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SOLUTION OF STOCHASTIC NON-AUTONOMOUS CHEN – GURTIN MODEL WITH MULTIPOINT INITIAL-FINAL CONDITION

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In this paper the authors investigate the solvability of a non-autonomous Chen – Gurtin model with a multipoint initial-final condition in the space of stochastic **K**-processes. To do this, we first consider the solvability of a multipoint initial-final problem for a non-autonomous Sobolev type equation in the case when the resolving family is a strongly continuous semiflow of operators. The Chen – Gurtin model refers to non-classical models of mathematical physics. Recall that non-classical are those models of mathematical physics whose representations in the form of equations or systems of partial differential equations do not fit within one of the classical types: elliptic, parabolic or hyperbolic. For this model, multipoint initial-final conditions, which generalizing the Cauchy and Showalter-Sidorov conditions, are considered.

Keywords: Sobolev type equations; resolving C_0-semiflow of operators; relatively spectral projectors; Nelson – Gliklikh derivative; space of stochastic **K***-processes.*

Introduction

Let $\Pi \subset \mathbb{C}^m$ be a bounded domain with boundary $\partial\Omega$ of C^∞ class. The parameters λ, $d \in \mathbb{R}$. On interval $(\tau_0, \tau_n) \subset \mathbb{R}$ consider the modified Chen – Gurtin equation [1]

$$(\lambda - \Delta)u_t(x,t) = \nu(t)(\Delta u(x,t) - id\Delta^2 u(x,t)) + g(x,t), \qquad (x,t) \in \Omega \times (\tau_0, \tau_n), \quad (1)$$

$$\Delta u(x,t) = u(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (\tau_0, \tau_n), \quad (2)$$

which allows us to take into account the change in system parameters over time and describes the process of thermal conductivity with «two temperatures» [1], as well as in the special case at $d = 0$ the dynamics of fluid pressure in a fractured-porous medium [2] and the process of moisture transfer in the soil [3]. In addition, if, in the case of $\lambda = 0$, we take Δu as the searched function, then a linearized classical Ginzburg –Landau equation can be obtained from this equation, taking into account diffraction and the absence of diffusion action [4].

The problem (1), (2) is reduced to a non-autonomous equation of the Sobolev type [5, 6] of the form

$$L\dot{u}(t) = a(t)Mu(t) + g(t), \quad (3)$$

where operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ (i.e. linear and continuous) and $M \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$ (i.e. linear, closed and densely defined in $\mathfrak{U}$) defined in some Banach spaces $\mathfrak{U}$, $\mathfrak{F}$. Equation (3) refers to equations unresolved with respect to the highest derivative (see more in [7]), since the operator with the derivative on the left side can be zero for some parameter values. Note that the non-autonomous model (1), (2) is described by the equation (3) which relating to the relatively p-radial case [5, 8], i.e. the operators L and M generates a strongly continuous resolving semigroup for a homogeneous autonomous equation (3) ($a(t) \equiv 1$). Note that such a class of equations was firstly considered in [8].

Fix $u_j \in \mathfrak{U}$, $j = \overline{0,n}$ and take $\tau_0 = 0$ и $\tau_j \in \mathbb{R}_+$ such as $\tau_{j-1} < \tau_j$, $j = \overline{1,n}$. Let us add the equation (3) with *a multipoint initial-final condition* [9]

$$\lim_{t \to \tau_0+} P_0(u(t) - u_0) = 0, \qquad P_j(u(\tau_j) - u_j) = 0, \quad j = \overline{1,n}, \tag{4}$$

where operators P_j are relatively spectral projectors [9]. Condition (4) generalizes the classical conditions of Cauchy and Showalter – Sidorov [10, 11].

Previously, the problem (3), (4) was considered in the deterministic case [12], and the stochastic equation (3) with Cauchy and Showalter–Sidorov conditions investigated in [13]. This article we consider a multipoint initial-final condition for the Chen – Gurtin model in the stochastic case. The article, in addition to the introduction, conclusion and list of references, contains four parts. The first part provides information on the solvability of classical problems for abstract deterministic Sobolev-type equations with relatively p-radial operators. The second part describes the space of stochastic **K**-processes, after which the third part describes the solution of the stochastic problem (3), (4). And in the last part, solutions of the stochastic non-autonomous Chen – Gurtin model with a multipoint initial-final condition are constructed.

1. Non-autonomous Sobolev-type Equations with Relatively p-Radial Operators

Let $\mathfrak{U}$ and $\mathfrak{F}$ are Banach spaces. Operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ (i.e. linear and continuous) and $M \in \mathcal{Cl}(\mathfrak{U};\mathfrak{F})$ (i.e. linear, closed and densely defined in $\mathfrak{U}$). By [5, 8] we call sets $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{F};\mathfrak{U})\}$ и $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ *L-resolvent set* and *L-spectrum* of operator M correspondingly. In [5, 8] It is shown that the L-resolvent set is open, and therefore the L-spectrum of the operator M is always closed. The L-resolvent set of the operator M can be an empty set, for example, if $\ker L \cap \ker M \neq \{0\}$. Assuming that $\rho^L(M) \neq \emptyset$, we introduce the operator-functions of a complex variable $(\mu L - M)^{-1}$, $R^L_\mu(M) = (\mu L - M)^{-1}L$, $L^L_\mu(M) = L(\mu L - M)^{-1}$ with the domain $\rho^L(M)$, which we call *L-resolvent, right* and *left L-resolvent* of operator M correspondingly. Similarly, the function operator $(p+1)$ of a complex variable type

