a stochastic **K**-process, denote by $\overset{\circ}{\eta}$ its Nelson – Glicklich derivative [1]. A large number of works (see, for example, [2–4]) are devoted to the study of the Cauchy and Showalter – Sidorov problems for equation (2). The paper [5] considers Sobolev type equations of higher order with additive «white noise». In [6], multipoint initial-final value problems for stochastic dynamical Sobolev type equations are studied. The stability of equation (2) is studied in [7]. In [8–10], numerical experiments are carried out to calculate stable and unstable solutions to equation (2).

A large number of papers (see, for example, the reviews [11–15]) are devoted to the stabilization of linear stationary deterministic systems. One of the problems is the problem of stabilizing a linear stationary system using feedback [13]. In [13], the study of the stabilizability of a stochastic system is reduced to the problem of optimal control to a deterministic system. In the deterministic case, the question of stabilization of solutions to the Cauchy problem for a parabolic equation was first considered by L.A. Lusternik and M.I. Vishik (see, for example, the review [14]).

The aim of the work is to stabilize a stochastic linear equation of the Sobolev type. Equation (2) is considered as a system of equations, one of which is defined on the stable invariant space $\mathbf{I}^s$, and the second one is defined on the unstable invariant space $\mathbf{I}^u$. By the stabilization of solutions, we mean the following problem. It is required to find a controlling effect on the second equation of the system such that the invariant space $\mathbf{I}^u$ becomes stable.

The article consists of Introduction, two sections and References. Section 1 considers static stabilization of a linear stochastic equation by stationary feedback. Section 2 contains the results of stabilization of the Barenblatt – Zheltov – Kochina equation and the results of a numerical experiment.

1. Stabilization of Solutions to Stochastic Sobolev Type Equation

Let $\mathfrak{U}$ ($\mathfrak{F}$) be a separable real Hilbert space, $\{\varphi_k\}(\{\psi_k\})$ be a basis in this space. The elements of the space $\mathbf{U_K L}_2$ ($\mathbf{F_K L}_2$) are the vectors

$$\xi = \sum_{k=1}^{\infty} \lambda_k \xi_k \varphi_k \quad \left(\zeta = \sum_{k=1}^{\infty} \lambda_k \zeta_k \psi_k \right),$$

where $\mathbf{K} = \{\lambda_k\} \subset \mathbb{R}_+$, $\sum\limits_{k=1}^{\infty} \lambda_k^2 < +\infty$, and $\{\xi_k\} \subset \mathbf{L}_2$ $(\{\zeta_k\} \subset \mathbf{L}_2)$, $\|\xi_k\|_{\mathbf{L}_2} \leq$ const $(\|\zeta_k\|_{\mathbf{L}_2} \leq \text{const})$.

Let the operators $L \in \mathcal{L}(\mathbf{U_K L}_2; \mathbf{F_K L}_2)$, $M \in Cl(\mathbf{U_K L}_2; \mathbf{F_K L}_2)$.

Lemma 1. *The operator $A \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ (linear and continuous) if and only if $A \in \mathcal{L}(\mathbf{U_K L}_2; \mathbf{F_K L}_2)$.*

The set $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{F}; \mathfrak{U})\}$ is called the *L-resolvent set of the operator M*, and the set $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ is called the *L-spectrum of the operator M*. If the operator M is (L, σ)-bounded, i.e. $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ is a bounded set, then there exist the projectors

$$P = \frac{1}{2\pi i} \int_{\gamma} (\mu L - M)^{-1} L d\mu \in \mathcal{L}(\mathbf{U_K L}_2), \ Q = \frac{1}{2\pi i} \int_{\gamma} L(\mu L - M)^{-1} d\mu \in \mathcal{L}(\mathbf{F_K L}_2). \quad (3)$$

The contour $\gamma \subset \mathbb{C}$ bounds an area containing $\sigma^L(M)$.

