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NUMERICAL SIMULATION OF NEA-BOTTOM PARTS OF A TORNADO AND A TROPICAL CYCLONE IN A STATIONARY CASE

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The paper presents mathematical and numerical simulation of gas flow in the nea-bottom part of a tornado. The simulation allows calculating the kinetic energy of a whirlwind. The paper describes the statement of a problem and theorems about necessary and sufficient conditions for solution to the considered problem. The work presents a mathematical method for determining gas dynamic parameters, develops a numerical method for determining gas dynamic parameters of a flow and kinetic energy for different tornado types according to Fujita scale. Efficient algorithms for finding radius of inflow and kinetic energy of ascending swirling flows are implemented. A number of computational experiments are conducted to simulate stationary gas current in the bottom part of a flow formed from the Earth's surface. The kinetic energy of a whirlwind is calculated for the presented computational experiments. The values of the kinetic energy of the flow can be used to cope with natural hazards which make the theoretical study of tornado flows highly relevant and useful.

Keywords: ascending swirling flow; gas dynamic parameters of flow; kinetic energy of whirlwind; Fujita scale; computational experiments.

Introduction

Whirlwinds, tornadoes and tropical cyclones are quite common. These flows start on the flat ground in warm weather conditions. The formed tornado is characterized by high flow rate and great destructive power. This leads to the necessity to simulate tornado mathematically taking into account all natural data of a flow. Only wind speed and track width of a tornado can really be measured. Certain gas dynamic parameters allow to determine the kinetic energy of a flow. This data can be used for developing ways to deal with natural phenomena at the initial stages.

Bautin's scientific school considers the flows which are formed at the surface of the Earth at the stage of origin, existence and decay. This paper is devoted to mathematical and numerical simulation of gas flow in the bottom part of a tornado which allows to determine kinetic energy of a flow in the context of the model under consideration. Further, recommendations for tornado detection and possible ways of its suppression are developed [1].

J. Houser and her team from the University of Oklahoma use a new type of mobile Doppler radar system which lead them to conclusion that tornado is formed at the surface of the Earth [2]. Radar data, photographs and videos prove that spins at the surface of the Earth happen before any swirls are detected in the air. All data sets under consideration show that none of the tornado is formed from top to bottom as it is generally considered. Appearing and constant functioning of ascending swirling flows are explained by the existence of natural power which is always presented and has rotational torque. Here we consider the Coriolis force which is caused by the rotation of the Earth around its axis [3].

Fig. 1 presents the surface of the Earth. The point O is the beginning of the Cartesian plane which rotates with the Earth. Here $\Omega = |\mathbf{\Omega}|$ is the absolute value of angular velocity of the Earth rotation; ψ is the latitude at which a tornado happens, $g = \text{const} > 0$ is the magnitude of acceleration of gravity.

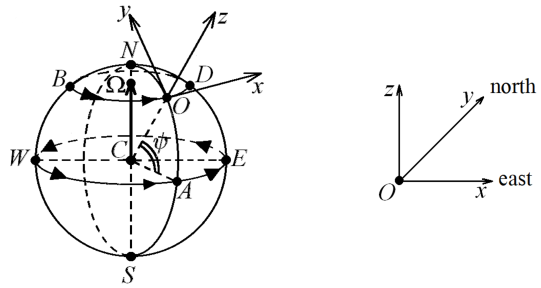


Fig. 1. The point O at the surface of the Earth is the beginning of the coordinate plane xOy.

1. Problem Statement

1.1. Bautin's Mathematical Model

For system of equations (1), the problem is set with initial data (2) on a plane $z = 0$ with the conditions for circumferential and radial gas velocity over a cylinder radius at the point $r = r_{in}$, Fig. 2.

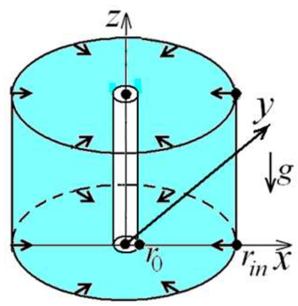


Fig. 2. Vertical cylinder

In system (1), r and φ are the polar radius and angle; $c = \rho^{(\gamma-1)/2}$ is the speed sound of the gas; $\gamma = \text{const} > 1$ is the gas polytropic index in the equation of state $p = \rho^\gamma/\gamma$, where p and ρ are the pressure and gas density, in computational experiments $\gamma = 1.4$; u, v, w are the radial, circumferential and vertical components of the gas velocity vector ; $a = 2\Omega \sin\psi$; $b = 2\Omega\cos\psi$. Further (1), (2) are called the Bautin's model in the stationary case.

$$\begin{cases} uc_r + \frac{v}{r}c_\varphi + wc_z + \dfrac{(\gamma-1)}{2}c\left(u_r + \frac{u}{r} + \frac{v_\varphi}{r} + w_z\right) = 0, \\ uu_r + \frac{v}{r}u_\varphi - \frac{v^2}{r} + wu_z + \dfrac{2}{(\gamma-1)}cc_r = av - bw\cos\varphi, \\ uv_r + \frac{uv}{r} + \frac{v}{r}v_\varphi + wv_z + \dfrac{2}{(\gamma-1)}\frac{c}{r}c_\varphi = -au + bw\sin\varphi, \\ uw_r + \frac{v}{r}w_\varphi + ww_z + \dfrac{2}{(\gamma-1)}cc_z = bu\cos\varphi - bv\sin\varphi - g. \end{cases} \quad (1)$$

$$\begin{cases} c(r,\varphi,z)|_{z=0} = c_0(r,\varphi), \\ u(r,\varphi,z)|_{z=0} = u_0(r,\varphi), \\ v(r,\varphi,z)|_{z=0} = v_0(r,\varphi), \\ w(r,\varphi,z)|_{z=0} = 0. \end{cases} \tag{2}$$

