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STRUCTURAL SYNTHESIS OF THE COMPUTER VISION AND ITS PARAMETRIC IDENTIFICATION WITH STATISTICAL ESTIMATION OF VARIATIONAL PARAMETERS

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The article presents the results of a study of the problem on structural synthesis of a computer vision and its parametric identification. We present the main classes of procedural transformations of images and describe a method for constructing a model of a vision system for measuring a deterministic feature using procedural and parametric transformations of modified descriptive image algebras. The function of measuring a deterministic quantitative characteristic of an object of observation is defined as a superposition of a procedural and parametric descriptive algebraic image transformation scheme. The parametric identification problem is formulated in the form of a nonlinear optimization problem. We give examples of the use of the described method in measuring the area of objects of various nature. Recommendations are formulated for the statistical processing of the values of the measured characteristic in order to clarify its value.

Keywords: image transformation; feature measurement; computer vision model; structural synthesis of model; parametric identification of model; nonlinear optimization; confidence interval.

Introduction

A modern computer vision (CV) is a software and hardware complex for processing, analyzing (measuring) and recognizing images. Various technical and applied problems are solved at the hardware and software levels. In order to increase the speed of operation, some of the problems of the software level can be implemented at the hardware level. Such problems include, for example, the problems of filtering, segmentation of primitives, restoration and improvement of images, etc. [1, 2].

The work [3] presents the results of the work performed on the study of the current state and dynamics of changes in the ice topography of the islands of the Russian Arctic carried out in 2013-2014 within the research of the Moscow State University of Geodesy and Cartography (MIIGAiK) scientific school in the Arctic. As the main source of spatial data to obtain up-to-date areal information, we use radar images obtained by Sentinel-1A spacecraft, while in order to assess vertical changes, we use altimetry measures data from Cryosat-2 and ICEsat-1 spacecraft. The ERS-2 radar data have a spatial resolution of 30 m at VV polarization. Sentinel-1A radar data have a spatial resolution of 50 m at HH and HV polarizations. For the multispectral optical data of the Landsat-5 satellite, a red channel with a spatial resolution of 30 m is chosen. The authors conclude that the applied approach to calculating areas corresponds to the scales of 1:100000 and 1:20000 of the used topographic maps.

The mathematical apparatus proposed in this paper allows to develop models for processing and analyzing images of the program level of the CV [4-6]. The article describes source data models; types of procedural transformations over images; forms of descriptive algebraic schemes for image transformation; structural synthesis of the CV model for measuring features that have a deterministic nature; statement of the problem of parametric identification of the CV model.

1. Structural Synthesis of CV Model

Suppose that the original color images captured by a surveillance camera are written in the form [4–6]

$$I_{color} = \| < r_{ij}, g_{ij}, b_{ij} > \| , \quad \langle r_{ij}, g_{ij}, b_{ij} \rangle \in \{0, \ldots, 255\}^3 \ , \tag{1}$$

where I_{color} is the finite-dimensional matrix of order $n \times m$ consisting of tuples of the form $< r_{ij}, g_{ij}, b_{ij} >$; the elements $r_{ij}, g_{ij}, b_{ij} \in \{0, \ldots, 255\}$ represent the brightness of the pixel $(i; j)$ in RGB space; $i = \overline{1, n}$ is the number of rows of the matrix I_{color}; $j = \overline{1, m}$ is the number of columns of the matrix I_{color}.

It is necessary to develop a function for measuring the deterministic feature p of the object of observation on the image I_{color} in the form of a superposition of descriptive algebraic image transformation schemes (T-DAITS and P-DAITS) [4-6]

$$p = R_P \left(R_T \left(I_{color}, \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \right), \langle \beta_1, \beta_2, \ldots, \beta_m \rangle \right), \tag{2}$$

where I_{color} is the initial color image represented as a matrix; $I_{bin} = R_T \left(I_{color}, \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \right) = O_T^n \left(O_T^{n-1} \left(...O_T^n \left(I_{color}, \alpha_1 \right), \alpha_{n-1} \right), \alpha_n \right)$ is the sequence of n procedural transformations with the variational parameters $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$; $S = R_P \left(I_{bin}, \langle \beta_1, \beta_2, \ldots, \beta_m \rangle \right)$ is the set of m parametric transformations with the variational parameters $\langle \beta_1, \beta_2, \ldots, \beta_m \rangle$.

T-DAITS represents a sequence of procedural transformations (T-transformations), each of which transforms one image into another image. Procedural transformations can be one of the following classes [4-6]:

1. $I_{color} = O_T^i \left(I_{color}, \alpha_i \right)$ converts the color image I_{color} to the color image I_{color};
2. $I_{gray} = O_T^i \left(I_{color}, \alpha_i \right)$ converts the color image I_{color} to the grayscale image I_{gray};
3. $I_{gray} = O_T^i \left(I_{gray}, \alpha_i \right)$ converts the grayscale image I_{gray} to the grayscale image I_{gray};
4. $I_{bin} = O_T^i \left(I_{gray}, \alpha_i \right)$ converts the grayscale image I_{gray} to the binary image I_{bin};
5. $I_{bin} = O_T^i \left(I_{bin}, \alpha_i \right)$ converts the binary image I_{bin} to the binary image I_{bin}.