$$R^L_{(\lambda,p)}(M) = \prod_{k=0}^{p} R^L_{\lambda_k}(M), \quad L^L_{(\lambda,p)}(M) = \prod_{k=0}^{p} L^L_{\lambda_k}(M), \qquad \lambda_k \in \rho^L(M) \quad (k = \overline{0,p})$$

with the domain $[\rho^L(M)]^{p+1}$ we call *right* and *left* (L,p)*-resolvent* of operator M correspondingly. Also, due to the results of [5, 8], all the presented operator-functions are holomorphic in their domain of definition.

Definition 1. [5] Operator M we call *p-radial relatively to the operator* L (for short (L,p)*-radial*) if

(i) $\exists \alpha \in \mathbb{R} : (\alpha, +\infty) \subset \rho^L(M)$;

(ii) $\exists K > 0 \ \forall \mu = (\mu_0, \mu_1, \ldots, \mu_p) \in (\alpha, +\infty)^{p+1} \ \forall n \in \mathbb{N}$

$$\max\{\|(R^L_{(\mu,p)}(M))^n\|_{\mathcal{L}(\mathfrak{U})}, \|(L^L_{(\mu,p)}(M))^n\|_{\mathcal{L}(\mathfrak{F})}\} \leq \frac{K}{\prod\limits_{k=0}^{p} (\mu_k - \alpha)^n}.$$

Denote $\mathfrak{U}^0 = \ker R^L_{(\mu,p)}(M)$, $\mathfrak{F}^0 = \ker L^L_{(\mu,p)}(M)$, $L_0 = L\Big|_{\mathfrak{U}^0}$, $M_0 = M\Big|_{\operatorname{dom} M\cap\mathfrak{U}^0}$. By symbols $\mathfrak{U}^1$ and $\mathfrak{F}^1$ we denote the closure of lineals $\operatorname{im} R^L_{(\mu,p)}(M)$ and $\operatorname{im} L^L_{(\mu,p)}(M)$. Note that under the condition of (L,p)-radiality of the operator M, there exists an operator $M_0^{-1} \in \mathcal{L}(\mathfrak{F}^0;\mathfrak{U}^0)$.

One-parameter family of operators $U^\bullet : \overline{\mathbb{R}}_+ \to \mathcal{L}(\mathfrak{U})$ we call *a strongly continuous semigroup* (*C_0-semigroup*) of operators if

(i) $U^sU^t = U^{s+t}$ $\forall s,t \in \overline{\mathbb{R}}_+$;

(ii) U^t strongly continuous for $t > 0$ and exists $\lim\limits_{t\to 0+} U^t u = u$ for all u from some lineal dense in $\mathfrak{U}$.

Semigroup $\{U^t \in \mathcal{L}(\mathfrak{U}) : t \in \overline{\mathbb{R}}_+\}$ we call *an exponentially bounded with constants C* and α if $\exists C > 0$ $\exists \alpha \in \mathbb{R}$ $\forall t \in \overline{\mathbb{R}}_+$ $\quad \|U^t\|_{\mathcal{L}(\mathfrak{U})} \le Ce^{\alpha t}$.

Theorem 1. [5] *Let the operator M be an (L,p)-radial ($p \in \{0\}\cup\mathbb{N}$). Then there exist degenerate C_0-semigroup $\{U^t \in \mathcal{L}(\tilde{\mathfrak{U}}) : t \in \overline{\mathbb{R}}_+\}$ и $\{F^t \in \mathcal{L}(\tilde{\mathfrak{F}}) : t \in \overline{\mathbb{R}}_+\}$ which are exponentially bounded with constants K, α from Definition 1. Here by $\tilde{\mathfrak{U}}$ and $\tilde{\mathfrak{F}}$ denote the closure of lineals $\mathfrak{U}^0 \dot{+} \operatorname{im} R^L_{(\mu,p)}(M)$ and $\mathfrak{F}^0 \dot{+} \operatorname{im} L^L_{(\mu,p)}(M)$ by norm of spaces $\mathfrak{U}$ and $\mathfrak{F}$.*

Remark 1. [5] Units of semigroups $\{U^t \in \mathcal{L}(\tilde{\mathfrak{U}}) : t \in \overline{\mathbb{R}}_+\}$ and $\{F^t \in \mathcal{L}(\tilde{\mathfrak{F}}) : t \in \overline{\mathbb{R}}_+\}$ are projectors $P = \lim\limits_{t\to 0+} U^t$ and $Q = \lim\limits_{t\to 0+} F^t$ along $\mathfrak{U}^0$ or $\mathfrak{F}^0$ on subspace $\mathfrak{U}^1$ or $\mathfrak{F}^1$ correspondingly.