The vertical component of the gas velocity is equal to zero $w = 0$. The plane $z = 0$ is impermeable to the gas. In gas dynamics, such type of surface is called contact. If the system of gas dynamic equations has two or three spatial variables the contact surface is considered to be a characteristic. As soon as gaps are not detected in tornadoes we study isentropic flows. For the isentropic flows the multiplicity of the contact characteristic equals 2. In order to obtain the situation when the stated problem with initial data on the plane $z = 0$ has a single solution, two conditions for radial and circumferential components of gas velocity vector are set at another surface of a cylinder at the point $r = r_{in}$, $r_{in} = \text{const} > 0$.

In detail we consider the case when the radial component of the velocity vector u is a small negative number $u_{in} = \text{const} < 0$ and the circumferential component of the gas velocity vector v equals zero. From the point of view of gas dynamics it means that constant radial inflow with no spin is set through the surface of a cylinder into its inner area.

1.2. Necessary and Sufficient Conditions for Unique Mathematic Solvability

Theorem 1. *In a stationary case, for solvability of Bautin's problem* (1), (2), *it is necessary to fulfill the following conditions:*

$$\begin{cases} u_0 u_{0r} + \dfrac{v_0 u_{0\varphi}}{r} - \dfrac{v_0^2}{r} + \dfrac{2}{(\gamma-1)} c_0 c_{0r} = a v_0, \\ u_0 v_{0r} + \dfrac{u_0 v_0}{r} + \dfrac{v_0 v_{0\varphi}}{r} + \dfrac{2}{(\gamma-1)} \dfrac{c_0 c_{0\varphi}}{r} = -a u_0. \end{cases} \tag{3}$$

These equations are sufficient conditions for solvability in case when the existence of problem solution is expected (1), (2). The coefficients c_0, u_0, v_0 describe the flat flow and at the same time they are terms of the series which set the spatial flow at the area of the impermeable plane $z = 0$.

Theorem 2. *If conditions* (4) *are satisfied:*

$$\begin{cases} u(r,\varphi,z)|_{r=r_{in}} = u^o(\varphi,z), \\ v(r,\varphi,z)|_{r=r_{in}} = v^o(\varphi,z); \quad r_{in} = const > 0, \end{cases} \tag{4}$$

and the analytical functions $u^o(\varphi,z)$, $v^o(\varphi,z)$, are agreed with conditions (2) *at the values $z = 0$, $r = r_{in}$:*

$$\begin{cases} u_0(r,\varphi)|_{r=r_{in}} = u^o(\varphi,z)|_{z=0}, \\ v_0(r,\varphi)|_{r=r_{in}} = v^o(\varphi,z)|_{z=0}. \end{cases} \tag{5}$$

then problem (1), (2), (4), (5) *has the unique mathematic solvability due to analytical input and implementation of the necessary solvability conditions. It is defined in a neighbourhood of the point M_0 with the coordinates ($r = r_{in}$, $\varphi = \varphi_0$, $z = 0$), where φ_0 is any value on the line segment* $[0, 2\pi]$.

It is proved that under these additional conditions the regarded problem has a unique local analytical solution which can be represented as an endless power series in z converged in the neighborhood of the plane $z = 0$. The coefficients of the series depend on the independent variables r, φ, z.

$$\mathbf{U}(r,\varphi,z) = \sum_{k=0}^{\infty} \mathbf{U}_k(r,\varphi)\frac{z^k}{k!}, \qquad \mathbf{U}_k(r,\varphi) = \left.\frac{\partial^k \mathbf{U}(r,\varphi,z)}{\partial z^k}\right|_{z=0}. \tag{6}$$

The solution to problem (1), (2), (4), (5) describes the gas flow in the neighbourhood of the impermeable plane $z = 0$ when the defined inflow is executed through the surface of a vertical cylinder with the non-zero radius $r = r_{in}$.

The constructed segments of series (6) have the following form:

$$\begin{aligned} c &= c_0 + c_1 z + c_2\frac{z^2}{2} + c_3\frac{z^3}{6} + c_4\frac{z^4}{24}, & u &= u_0 + u_1 z + u_2\frac{z^2}{2} + u_3\frac{z^3}{6}, \\ v &= v_0 + v_1 z + v_2\frac{z^2}{2} + v_3\frac{z^3}{6}, & w &= w_0 + w_1 z + w_2\frac{z^2}{2} + w_3\frac{z^3}{6} + w_4\frac{z^4}{24}. \end{aligned} \tag{7}$$

Regarding the particular case of Bautin's model for radial flows, it is sufficient to add low of conservation of mass in the differential form for spatial flow with the parameters c_0, u_0, v_0 positioned at the plane $z = 0$.

$$\begin{cases} u_0 c_{0r} + \dfrac{v_0 c_{0\varphi}}{r} + \dfrac{(\gamma-1)c_0}{2}\left(u_{0r} + \dfrac{u_0}{r} + \dfrac{v_{0\varphi}}{r}\right) = 0, \\ u_0 u_{0r} + \dfrac{v_0 u_{0\varphi}}{r} - \dfrac{v_0^2}{r} + \dfrac{2}{(\gamma-1)} c_0 c_{0r} = a v_0, \\ u_0 v_{0r} + \dfrac{u_0 v_0}{r} + \dfrac{v_0 v_{0\varphi}}{r} + \dfrac{2}{(\gamma-1)}\dfrac{c_0 c_{0\varphi}}{r} = -a u_0. \end{cases} \tag{8}$$