Projectors (3) split the spaces $\mathbf{U_K L}_2 = \mathbf{U^0_K L}_2 \oplus \mathbf{U^1_K L}_2$ and $\mathbf{F_K L}_2 = \mathbf{F^0_K L}_2 \oplus \mathbf{F^1_K L}_2$, where $\mathbf{U^0_K L}_2$ $(\mathbf{U^1_K L}_2) = \ker P$ $(\text{im} P)$, $\mathbf{F^0_K L}_2$ $(\mathbf{F^1_K L}_2) = \ker Q$ $(\text{im} Q)$, The operators L_k, M_k are the restrictions of the operators L, M on the spaces $\mathbf{U^k_K L}_2$, $k = 0, 1$. The operators $L_k(M_k) \in \mathcal{L}(\mathbf{U^k_K L}_2, \mathbf{F^k_K L}_2)$, $k = 0, 1$, there exist the operators $M_0^{-1} \in \mathcal{L}(\mathbf{F^0_K L}_2, \mathbf{U^0_K L}_2)$, $L_1^{-1} \in \mathcal{L}(\mathbf{F^1_K L}_2, \mathbf{U^1_K L}_2)$. Consider the operators $H = L_0^{-1} M_0$ and $S = L_1^{-1} M_1$. Let the operator M be (L, σ)-bounded and $H \equiv \mathbb{O}$, $p = 0$ or $H^p \not\equiv \mathbb{O}$, $H^{p+1} \equiv \mathbb{O}$, then it is called *$(L, p)$-bounded* operator.

Let the condition

$$\begin{gathered} \sigma^L(M) = \sigma^L_l(M) \oplus \sigma^L_r(M), \\ \sigma^L_l(M) = \{\mu \in \sigma^L(M) : \text{Re } \mu < 0\} \neq \oslash, \\ \sigma^L_r(M) = \{\mu \in \sigma^L(M) : \text{Re } \mu > 0\} \neq \oslash \end{gathered} \quad (4)$$

be fulfilled, then there exist the projectors

$$P_{l(r)} = \frac{1}{2\pi i} \int_{\Gamma_{l(r)}} (\mu L - M)^{-1} L d\mu, \quad Q_{l(r)} = \frac{1}{2\pi i} \int_{\Gamma_{l(r)}} L(\mu L - M)^{-1} d\mu,$$

where the contour $\Gamma_{l(r)}$ belongs to the left (right) half-plane and bounds an area containing that part of L-spectrum of the operator M, which is located in this half-plane. Note that $P_{l(r)}P = PP_{l(r)} = P_{l(r)}$, $Q_{l(r)}Q = QQ_{l(r)} = Q_{l(r)}$ and $P_lP_r = P_rP_l = \mathbb{O}$, $Q_lQ_r = Q_rQ_l = \mathbb{O}$. Let $\mathbf{I}^{s(u)} = \mathrm{im}P_{l(r)}$, $\mathbf{F}^{s(u)}_{\mathbf{K}}\mathbf{L}_2 = \mathrm{im}Q_{l(r)}$. The operators M_l (L_l) and M_r (L_r) are the restrictions of the operator M (L) on the spaces $\mathbf{I}^s$ and $\mathbf{I}^u$.

Consider linear equation (2) with the initial condition

$$\eta(0) = \eta_0, \tag{5}$$

where $\eta_0 = \sum\limits_{k=1}^{\infty} \lambda_k\xi_k\varphi_k$. Let condition (4) be fulfilled, then we consider problem (2), (5) as the system