Introduce the condition

$$\mathfrak{U} = \mathfrak{U}^0 \oplus \mathfrak{U}^1, \qquad \mathfrak{F} = \mathfrak{F}^0 \oplus \mathfrak{F}^1. \tag{5}$$

By symbols L_k and M_k denote the restrictions of L or M on subspace $\mathfrak{U}^k$ or $\operatorname{dom} M_k \cap \mathfrak{U}^k$ ($k = 0,1$). And introduce one more condition as

$$\text{there exists an operator } \; L_1^{-1} \in \mathcal{L}(\mathfrak{F}^1;\mathfrak{U}^1). \tag{6}$$

Remark 2. A sufficient condition for fulfilling the conditions (5) and (6) is, for example, the strong (L,p)-radiality of the operator M ($p \in \mathbb{N}_0$) [5, Chapter 2]. Here and further $\mathbb{N}_0 \equiv \{0\}\cup\mathbb{N}$.

Theorem 2. [5] *Let the operator M be an (L,p)-radial ($p \in \mathbb{N}_0$) and conditions (5), (6) are fulfilled. Then*

(i) $L_k \in \mathcal{L}(\mathfrak{U}^k;\mathfrak{F}^k)$, $\;M_k \in \mathcal{C}l(\mathfrak{U}^k;\mathfrak{F}^k)$, $\operatorname{dom} M_k = \operatorname{dom} M \cap \mathfrak{U}^k$, $k = 0,1$;

(ii) *an operator $H = M_0^{-1}L_0 \in \mathcal{L}(\mathfrak{F}^0)$ is nilpotent degree not higher than p;*

(iii) *an operator $S = L_1^{-1}M_1 \in \mathcal{C}l(\mathfrak{U}^1)$ generates C_0-semigroup of resolving operators for equation $\dot{u} = Su$.*

On interval $(\tau,T] \subset \overline{\mathbb{R}}_+$ we consider the Cauchy problem

$$\lim_{t\to\tau+} u(t) = u_\tau \tag{7}$$

for a homogeneous non-autonomous equation

$$L\dot{u}(t) = a(t)Mu(t), \tag{8}$$

where function $a : [\tau,T] \to \mathbb{R}_+$ will be defined below.

Definition 2. [5] *A solution* of equation (8) we call the vector-function $u \in C([\tau, T]; \mathfrak{U}) \cap C^1((\tau, T]; \mathfrak{U})$ which is satisfying this equation on $(\tau, T]$. The solution of equation (8) we call *a solution of Cauchy problem* (7), (8), if it additionally satisfies the condition (7).

Definition 3. [5] Closed set $\mathfrak{P} \subset \mathfrak{U}$ are called *a phase space* of equation (8) if

(i) any solution $u(t)$ of equation (8) in $\mathfrak{P}$ for all t;

(ii) for arbitrary u_τ from $\mathfrak{P}$ there exists the unique solution of the Cauchy problem (7) for equation (8).

Theorem 3. [12] *Let the operator M be an (L, p)-radial ($p \in \mathbb{N}_0$), the conditions* (5), (6) *are fulfilled and the function $a \in C(\mathbb{R}, \mathbb{R}_+)$. Then the phase space of equation* (8) *is a subspace $\mathfrak{U}^1$.*

Remark 3. Under the conditions of Theorem 3 for an arbitrary $u_\tau \in \mathfrak{U}^1$ there is a unique solution to the Cauchy problem (7), (8) of the form $u(t) = U(t, \tau)u_\tau$. If $u_\tau \notin \mathfrak{U}^1$, then the Cauchy problem is fundamentally unsolvable [10]. Here $U(t, \tau)$ is a semiflow of resolving operators of the equation (8) (more see in [12]).

2. Sobolev-type Equations in the Space of Complex-valued Stochastic K-Processes

Let $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ be a complete probability space with probability measure $\mathbf{P}$ associated with the σ-algebra $\mathcal{A}$ of subsets of the set Ω, and $\mathbb{C}$ is a set of complex numbers endowed with a Borel σ-algebra. The measurable mapping $\xi : \Omega \to \mathbb{C}$ is called a *random variable*. A set of random variables with zero mathematical expectation and finite variance forms a Hilbert space $\mathbf{L}_2 = \mathbf{L}_2(\Omega; \mathbb{C}) = \{\xi : \mathbf{E}\xi = 0,\ \mathbf{D}\xi < +\infty\}$ with a scalar product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\overline{\xi_2}$ and the norm $\|\xi\|^2_{\mathbf{L}_2} = \mathbf{D}\xi$.

Let us take the set $\mathfrak{I} \subset \mathbb{R}$ and consider two mappings: $f : \mathfrak{I} \to \mathbf{L}_2$, which matches each $t \in \mathfrak{I}$ with a random variable $\xi \in \mathbf{L}_2$, and $g : \mathbf{L}_2 \times \Omega \to \mathbb{C}$, which each pair (ξ, ω) matches the point $\xi(\omega) \in \mathbb{C}$. Mapping $\eta : \mathfrak{I} \times \Omega \to \mathbb{C}$ (or what is the same $\eta : \mathfrak{I} \to \mathbf{L}_2$), having the form $\eta = \eta(t, \omega) = g(f(t), \omega)$, we call *a complex-valued stochastic process*. A stochastic process $\eta = \eta(t)$ is continuous on the interval $\mathfrak{I}$ if all its trajectories are continuous (almost surely) (i.e. with a.a. (almost all) $\omega \in \mathcal{A}$ trajectories $\eta(\cdot, \omega)$ are continuous functions). The set of continuous stochastic processes $\eta : \mathfrak{I} \to \mathbf{L}_2$ forms a Banach space with a standard sup-norm, which we denote by the symbol $C(\mathfrak{I}; \mathbf{L}_2)$.