1.3. Bautin's Model for Radial Flows

For Bautin's model in the stationary case, consider the particular case for radial flows, then model (8) has the unique solution [3]:

$$c_0(r) = \left[\frac{A}{r u_0(r)}\right]^{(\gamma-1)/2},$$

$$F(r,u_0) \equiv \frac{2}{(\gamma-1)}\left(\frac{A}{r u_0}\right)^{(\gamma-1)} + u_0^2 - B + \frac{a^2 r_{in}^4}{4r^2} + \frac{a^2}{4}r^2 = 0,$$

$$v_0(r) = \frac{a(r_{in}^2 - r^2)}{2r}, \quad v(r_{in}) = 0, \quad v_0(r) \neq 0 \ \text{при} \ r \neq r_{in},$$

at the same time, the form of the coefficient c_1 is determined as follows:

$$c_1 = c_1(r,\varphi) = c_{10}(r) + c_{11}(r)\cos\varphi + c_{12}(r)\sin\varphi,$$

where

$$c_{10} = -g\frac{(\gamma-1)}{2}\frac{1}{c_0(r)}, \qquad c_{11} = b\frac{(\gamma-1)}{2}\frac{u_0(r)}{c_0(r)}, \qquad c_{12} = -b\frac{(\gamma-1)}{2}\frac{v_0(r)}{c_0(r)}.$$

1.4. Mathematical Method for Obtaining Gas Dynamic Parameters of Flow

Stage 1. The first coefficients of the series are determined. Bautin's model is numerically solved in the stationary case for radial flows. The solutions for the functions c_0, u_0, v_0 are defined according to the earlier determined inflow radius r_{in}.

Stage 2. The second coefficients of the series are determined after differentiation of system (1) by the variable z, when $z = 0$ and $\mathbf{U}_0$, c_1, w_1 are determined at Stage 1.

Stage 3. The third coefficients of the series are determined after double differentiation of system (1) by the variable z, when $z = 0$ and $\mathbf{U}_0$, $\mathbf{U}_1$, c_2, w_2 are determined at Stage 2.

Stage 4. The forth coefficients of the series are determined after triple differentiation of system (1) by the variable z, when $z = 0$ and $\mathbf{U}_0$, $\mathbf{U}_1$, $\mathbf{U}_2$, c_2, w_2, c_3, w_3 are determined at Stage 3.

Mathematical models obtained at each stage are numerically solved by the Runge - Kutta method.

Theorem 3. *Suppose that the coefficients* c_0, u_0, v_0, c_1, w_1 *of series* (6) *are uniquely defined then the following coefficients can be represented as final trigonometric sums of type* (9)*, the coefficients in front of harmonics depend on* r*:*

$$\begin{aligned} w_k(r,\varphi) &= w_{k,0}(r) + \sum_{j=1}^{k-1} \left[w_{k,2j-1}(r)\cos(j\varphi) + w_{k,2j}(r)\sin(j\varphi) \right], \quad k = 2,3,4; \\ f_k(r,\varphi) &= f_{k,0}(r) + \sum_{j=1}^{k} \left[f_{k,2j-1}(r)\cos(j\varphi) + f_{k,2j}(r)\sin(j\varphi) \right], \quad k = 1,2,3; \end{aligned} \tag{9}$$

where $f = \{c, u, v\}$.

After numerical determination of gas dynamic parameters of a flow, the kinetic energy of a whirlwind for the moving gas in the area

$$(D): \quad \{r_0 \le r \le r_{in}; \quad 0 \le \varphi \le 2\pi; \quad 0 \le z \le h\}$$

is defined by the formula:

$$W = \frac{1}{2} \iiint\limits_{(D)} \rho(x,y,z)\vec{V}^2 dxdydz = \frac{1}{2} \int_0^h \left\{ \int_0^{2\pi} \left[\int_{r_0}^{r_{in}} \rho(r,\varphi)\vec{V}^2(r,\varphi) r dr \right] d\varphi \right\} dz, \tag{10}$$

where $\vec{V}$ is the gas velocity vector. Analytical procedure for determining the kinetic energy is described in detail in [4].

2. Numerical Methods of Determining Gas Dynamic Parameters of Flow, Radius of Inflow and Kinetic Energy of Whirlwind

2.1. Numerical Method of Determining First Coefficients of Series and Radius of Inflow

In order to determine the first coefficients of a series, we perform numerical calculation of Bautin's model in a stationary case for radial flows. The solution for the functions c_0, u_0, v_0 is determined by the earlier found radius of inflow r_{in}.

The program “Tornado, definition of the radius of an inflow” is applied to determine r_{in} under fulfillment of the following condition at a given flow radius at the point $r = r_0$: $|\vec{V}| = \sqrt{u^2 + v^2}$. Values of radial and circumferential velocities are presented at the point $r = r_0$ in dimensional units m/sec. Fujita scale [5] is applied as reference information for determining gas dynamic parameters. The scale contains data about field observation of tornado: the radius r_0 of tornado flow, the wind velocity $|\vec{V}|$. The grade of a tornado is selected according to the proposed list and the latitude where the ascending flow functions.