$$H \overset{\circ}{\eta}{}^0 = \eta^0,\ \eta^0(0) = \eta^0_0, \tag{6}$$

$$L_l \overset{\circ}{\eta}_l = M_l\eta_l,\ \eta_l(0) = \eta_{l0}, \tag{7}$$

$$L_r \overset{\circ}{\eta}_r = M_r\eta_r,\ \eta_r(0) = \eta_{r0}. \tag{8}$$

Here $\eta^0_0 = (\mathbb{I}-P)\eta_0 = \sum\limits_{k=\{k:\ker L\neq\{0\}\}} \lambda_k\xi_k\varphi_k$, where l_0 is the kernel dimension of the operator L, and $\eta_{l0} = P_l\eta_0 = \sum\limits_{k=\{k:\mathrm{Re}\ \mu_k<0\}} \lambda_k\xi_k(t)\varphi_k$, $\eta_{r0} = P_r\eta_0 = \sum\limits_{k=\{k:\mathrm{Re}\ \mu_k>0\}} \lambda_k\xi_k(t)\varphi_k$. The existence of solutions to problem (2), (5) is discussed in [2]. It is shown that if $\eta_0 \in \mathbf{U}^1_{\mathbf{K}}\mathbf{L}_2$, then there exists a unique solution to problem (2), (5). In [7], it is shown that there exist the holomorphic groups

$$U^t_l = \frac{1}{2\pi i}\int\limits_{\gamma_l}(\mu L_l - M_l)^{-1}L_le^{\mu t}d\mu,\ U^t_r = \frac{1}{2\pi i}\int\limits_{\gamma_r}(\mu L_r - M_r)^{-1}L_re^{\mu t}d\mu.$$

It is proved that the solutions $\eta_l = \eta_l(t) = U^t_l\eta_{l0}$ to problem (7) belong to the stable invariant space $\mathbf{I}^s$, and the solutions $\eta_r = \eta_r(t) = U^t_r\eta_{r0}$ to problems (8) belong to the unstable invariant space $\mathbf{I}^u$. The stochastic process $\eta = \eta_l + \eta_r$ is a solution to problem (2), (5).

The solution to problem (9) has the form $\eta_l = U^t_l\eta_{l0}$. Since $\lim\limits_{t\to+\infty}\|\eta_l(t)\|_{\mathbf{U_K L_2}} = 0$, then consider the following stabilization problem. It is required to find the stochastic process $\chi \in \mathbf{F}^r_{\mathbf{K}}\mathbf{L}_2$ such that the solutions to the system

$$L_l \overset{\circ}{\eta}_l = M_l\eta_l,\ \eta_l(0) = \eta_{l0}, \tag{9}$$

$$L_r \overset{\circ}{\eta}_r = M_r\eta_r + \chi,\ \eta_r(0) = \eta_{r0} \tag{10}$$

are stabilized, i.e.

$$\lim_{t\to+\infty}\|\eta_r(t)\|_{\mathbf{U_K L_2}} = 0. \tag{11}$$

For (5), we find a static output feedback of the form

$$\chi = B\eta_r \tag{12}$$

such that a system closed by feedback (12) is asymptotically stable. Here B is some linear bounded operator. So, for the equation

$$L_r \overset{\circ}{\eta}_r = M_r\eta_r + B\eta_r = (M_r + B)\eta_r, \tag{13}$$

we need to find the operator B such that the spectrum $\sigma^{L_r}(M_r + B)$ belongs to the left half-plane of the complex plane.

2. Stabilization of Solutions to Stochastic Barenblatt – Zheltov – Kochina Equation

Let

$$\mathfrak{U} = \{z \in W_2^{l+2}(-\pi,\pi) : z(-\pi) = z(\pi) = 0\}, \ \mathfrak{F} = W_2^l(-\pi,\pi),$$

where $l \in \{0\} \cup \mathbb{N}$. The sequence $\{\sin kx\}$ of the eigenfunctions of the Laplace operator Δ is a basis in the Hilbert space $W_p^{l+2}(-\pi,\pi)$, and the spectrum of the operator Δ is $\sigma(\Delta) = -k^2$. Let the sequence $\{\chi_k\} \subset \mathbf{L_2}$ ($\{\zeta_k\} \subset \mathbf{L_2}$) be a uniformly bounded sequence, the sequence $\{\lambda_k\}$ be such that $\sum\limits_{k=1}^{\infty} \lambda_k < +\infty$. The elements of the space $\mathbf{U_K L}_2$ ($\mathbf{F_K L}_2$) are the vectors

$$\chi = \sum_{k=1}^{\infty} \lambda_k \chi_k \sin kx \ \left(\zeta = \sum_{k=1}^{\infty} \lambda_k \zeta_k \sin kx\right).$$