Let $\mathfrak{H}$ be a complex separable Hilbert space with an orthonormal basis $\{\varphi_k\}$, a monotone numerical sequence $\mathbf{K} = \{\lambda_k\} \subset \mathbb{R}_+$ is, what is $\sum\limits_{k=1}^{\infty} \lambda_k^2 < +\infty$, and the sequence $\{\xi_k\} = \xi_k(\omega) \subset \mathbf{L}_2$ of random variables such, what is $\|\xi_k\|_{\mathbf{L}_2} \leq \mathrm{C}$ with some constant $\max \mathrm{C} \in \mathbb{R}_+$ and for all $k \in \mathbb{N}$. Let's construct *$\mathfrak{H}$-valued random $\mathbf{K}$-value* $\xi(\omega) = \sum\limits_{k=1}^{\infty} \lambda_k \xi_k(\omega)\varphi_k$. Completion of the linear span of the set $\{\lambda_k \xi_k \varphi_k\}$ according to the norm $\|\eta\|_{\mathfrak{H}_\mathbf{K}\mathbf{L}_2} = \left(\sum\limits_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\xi_k\right)^{1/2}$ is called *space of $\mathfrak{H}$-valued random $\mathbf{K}$-value* and are denoted by $\mathfrak{H}_\mathbf{K}\mathbf{L}_2$. That is clear that the space $\mathfrak{H}_\mathbf{K}\mathbf{L}_2$ is a Hilbert space and the random $\mathbf{K}$-value $\xi = \xi(\omega) \in \mathfrak{H}_\mathbf{K}\mathbf{L}_2$.

Mapping $\eta : \mathfrak{I} \to \mathfrak{H}_{\mathbf{K}}\mathbf{L}_2$, which is defined as $\eta(t) = \sum\limits_{k=1}^{\infty} \lambda_k \eta_k(t)\varphi_k$, where $\{\eta_k\} \subset C(\mathfrak{I}; \mathbf{L}_2)$, are called *a continuous* $\mathfrak{H}$*-valued stochastic* $\mathbf{K}$*-process*, if the series in this equality converges uniformly according to the norm $\|\cdot\|_{\mathfrak{H}_{\mathbf{K}}\mathbf{L}_2}$ on any compact in $\mathfrak{I}$ and process trajectories $\eta = \eta(t)$ are continuous almost surely. The $\mathfrak{H}$-valued stochastic $\mathbf{K}$-process is *differentiating by Nelson – Gliklikh* [13, 14], if the series in $\overset{\circ}{\eta}(t) = \sum\limits_{k=1}^{\infty} \lambda_k \overset{\circ}{\eta}_k(t)\varphi_k$ converges uniformly according to the norm $\|\cdot\|_{\mathfrak{H}_{\mathbf{K}}\mathbf{L}_2}$ on any compact in $\mathfrak{I}$ and process trajectories $\overset{\circ}{\eta}=\overset{\circ}{\eta}(t)$ are continuous almost surely. Here $\overset{\circ}{\eta}_k$ is a Nelson – Gliklikh derivation of stochastic process $\eta_k : \mathfrak{I} \to \mathbf{L}_2$. By the symbol $C(\mathfrak{I}; \mathfrak{H}_{\mathbf{K}}\mathbf{L}_2)$ we denote the space of continuous $\mathfrak{H}$-valued stochastic $\mathbf{K}$-processes and analogously by the symbol $C^{\ell}(\mathfrak{I}; \mathfrak{H}_{\mathbf{K}}\mathbf{L}_2)$ we denote the space $\mathfrak{H}$-valued stochastic $\mathbf{K}$-processes, which are continuous differentiable in Nelson – Gliklikh sense up to and including the order of $\ell \in \mathbb{N}$.

Now let $\mathfrak{U}$ and $\mathfrak{F}$ be complex separable Hilbert spaces with an orthonormal basis $\{\varphi_k\}$ and $\{\psi_k\}$ correspondingly. By symbols $\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ and $\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$ we denote the Hilbert spaces, which are completion of linear span of *random* $\mathbf{K}$*-values*

$\xi = \sum\limits_{k=1}^{\infty} \lambda_k \xi_k \varphi_k$ ($\xi_k \in \mathbf{L}_2$) and $\zeta = \sum\limits_{k=1}^{\infty} \mu_k \zeta_k \psi_k$ ($\zeta_k \in \mathbf{L}_2$) according to the norm $\|\eta\|^2_{\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2} = \sum\limits_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\xi_k$ and $\|\omega\|^2_{\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2} = \sum\limits_{k=1}^{\infty} \mu_k^2 \mathbf{D}\zeta_k$ correspondingly. Note that in different spaces ($\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ and $\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$) the sequence $\mathbf{K}$ can be different ($\mathbf{K} = \{\lambda_k\}$ in $\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ and $\mathbf{K} = \{\mu_k\}$ in $\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$), however, all sequences marked with the symbol $\mathbf{K}$ must be monotonic and summable with a square. All results, generally speaking, will be true for different sequences of $\{\lambda_k\}$ and $\{\mu_k\}$, but for simplicity's sake we will limit ourselves to the case $\lambda_k = \mu_k$.