The efficiency of numerical method for determining radius of an inflow lies in the reduction of number of operations at the area $[r_0, r_{in}]$ to determine appropriate values for radial and circumferential components of the gas velocity vector. Runge-Kutta algorithm is executed and provides more accurate results (by the determined ϵ) which coincide with the data of natural tornadoes.

2.2. Numerical Method for Calculating Gas Dynamic Parameters and Kinetic Energy of Flow

The numerical method is based on the mathematical method of calculation of gas dynamic parameters of ascending swirling flow. At each stage, Bautin’s model is differentiated and transited to the systems of ordinary differential equations which are numerically solved by the Runge-Kutta method of the 4-th order. The kinetic energy of a flow is numerically determined after obtaining the values of gas dynamic parameters of a flow considering several coefficients of a series.

3. Results of Computational Experiments

3.1. Computational Experiments for Radial Flows of Bautin Model

3.1.1. Gas Sound Speed

Fig. 3 shows the gas dynamic distribution surfaces for the gas sound speed c_0 in the Cartesian coordinate system xOy depicted on the gas particle trajectories for the tornado F10 and the tropical cyclone of moderate intensity.

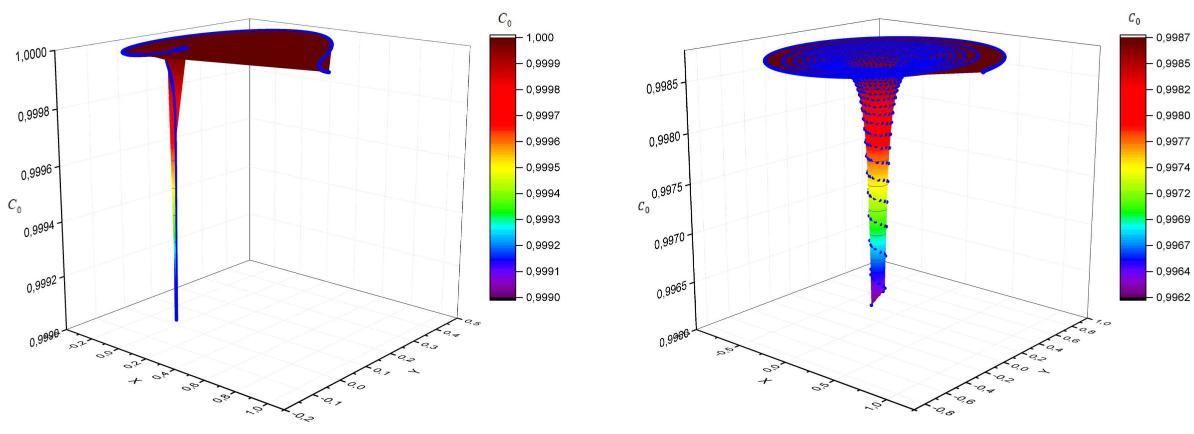


Fig. 3. Class F10 tornado and tropical cyclone, c_0 on the particle trajectory

For the gas dynamic parameter, i.e. gas sound speed c_0, computational experiments within the model confirm that there is a low-pressure area near the runoff radius in the center of the vortex flow. As the tornado class increases, the pressure decreases by a larger value.

3.1.2. Radial Speed of Gas

Fig. 4 shows the surfaces of gas dynamic distributions for the radial gas velocity u_0 in the Cartesian coordinate system xOy with highlighted current lines for the tornado F10 and a tropical cyclone of moderate intensity. The negative sign of the radial component

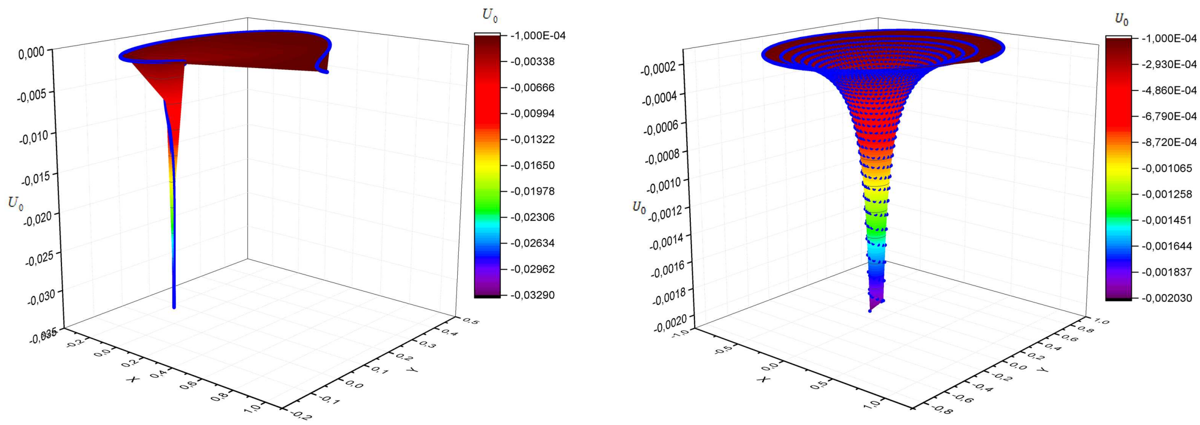


Fig. 4. F10 class tornado and tropical cyclone, u_0 on the particle trajectory

of the velocity vector indicates that the radial gas velocity is directed towards the center, towards the tornado sink r_0. A low-pressure area contributes to the direction of the gas toward the center of the flow. As the tornado class increases, the value of the radial gas velocity decreases.

3.1.3. Circumferential Gas Velocity

Fig. 5 shows gas dynamic distribution surfaces for the circumferential gas velocity v_0 in the Cartesian coordinate system xOy with the current lines shown for the tornado F10 and a tropical cyclone of moderate intensity.