The formulas

$$L = (\lambda - \Delta), \quad M = \alpha\Delta \tag{14}$$

define the operators $L,\ M : \mathbf{U_K L}_2 \to \mathbf{F_K L}_2$. Then we consider stochastic equation (1) in the form (2). The phase space of equation (2) is the space [3]

$$\mathbf{U^1_K L}_2 = \begin{cases} \mathbf{U_K L}_2, \quad \text{if } \lambda \neq -k^2, \quad k \in \mathbb{N}; \\ \eta \in \mathbf{U_K L}_2 : \ (\eta, \sin kx) = 0, \ \text{if } \lambda = -k^2. \end{cases}$$

Let $\alpha \in \mathbb{R}_+$, $\lambda \in \mathbb{R}_-$. Then

$$\sigma^L(M) = \sigma^L_+(M) \cup \sigma^L_-(M),$$

where

$$\sigma^L_+(M) = \left\{\frac{-\alpha k^2}{\lambda + k^2} : \ -k^2 > \lambda\right\}, \ \sigma^L_-(M) = \left\{\frac{-\alpha k^2}{\lambda + k^2} : \ -k^2 < \lambda\right\}.$$

The spaces

$$\mathbf{I}^u = \left\{\eta \in \mathbf{U_K L}_2 : (\eta, \sin kx) = 0, \ -k^2 > \lambda\right\}, \tag{15}$$

$$\mathbf{I}^s = \left\{\eta \in \mathbf{U_K L}_2 : (\eta, \sin kx) = 0, -k^2 < \lambda\right\} \tag{16}$$

are unstable and stable invariant spaces. The dimension of the space $\mathbf{I^u}$ is equal to $m = \max\limits_k\{k : -k^2 > \lambda\}$, and $\operatorname{codim}\mathbf{I}^s = m + \dim\ker L$.