Let $A : \mathfrak{U} \to \mathfrak{F}$ be a linear operator. By the formula $A\xi = \sum\limits_{k=1}^{\infty} \lambda_k \zeta_k A\varphi_k$ we define linear operator $A : \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2 \to \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$, moreover, if the series on the right side of this equality converges (in the metric $\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$), then $\xi \in \operatorname{dom} A$, and if it diverges, then $\xi \notin \operatorname{dom} A$. Traditionally , we define spaces of linear continuous operators $\mathcal{L}(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$ and linear closed densely defined operators $\mathcal{C}l(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$.

Lemma 1. (i) *The operator* $A \in \mathcal{L}(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$ *exactly when* $A \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$.
(ii) *The operator* $A \in \mathcal{C}l(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$ *exactly when* $A \in \mathcal{C}l(\mathfrak{U}; \mathfrak{F})$.

Lemma 2. (i) *The operator* $M \in \mathcal{C}l(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$ *be a p-radial relatively to the operator* $L \in \mathcal{L}(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$ *exactly when the operator* $M \in \mathcal{C}l(\mathfrak{U}; \mathfrak{F})$ *be a p-radial relatively to the operator* $L \in \mathcal{L}(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$.
(ii) *L-spectrums of operators M match in both cases.*
(iii) *Conditions* (5), (6) *are fulfilled in spaces* $\mathfrak{U}$, $\mathfrak{F}$ *exactly when they are fulfilled in spaces* $\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$, $\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$.

The proofs of these lemmas are based on the obvious equality

$$\|A\xi\|_{\mathfrak{F}} \leq \sum_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\xi_k \|A\varphi_k\|^2_{\mathfrak{F}} \leq \text{const} \sum_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\xi_k = \text{const}\, \|\xi\|_{\mathfrak{U}}.$$

So, let the operators $L \in \mathcal{L}(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$ and $M \in \mathcal{C}l(\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2; \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2)$ such that the operator M is (L, p)-radial ($p \in \{0\} \cup \mathbb{N}$), and conditions (5), (6) are fulfilled. Consider a linear evolutionary stochastic equation

$$L \overset{\circ}{\eta} = M\eta. \tag{9}$$

The process $\eta \in \mathbf{C}^1(\mathbb{R}; \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2)$ we call *solution of equation* (9), if when substituting it in (9) it turns this equation into an identity almost surely. Solution $\eta = \eta(t)$ of equation (9) we call *solution of Cauchy problem*

$$\lim_{t \to \tau+} (\eta(t) - \eta_\tau) = 0 \tag{10}$$

if it is fulfilled for this function and some random $\mathbf{K}$-value $\eta_\tau \in \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$. Analogously, the *solution of the Showalter – Sidorov problem*

$$\lim_{t \to \tau+} P(\eta(t) - \eta_\tau) = 0 \tag{11}$$

for equation (9) is defined.

Definition 4. Set $\mathfrak{P}_{\mathbf{K}}\mathbf{L}_2 \subset \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ we call *stochastic phase space* of equation (9) if

(i) almost surely each trajectory of the solution $\eta = \eta(t)$ of the equation (9) in $\mathfrak{P}_{\mathbf{K}}\mathbf{L}_2$, i.e. $\eta(t) \in \mathfrak{P}_{\mathbf{K}}\mathbf{L}_2, t \in \mathbb{R}$, for almost all trajectories;

(ii) for almost all $\eta_\tau \in \mathfrak{P}_{\mathbf{K}}\mathbf{L}_2$ there is a unique solution to the Cauchy problem (10) for equation (9).

Theorem 4. *Let the operator M be an (L, p)-radial ($p \in \mathbb{N}_0$) and condition* (5), (6) *are fulfilled. Then the stochastic phase space for equation* (9) *is a space $\mathfrak{U}^{\mathbf{1}}_{\mathbf{K}}\mathbf{L}_2$.*

Remark 4. As above in the remark 3, we note that the Cauchy problem (9), (10) is solvable only for $\eta_\tau \in \mathfrak{U}^{\mathbf{1}}_{\mathbf{K}}\mathbf{L}_2$, but the Showalter – Sidorov problem(9), (11) is solvable for any $\eta_\tau \in \mathfrak{U}^{\mathbf{1}}_{\mathbf{K}}\mathbf{L}_2$. This solution has the form $\eta(t) = U^{t-\tau}P\eta_\tau$, where $U^{t-\tau}$ semigroup of solving operators for a stochastic equation (9), which exists by virtue of Theorem 1 with taking into account Lemmas 1 and 2, and the projector P from Remark 1 with taking into account the same lemmas (see more in [13]).