A positive value of the circumferential velocity indicates that the gas twist is counterclockwise, which corresponds to the location of the flow in the northern hemisphere. As the tornado class increases, the circumferential velocity of the gas increases.

The results of computational experiments for the first coefficients of the series demonstrated that as the tornado class increases, the gas sound speed, density and pressure values at the tornado runoff radius r_0 decrease. For the Bautin model in the stationary case for radial currents, computational experiments confirmed the presence of a low-pressure area. As the tornado class increases, the magnitude of the circumferential velocity increases and the radial gas velocity near the runoff radius r_0 decreases.

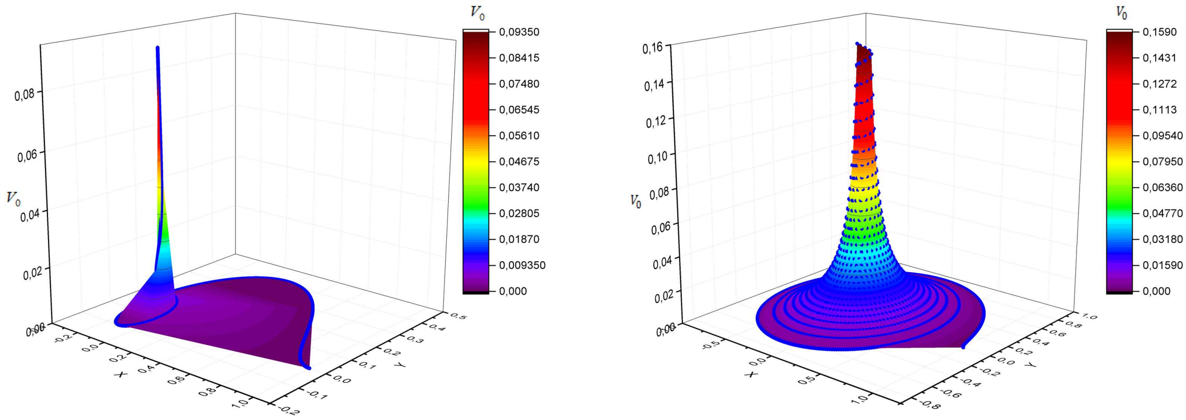


Fig. 5. F10 class tornado and tropical cyclone, v_0 on the particle trajectory

3.2. Computational Experiments for Bautin Model when Considering Two Coefficients of Power Series

Further, we present the results of computational experiments of gasdynamic parameters taking into account the first two coefficients of a series (with index 0 and 1). Below, we describe comparative computational experiments for c, u, v when varying the height of the bottom part of the tornado z. The computational experiments including the first coefficients of the series are thereafter referred to as basic computational experiments. We also consider the results of computational experiments of gasdynamic parameters at the height $z = 10$, i.e. the meteorological standard height of the wind vane [6] taking into account all roughnesses of the Earth surface, and $z = 15$, i.e. the height without taking into account the dependence on the surface.

3.2.1. Sound Velocity of Gas

In Fig. 6, the current lines show the distribution of the gas sound velocity for the tornado class F10 and a tropical cyclone of moderate intensity. The dimensionless value of the runoff radius $r_0 = 0.00305$ for F10 is 8 m. The dimensionless value of the runoff radius $r_0 = 0.05$ for a tropical cyclone of moderate intensity is 3650 m. The value of gas sound velocity for the two coefficients of series $c_0 + c_1 z$ is denoted by C_{h_1} at $z = 10$ m and C_{h_2} at $z = 15$ m. Taking into account additional terms and varying the height of the nea-bottom part, some change in the value of the gas sound velocity is noted. The relative error between the basic computational experiments and the computational experiments at the wind vane height for the main tornado classes does not exceed 0.02%. The relative error between the baseline computational experiments and the computational experiments for $z = 15$ does not exceed 0.027%.

3.2.2. Radial Gas Velocity

In Fig. 7, the current lines show the radial gas velocity for the F10 class tornado and a medium-intensity tropical cyclone. The value of the radial gas velocity for the two coefficients of the series $u_0 + u_1 z$ is denoted by U_{h_1} at $z = 10$ m and U_{h_2} at $z = 15$ m.