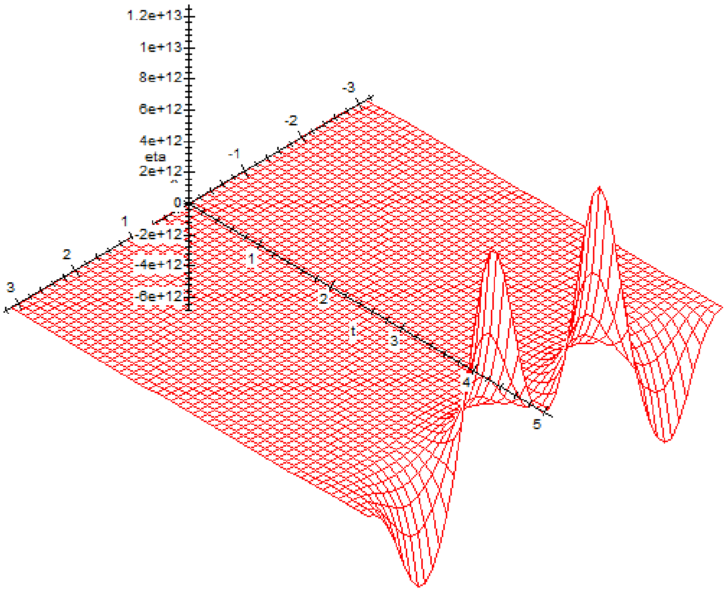

Fig. 1. Unstable solution

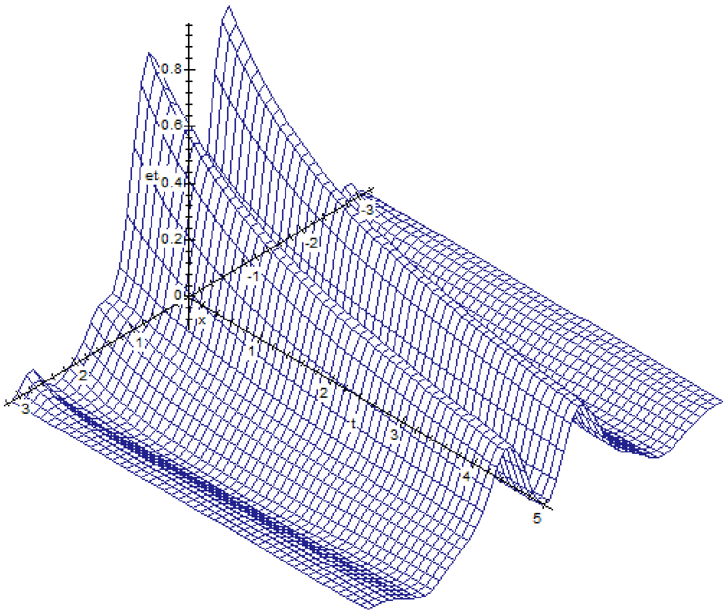

Fig. 2. Stabilization of the solution

Let $B = \alpha(\varepsilon + m^2)\mathbb{I}$, $\varepsilon > 0$ can be chosen arbitrarily small. Then

$$\sigma^{L_r}(M_r + K) = \left\{ \frac{-\alpha k^2 + \alpha(\varepsilon + m^2)}{\lambda + k^2} \right\} < 0.$$

The solution to problem (13) has the form

$$\eta_r = \sum_{k=1}^{m} \exp\left(\frac{-\alpha k^2 + \alpha(\varepsilon + m^2)}{\lambda + k^2} t \right) \left(\sum_{k=1}^{m} \lambda_k \xi_k \sin kx, \sin kx \right) \sin kx.$$

The number of random **K**-values is chosen equal to 5, the parameters $\lambda = -5$, $\alpha = 1.5$ for a numerical experiment. Following the algorithm from [8], we calculate the solutions η_l

and η_r. Calculate $\eta = \eta_l + \eta_r$. Fig. 1 shows a graph of the solution to the equation before stabilization, Fig. 2 shows a graph of the solution of the equation after stabilization at $t \in [0.5]$ and $\xi_1 = 0.23539$, $\xi_2 = -1.07919$, $\xi_3 = -0.73045$, $\xi_4 = 0.86707$, $\xi_5 = 0.15989$.

Conclusion

In the future, we intend to consider the stabilization of semilinear stochastic Sobolev type equations [16, 17].

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СТАБИЛИЗАЦИЯ СТОХАСТИЧЕСКОГО УРАВНЕНИЯ БАРЕНБЛАТТА – ЖЕЛТОВА – КОЧИНОЙ

О. Г. Китаева

Статья посвящена стабилизации решений стохастического уравнения Баренблатта – Желтова – Кочиной. Уравнение Баренблатта – Желтова – Кочиной является моделью фильтрации вязкой жидкости в пористой среде. Это уравнение также моделирует процессы переноса влаги в почве. Рассматривается задача для уравнения Баренблатта – Желтова – Кочиной со случайными начальными данными. Уравнение рассматривается в виде системы уравнений, заданных на устойчивом и неустойчивом инвариантных пространствах. Задача стабилизации состоит в следующем. Требуется найти управляющее воздействие на систему, чтобы ее решения стали асимптотики устойчивыми. Для стохастического уравнения Баренблатта – Желтова – Кочиной найдена такая обратная связь, что замкнутая система была асимптотически устойчивой. Найдены численные решения стохастического уравнения Баренблатта – Желтова – Кочиной и стабилизированного уравнения. Построены графики решений.

Ключевые слова: уравнения соболевского типа; стохастические уравнения; дифференциальные формы; экспоненциальные дихотомии.

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