3. Solution of a Multipoint Initial-Final Problem for a Non-autonomous Evolutionary Equation

Let $\mathfrak{U}$, $\mathfrak{F}$ be complex separable Hilbert spaces, operators $L \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$, $M \in \mathcal{C}l(\mathfrak{U}; \mathfrak{F})$ such that the operator M is (L, p)-radial ($p \in \mathbb{N}_0$), and the conditions (5), (6) are satisfied. On the interval $(\tau_0, \tau_n]$ consider an inhomogeneous stochastic equation

$$L \overset{\circ}{\eta}(t) = a(t)M\eta(t) + \varpi(t), \tag{12}$$

where $\eta = \eta(t)$ is the searched stochastic $\mathbf{K}$ process, $t \in (\tau_0, \tau_n]$, and $\varpi : (\tau_0, \tau_n) \to \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$ is a given one. Introduce an additional condition

$$\left\{ \begin{array}{l} \sigma^L(M) = \bigcup\limits_{j=0}^{n} \sigma^L_j(M),\ n \in \mathbb{N},\ \text{such that } \sigma^L_j(M) \neq \emptyset \text{ contained in a limited} \\ \text{area } D_j \subset \mathbb{C} \text{ with a piecewise smooth border } \partial D_j = \gamma_j \subset \mathbb{C}. \text{ Moreover,} \\ \overline{D_j} \cap \sigma^L_0(M) = \emptyset \text{ and } \overline{D_k} \cap \overline{D_l} = \emptyset \text{ for all } j, k, l = \overline{1, n}, k \neq l. \end{array} \right. \tag{13}$$

Due to the holomorphism of relative resolvents the condition (13) guarantied that there exist projectors, that have the form [9]

$$P_j = \frac{1}{2\pi i}\int_{\gamma_j} R^L_\mu(M)d\mu \in \mathcal{L}(\mathfrak{U}), \quad Q_j = \frac{1}{2\pi i}\int_{\gamma_j} L^L_\mu(M)d\mu \in \mathcal{L}(\mathfrak{F}), \quad j = \overline{1,n},$$

$P_0 = P - \sum\limits_{j=1}^{n} P_j$, $\quad Q_0 = Q - \sum\limits_{j=1}^{n} Q_j$, where P and Q are defined in Remark 1.

Introduce the subspaces $\mathfrak{U}^{1j} = \operatorname{im} P_j$, $\mathfrak{F}^{1j} = \operatorname{im} Q_j$, $j = \overline{0,n}$. Then by the contraction we have

$$\mathfrak{U}^1 = \bigoplus_{j=0}^{n} \mathfrak{U}^{1j} \qquad \text{and} \qquad \mathfrak{F}^1 = \bigoplus_{j=0}^{n} \mathfrak{F}^{1j}.$$

By symbols L_{1j} we denote the restrictions of operator L on $\mathfrak{U}^{1j}$, $j = \overline{0,n}$, and by symbols M_{1j} denote the restrictions of operator M on $\operatorname{dom} M \cap \mathfrak{U}^{1j}$, $j = \overline{0,n}$. That is clear that $P_j\varphi \in \operatorname{dom} M$ for $\varphi \in \operatorname{dom} M$, that's why domain $\operatorname{dom} M_{1j} = \operatorname{dom} M \cap \mathfrak{U}^{1j}$ is dense in $\mathfrak{U}^{1j}$, $j = \overline{0,n}$.

Theorem 5. (Generalized spectral theorem) [9]. *Let operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ and $M \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$ such that operator M is an (L,p)-radial* ($p \in \mathbb{N}_0$)*, and condition* (5), (6), (13) *are satisfied. Then*

(i) *operators $L_{1j} \in \mathcal{L}(\mathfrak{U}^{1j};\mathfrak{F}^{1j})$, $M_{1j} \in \mathcal{C}l(\mathfrak{U}^{1j};\mathfrak{F}^{1j})$, $j = \overline{0,n}$;*

(ii) *there exist operatprs $L^{-1}_{1j} \in \mathcal{L}(\mathfrak{F}^{1j};\mathfrak{U}^{1j})$, $j = \overline{0,n}$.*

Fix $\eta_j \in \mathfrak{U}_\mathbf{K}\mathbf{L}_2$ ($j = \overline{0,n}$) and take $\tau_0 = 0$ and $\tau_j \in \mathbb{R}_+$ such that $\tau_{j-1} < \tau_j$, $j = \overline{1,n}$. For them, consider a multipoint initial-final problem [9, 11, 12]

$$\lim_{t\to\tau_0+} P_0(\eta(t) - \eta_0) = 0, \qquad P_j(\eta(\tau_j) - \eta_j) = 0, \quad j = \overline{1,n} \tag{14}$$

for equation (12). Let's act on the equation (12) sequentially by projectors $\mathbb{I} - Q$ and Q_j, $j = \overline{0,n}$, and we get a system

$$\begin{cases} H \overset{\circ}{\eta}{}^0(t) = a(t)\eta^0(t) + M_0^{-1}\varpi^0(t), \\ \overset{\circ}{\eta}{}^{1j}(t) = a(t)S_j\eta^{1j}(t) + L_{1j}^{-1}\varpi^{1j}(t), \ j = \overline{0,n}, \end{cases}$$

which equivalent to this equation. Here operator $H = M_0^{-1}L_0 \in \mathcal{L}(\mathfrak{U}^0)$ is nilpotent degree not higher than $p \in \{0\} \cup \mathbb{N}$, and operators $S_j = L^{-1}_{1j}M_{1j} \in \mathcal{C}l(\mathfrak{U}^{1j})$ such that $\sigma(S_j) = \sigma^L_j(M)$; $\varpi^0 = (\mathbb{I} - Q)\varpi$, $\varpi^{1j} = Q_j\varpi$, $\eta^0 = (\mathbb{I} - P)\eta$, $\eta^{1j} = P_j\eta$, $j = \overline{0,n}$.