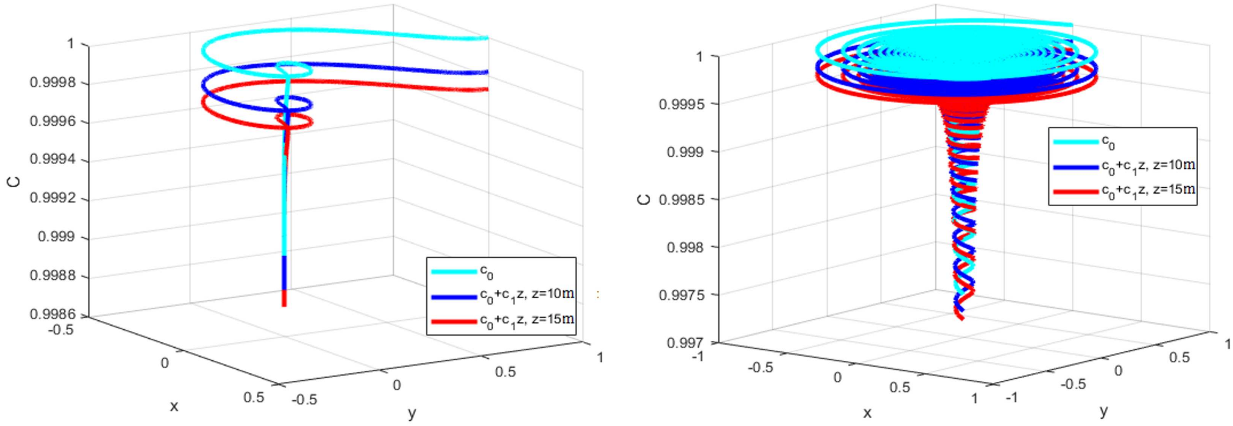


Fig. 6. F10 class tornado and tropical cyclone: c_0, C_{h_1}, C_{h_2}

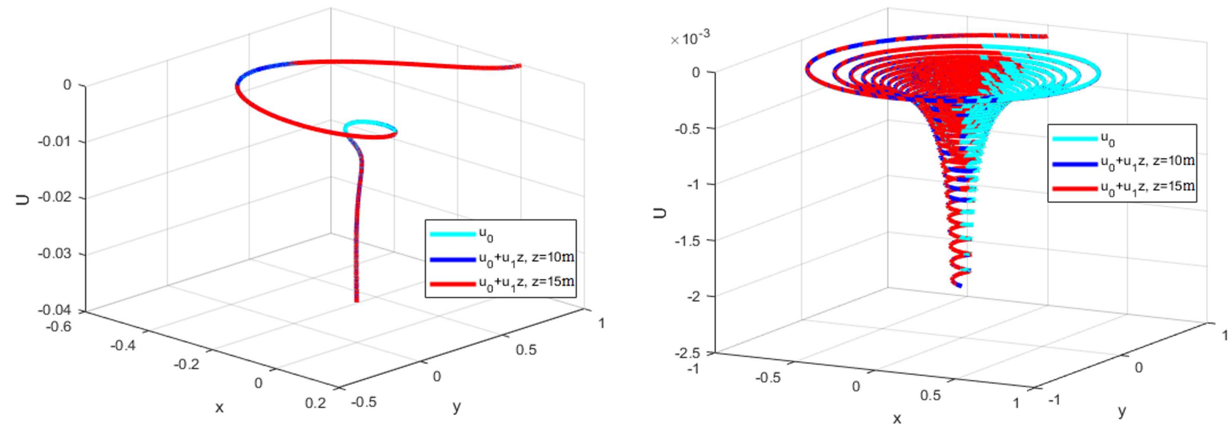


Fig. 7. F10 class tornado and tropical cyclone: u_0, U_{h_1}, U_{h_2}

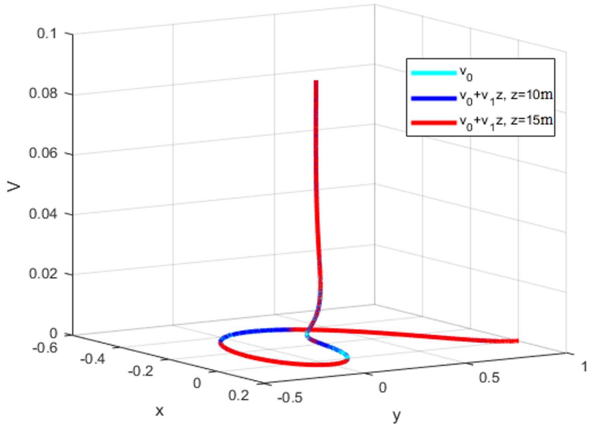
Taking into account additional summands and change of the near-bottom height, there is an insignificant change in the radial gas velocity relative to the basic computational experiments.

The relative error between the baseline computational experiments and the computational experiments for the wind vane height for the main tornado classes does not exceed 0.08%. The relative error between the basic computational experiments and the computational experiments for $z = 15$ m also does not exceed 0.08%.

3.2.3. Circumferential Gas Velocity

Fig. 8 shows the circumferential gas velocity for the F10 class tornado and a medium-intensity tropical cyclone on the particle trajectories. The value of the circumferential gas velocity for the two coefficients of the series $v_0 + v_1 z$ is denoted by V_{h_1} at $z = 10$ m and V_{h_2} at $z = 15$ m.

Taking into account the additional terms and the change in height of near-bottom part of a tornado, the value of the circumferential gas velocity changes insignificantly. The relative error between the basic computational experiments and the computational



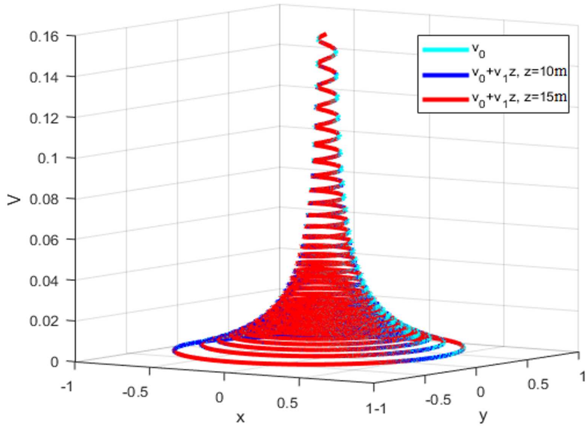


Fig. 8. F10 class tornado and tropical cyclone: v_0, V_{h_1}, V_{h_2}

experiments for the wind vane height for the corresponding tornado class does not exceed 0.01%. The relative error between the basic computational experiments and the computational experiments for $z = 15$ m for the corresponding tornado class does not exceed 0.04%.

The flow parameters and the difference between them are compared. For wind vane heights of 10 m and 15 m, the parameter values in the solutions found for large tornado classes are almost exactly the same. Studies were carried out for the height of the near-bottom part of the flow equal to 50 m [7]. As the height increases, the discrepancy between the segments of the series turned out to be quite significant.

3.3. Calculation Experiments for Bautin's Model Considering Three Coefficients of Power Series

3.3.1. Sound Velocity of Gas

Fig. 9 shows the values of the gas sound velocity for the tornado F10 and a tropical cyclone of moderate intensity (for further details, consider F60). The height of the near-bottom part is three meters.

Computational experiments considering two row coefficients at the height $z = 3$ m are denoted by C_{h_3}, those considering three row coefficients at the height $z = 3$ m are denoted by C_{h_4}.

For the gas sound velocity and the near-bottom height of three meters, the relative error between the basic computational experiments and the computational experiments at the wind vane height for the main tornado classes does not exceed 0.006%. The obtained solution falls in the convergence area near the plane $z = 0$.

3.3.2. Radial Gas Velocity

Fig. 10 shows the value of the radial gas velocity parameter. The height of the near-bottom region is three meters. Computational experiments considering two series coefficients at $z = 3$ m are denoted by U_{h_3}, those considering three series coefficients at $z = 3$ m are denoted by U_{h_4}. For the radial gas velocity and a ground level height of

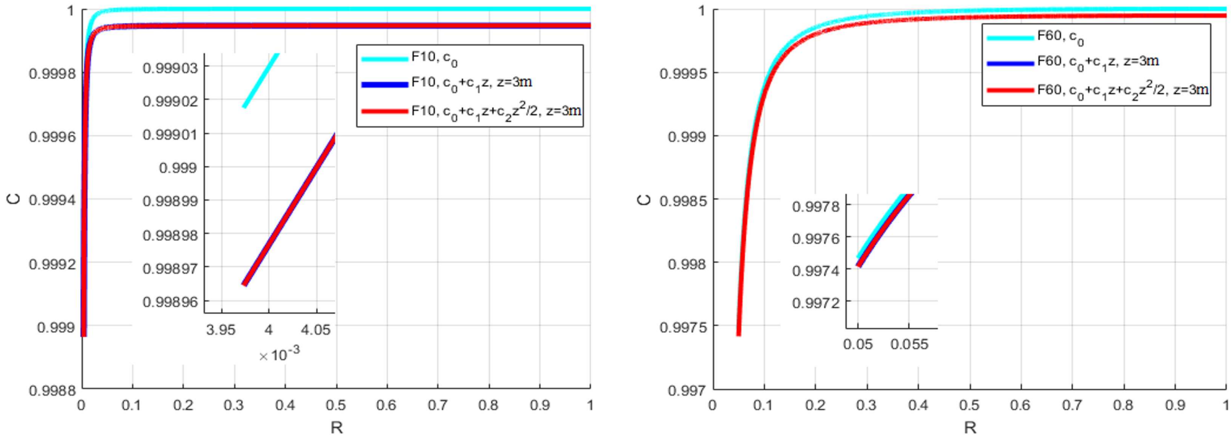


Fig. 9. F10 class tornado and tropical cyclone: c_0, C_{h_3}, C_{h_4}

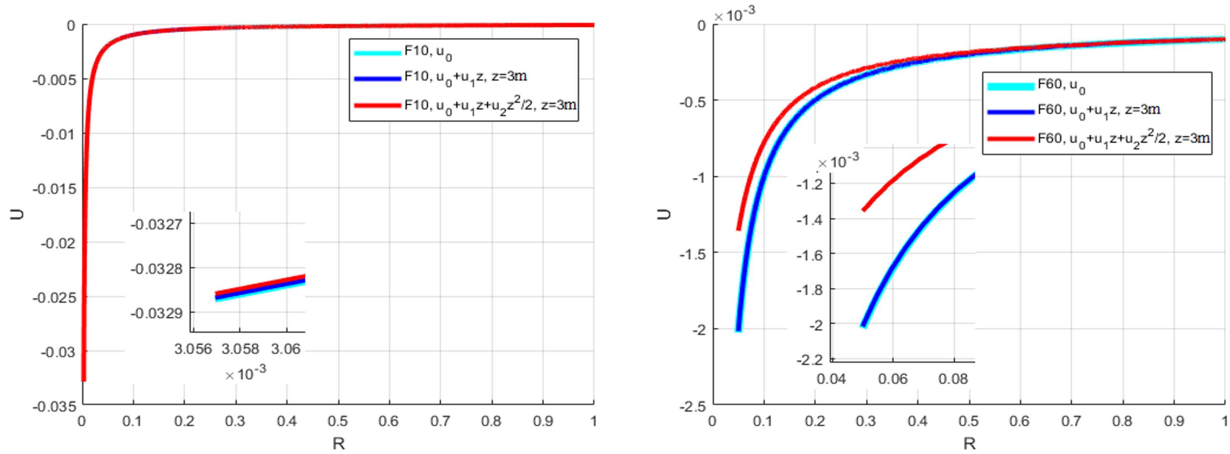


Fig. 10. F10 class tornado and tropical cyclone: u_0, U_{h_3}, U_{h_4}

three meters, the relative error between the three series coefficient calculations and the basic calculations begins to increase with increasing tornado class. The radial gas velocity begins to decrease significantly with increasing tornado class, causing the divergence in this parameter to be most pronounced. The largest value of the radial gas velocity divergence is for the tropical cyclone, the most destructive classes represented in the computational experiments. For the tropical cyclone, the relative error is almost 33%.