Definition 5. The process $\eta \in C([\tau_0,\tau_n];\mathfrak{U}_\mathbf{K}\mathbf{L}_2) \cap C^1((\tau_0,\tau_n];\mathfrak{U}_\mathbf{K}\mathbf{L}_2)$ we call *solution of equation* (12), if it almost surely turns it into an identity on (τ_0,τ_n). Solution $\eta = \eta(t)$ of equation (12) are called *solution of a multipoint initial-final problem* (12), (14), if it almost surely satisfies the conditions (14).

Theorem 6. *Let the scalar function $a \in C^{p+1}([\tau_0,\tau_n];\mathbb{R}_+)$, the operator M is an (L,p)-radial* ($p \in \mathbb{N}_0$)*, and conditions* (5), (6), (13) *are satisfied. Then for arbitrary random* $\mathbf{K}$*-value $\eta_j \in \mathfrak{U}_\mathbf{K}\mathbf{L}_2$ ($j = \overline{0,n}$) that are independent from $\mathfrak{U}$-valued* $\mathbf{K}$*-processes $L_1^{-1}Q\varpi : (\tau_0,\tau_n) \to \mathfrak{U}_\mathbf{K}\mathbf{L}_2$ such that $Q\varpi \in C((\tau_0,\tau_n),\mathfrak{F}^\mathbf{1}_\mathbf{K}\mathbf{L}_2)$ and $(\mathbb{I}_\mathfrak{F} - Q)\varpi \in C^{p+1}((\tau_0,\tau_n),\mathfrak{F}^\mathbf{0}_\mathbf{K}\mathbf{L}_2)$,* *there exists almost surely the unique solution* $\eta \in C([\tau_0, \tau_n]; \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2) \cap C^1((\tau_0, \tau_n]; \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2)$ *of problem* (12), (14), *which has the form*

$$\eta(t) = -\sum_{k=0}^{p} H^k M_0^{-1}\left(\frac{1}{a(t)}\frac{\textcircled{d}}{dt}\right)^k \frac{\varpi^0(t)}{a(t)} + \sum_{j=0}^{n}\left(U(t,\tau_j)P_j\eta_j - \int_t^{\tau_j} U(t,s)P_j L_{1j}^{-1}\varpi^{1j}(s)ds\right).$$

Here the symbol $\dfrac{\textcircled{d}}{dt}$ *denote the Nelson – Gliklikh derivation, and operator* $U(t,\tau)$ *is from the Remark* 4.

The statement of this theorem is a consequence of the corresponding theorem from [12] taking into account Lemmas 1 and 2.

4. Solutions of a Multipoint Initial-Final Problem for a Non-autonomous Chen – Gurtin Model with Complex Coefficients

Let $\Pi \subset \mathbb{C}^m$ be a bounded domain with boundary $\partial\Pi$ of class C^∞. Transform the stochastic equation

$$(\lambda - \Delta)\overset{\circ}{\eta}(x,t) = \nu(t)(\Delta - id\Delta^2)\eta(x,t) + \varpi(x,t), \qquad (x,t) \in \Pi \times (\tau_0, \tau_n) \tag{15}$$

with parameters $\lambda, d \in \mathbb{R}$ and boundary conditions

$$\Delta\eta(x,t) = \eta(x,t) = 0, \qquad (x,t) \in \partial\Pi \times (\tau_0, \tau_n) \tag{16}$$

to equation (12). To do this let's take functional spaces $\mathfrak{U} = W_2^2(\Pi) \cap \overset{\circ}{W}{}_2^1(\Pi)$, $\mathfrak{F} = L_2(\Pi)$, where $W_2^2(\Pi)$, $\overset{\circ}{W}{}_2^1(\Pi)$ are Sobolev spaces. Define operators $L \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ and $M \in \mathcal{C}l(\mathfrak{U}; \mathfrak{F})$ by formulas $L = \lambda - \Delta$ and $M = \Delta - id\Delta^2$ with domain of definition $\operatorname{dom} M = \{u \in W_2^4(\Pi) : u(x) = \Delta u(x) = 0,\ x \in \partial\Pi\}$.