3.3.3. Circumferential Gas Velocity

Fig. 11 shows the peripheral gas velocity for F10 and a tropical cyclone of moderate intensity. The height of the near-bottom region is three meters. Computational experiments considering two series coefficients at the height $z = 3$ m are denoted by V_{h_3}, those considering three series coefficients at $z = 3$ m are denoted by V_{h_4}.

For the circumferential gas velocity and the near-bottom height value equal to three meters, the relative error between the calculation experiments considering three series coefficients and the basic calculations does not exceed 0.006%. This indicates that the obtained approximate solutions are within the convergence range of the series.

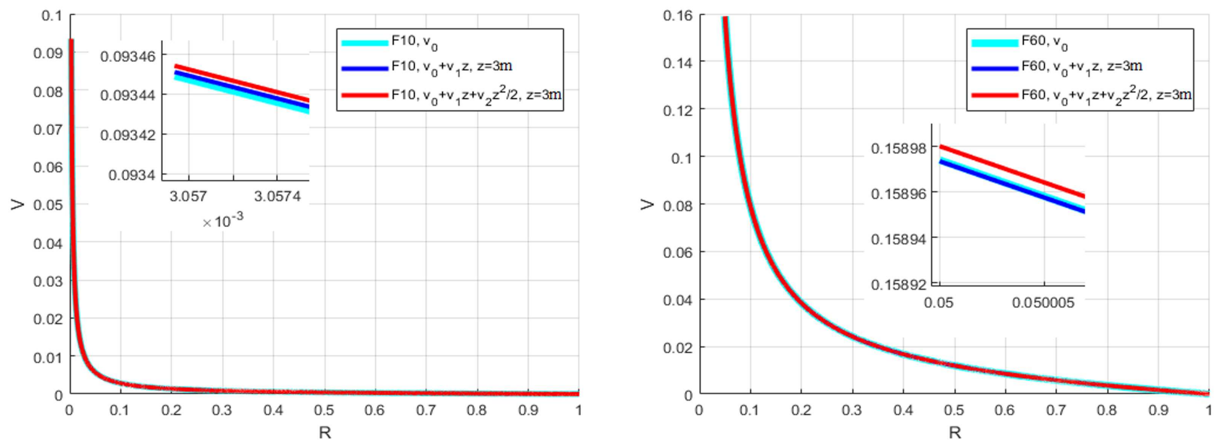


Fig. 11. F10 class tornado and tropical cyclone: v_0, V_{h_3}, V_{h_4}

3.3.4. Kinetic Energy

Table 1 shows the results of computational experiments for the kinetic energy of the three parts of the flow for all classes of tornadoes from the extended Fujita table. For basic calculation experiments, the kinetic energy of the circumferential part of the flow accounts for the majority of the kinetic energy from 50% to 99% as the tornado class increases, the kinetic energy of the radial part of the flow has the highest value at the minimum tornado class. For a tornado to acquire destructive power, the ratio of the kinetic energy of the circumferential part of the flow to the total part must be half or more.

Table 1

Kinetic energy for basic calculation, the width is $\pi/6$

Tornado class	W_U MJ	W_V MJ	W MJ	W_V/W
F00	2.560E-01	2.540E-01	5.100E-01	0.4980
F01	8.220E-01	2.658E+00	3.480E+00	0.7638
F10	1.787E+00	1.268E+01	1.447E+01	0.8763
F11	4.615E+00	8.746E+01	9.207E+01	0.9499
F20	8.690E+00	3.150E+02	3.237E+02	0.9731
F21	2.040E+01	1.809E+03	1.830E+03	0.9885
F30	3.572E+01	5.655E+03	5.690E+03	0.9939
F31	8.049E+01	3.008E+04	3.016E+04	0.9974
F40	1.447E+02	9.988E+04	1.000E+05	0.9988
F41	2.811E+02	3.919E+05	3.922E+05	0.9992
F50	4.873E+02	1.211E+06	1.212E+06	0.9992
F60	7.205E+02	3.417E+06	3.418E+06	0.9997
F51	1.026E+03	5.658E+06	5.659E+06	0.9998
F52	1.636E+03	1.472E+07	1.472E+07	0.9999

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ПРИДОННЫХ ЧАСТЕЙ ТОРНАДО И ТРОПИЧЕСКОГО ЦИКЛОНА В СТАЦИОНАРНОМ СЛУЧАЕ

О. В. Опрышко

В данной статье представлено математическое и численное моделирование течения газа в придонной области потока, которое позволяет определить кинетическую энергию вихря. Описана постановка задачи, теоремы о необходимых и достаточных условиях разрешимости поставленной задачи. Представлен математический метод определения газодинамических параметров, разработан численный метод определения газодинамических параметров потока и кинетической энергии для разных классов торнадо из таблицы Фудзиты. Реализованы эффективные алгоритмы нахождения радиуса притока и кинетической энергии восходящих закрученных потоков. Проведены вычислительные эксперименты для моделирования стационарного течения газа в придонной части потока при формировании торнадо от поверхности Земли. Для представленных вычислительных экспериментов рассчитана кинетическая энергия вихря. Информацию о значении кинетической энергии потока целесообразно применить для борьбы с природным явлением, поэтому теоретическое исследование потоков типа торнадо актуально и востребовано на сегодняшний день.

Ключевые слова: восходящий закрученный поток; газодинамические параметры потока; кинетическая энергия вихря; шкала Фудзиты; вычислительные эксперименты.

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