Lemma 3. [13] *For arbitrary* $\lambda \in \mathbb{R} \setminus \{0\}$, $d \in \mathbb{R}$, *the operator* M *is an* $(L, 0)$*-radial and conditions* (5), (6) *are satisfied.*

The statement of the Lemma 3 follows from [13] taking into account the Remark 2. Denote by $\{\lambda_k\}$ the sequence of eigenvalues of a homogeneous Dirichlet problem for the operator Laplace Δ in the domain of Π. Let the sequence $\{\lambda_k\}$ be numbered by non-increment, taking into account multiplicity. Denote by $\{\phi_k\}$ orthonormal (in the sense of $L_2(\Pi)$) a sequence of corresponding eigenfunctions $\phi_k \in C^\infty(\Pi)$, $k \in \mathbb{N}$. The L-spectrum of operator M has the form

$$\sigma^L(M) = \left\{\mu_k = \frac{\lambda_k - id\lambda_k^2}{\lambda - \lambda_k},\ k \in \mathbb{N} \setminus \{l : \lambda_l = \lambda\}\right\}.$$

In order for the contour $\gamma \subset \mathbb{C}$ to satisfy the condition (13), it is enough to take $\gamma_j = \partial D_j$ ($j = \overline{0,n}$) so that $\bigcup_{j=0}^{n} D_j \supset \sigma^L(M)$ and each of the area D_j ($j = \overline{1,n}$) contained a finite number of points from $\sigma^L(M)$. Denote $\sigma^L_j(M) = \sigma^L(M) \cap D_j$ and construct projectors

$$P_j = \sum_{k:\mu_k \in \sigma^L_j(M)} \langle \cdot, \phi_k \rangle \, \phi_k, \qquad j = \overline{0, n}.$$

Fix $\eta_j \in \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$, $j = \overline{0, n}$ and take $\tau_0 = 0$ and $\tau_j \in \mathbb{R}_+$ such that $\tau_{j-1} < \tau_j$, $j = \overline{1, n}$. In the cylinder $\Pi \times (\tau_0, \tau_n)$ find a solution to the equation (15) satisfying the boundary condition (16) and the conditions

$$P_j(\eta(x, \tau_j) - \eta_j(x)) = \sum_{k:\mu_k \in \sigma^L_j(M)} \langle (\eta(\tau_j) - \eta_j), \phi_k \rangle \, \phi_k(x) = 0, \quad j = \overline{0, n} \tag{17}$$

of multipoint initial-final problem.

The following statement follows from Theorem 6 and Lemma 3.

Theorem 7. *For an arbitrary $\lambda \in \mathbb{R} \setminus \{0\}$, $d \in \mathbb{R}$, $\nu \in C^1((\tau_0, \tau_n); \mathbb{R}_+)$, and for any random $\mathbf{K}$-value $\eta_j \in \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ ($j = \overline{0, n}$) which independent from $\mathfrak{U}$-valued $\mathbf{K}$-process $L_1^{-1}Q\varpi : (\tau_0, \tau_n) \to \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ such that the conditions*

$$(\mathbb{I}_{\mathfrak{F}} - Q)\varpi \in C^1((\tau_0, \tau_n); \mathfrak{F}^0), \quad Q_j\varpi \in C((\tau_0, \tau_n); \mathfrak{F}^{1j}), \quad j = \overline{0, n}$$

are satisfied, there exist almost surely the unique solution $\eta \in C^1((\tau_0, \tau_n); \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2)$ of multipoint initial-final problem (15), (16), (17), *which has the form*

$$\eta(x, t) = -\sum_{\lambda_k = \lambda} \frac{\langle \varpi(t), \phi_k \rangle}{\nu(t)(\lambda_k - id\lambda_k^2)} \, \phi_k(x) +$$

$$+ \sum_{j=0}^{n} \left[\sum_{k:\mu_k \in \sigma^L_j(M)} \exp\left(\frac{\lambda_k - id\lambda_k^2}{\lambda - \lambda_k} \int_{\tau_j}^{t} \nu(\zeta) d\zeta \right) \langle \eta_j, \phi_k \rangle \phi_k(x) + \right.$$

$$\left. + \sum_{k:\mu_k \in \sigma^L_j(M)} \int_{\tau_j}^{t} \exp\left(\frac{\lambda_k - id\lambda_k^2}{\lambda - \lambda_k} \int_{s}^{t} \nu(\zeta) d\zeta \right) \frac{\langle \varpi(s), \phi_k \rangle}{\lambda_k - id\lambda_k^2} \, \phi_k(x) \, ds \right].$$

Conclusion

In the future, it is planned to develop methods for numerical research of the problems discussed in this article.

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РЕШЕНИЕ СТОХАСТИЧЕСКОЙ НЕАВТОНОМНОЙ МОДЕЛИ ЧЕНА – ГЕТИНА С МНОГОТОЧЕЧНЫМ НАЧАЛЬНО-КОНЕЧНЫМ УСЛОВИЕМ

М. А. Сагадеева, С. А. Загребина

В статье исследуется разрешимость неавтономной модели Чена – Гетина с многоточечным начальным-конечным условием в пространстве стохастических **K**-процессов. Для этого сначала рассматривается разрешимость многоточечной начально-конечной задачи для неавтономного уравнения соболевского в случае когда разрешающим семейством является сильно непрерывный полупоток операторов. Модель Чена –Гетина относится к неклассическим моделям математической физики. Напомним, что неклассическими называют те модели математической физики, чьи представления в виде уравнений или систем уравнений в частных производных не укладываются в рамках одного из классических типов – эллиптического, параболического или гиперболического. Для этой модели рассмотрены многоточечные начально-конечные условия, обобщающие условия Коши и Шоуолтера – Сидорова.

Ключевые слова: уравнения соболевского типа; разрешающие C_0-полупотоки операторов; относительно спектральные проекторы; производная Нельсона – Гликлиха; пространство стохастических **K***-процессов.*

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