By definition, T-DAITS [4-6] can be written in the recursive form

$$R_T \left(I_{color}, \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \right) = O_T^n \left(R_T \left(I_{color}, \langle \alpha_1, \ldots, \alpha_{n-1} \rangle \right), \alpha_n \right), \tag{3}$$

where $O_T^n(I_{bin}, \alpha_n)$ is the last procedural transformation from the list of acceptable image transformation methods with the variational parameter α_n.

P-DAITS [4-6] represents a set of parametric transformations (P-transformations), on the basis of which the algorithm for measuring the deterministic quantitative feature p of the object of observation is constructed. By definition, P-DAITS is written as

$$p = R_P\left(I_{bin}, \langle \beta_1, \beta_2, \ldots, \beta_m \rangle\right) = \mathop{\circ}_{i=1}^{m} O_P^i\left(I_{bin}, \beta_i\right), \tag{4}$$

where $O_P^i(I_{bin}, \beta_i)$ are P-transformations representing the methods for measuring the desired feature p on the binary image I_{bin} with the variational parameters β_i, and p is the measured feature, $\circ$ is the operation of serial or parallel execution of P-transformations.

2. Parametric Identification of CV Model

Parametric identification of a mathematical model involves calculation of all admissible sets of values of variational parameters for the CV measurement function. The problem of parametric identification is formulated as a problem of nonlinear optimization, the objective function of which contains the CV measurement function [4-6].

$$\begin{array}{l} F(\overline{\alpha}, \overline{\beta}) = \left|p^* - R_P\left(R_T\left(I_{color}, \langle \alpha_1, \alpha_2, ..., \alpha_n \rangle\right), \langle \beta_1, \beta_2, ..., \beta_m \rangle\right)\right| \to \min \\ \begin{cases} \alpha_i^{\min} \le \alpha_i \le \alpha_i^{\max}, & i = \overline{1, n} \\ \beta_j^{\min} \le \beta_j \le \beta_j^{\max}, & j = \overline{1, m} \end{cases} \end{array} \tag{5}$$

where $F(\overline{\alpha}, \overline{\beta})$ is the objective function with the variational parameters $\overline{\alpha} = \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ and $\overline{\beta} = \langle \beta_1, \beta_2, \ldots, \beta_m \rangle$; $\alpha_i^{\min} \le \alpha_i \le \alpha_i^{\max}$, $i = \overline{1, n}$ are restrictions on the variational parameters α_i of T-DAITS transformations;

$$\beta_j^{\min} \le \beta_j \le \beta_j^{\max}, \quad j = \overline{1, m}$$

are restrictions on the variational parameters β_j of P-DAITS transformations.

To solve a nonlinear programming problem, it is necessary to first estimate the value p^* of the desired feature p of the object of observation for one of the objects of observation in the image. The set of admissible sets of values of the variational parameters $\{\langle \alpha_1^*, \alpha_2^*, ..., \alpha_n^* \rangle\}$ and $\{\langle \beta_1^*, \beta_2^*, ..., \beta_n^* \rangle\}$ of the objective function $F(\overline{\alpha}, \overline{\beta})$ allow to measure the desired feature p of the object of observation with a given accuracy $\varepsilon > 0$, which means that condition (6) [4-6] holds.

$$\left|p^* - R_P\left(R_T\left(I_{color}, \langle \alpha_1^*, \alpha_2^*, ..., \alpha_n^* \rangle\right), \langle \beta_1^*, \beta_2^*, ..., \beta_m^* \rangle\right)\right| < \varepsilon. \tag{6}$$

Let us assume that the image is captured under unknown external factors that change insignificantly over a short period of time. Therefore, using the same sets of values of the variational parameters, it is possible to measure the desired feature p of the object of observation with the same specified accuracy $\varepsilon > 0$ for the rest of the observed objects in the given image. Let us consider two examples of the development of mathematical models of CV for measuring the areas of aircrafts and lakes and setting parametric identification problems for them [7, 8].

In the work [7], a CV model is developed to measure the total area of a Boeing 737-300 aircraft. The area measurement function has the form (7).

$$
\begin{array}{l}
S = R_P\left(R_T\left(I_{color}, < n, \theta >\right), \cdot\right) \\
\{\ I_{color} = \left\|\langle r_{ij}, g_{ij}, b_{ij}\rangle\right\|,\ \ r_{ij}, g_{ij}, b_{ij} \in \{0, ..., 255\}; \\
I^1_{gray} = \left\|x_{ij}\right\|,\ \ x_{ij} \in \{0, ..., 255\}; \\
I^2_{gray} = \left\|y_{ij}\right\|,\ \ y_{ij} \in \{0, ..., 255\}; \\
I_{bin} = \left\|z_{ij}\right\|,\ \ z_{ij} \in \{0, 1\}; \\
I^1_{gray} = O_T^{color\to gray}(I_{color}, \cdot);\ x_{ij} = \left\lceil \frac{r_{ij}+g_{ij}+b_{ij}}{3} \right\rceil; \\
I^2_{gray} = O_T^{median}(I^1_{gray}, (n, n)); \\
I_{bin} = O_T^{gray\to bin}(255 - I^2_{gray}, \theta);\ z_{ij} = \left\{ \begin{array}{l} 0, 255 - y_{ij} < \theta, \\ 1, 255 - y_{ij} \geq \theta; \end{array} \right. \\
S = \sum\limits_i \sum\limits_j z_{ij}\ \},
\end{array} \tag{7}
$$

where $I_{color}, I^1_{gray}, I^2_{gray}, I_{bin}$ are the types of images used in the mathematical model; $I^1_{gray} = O_T^{color\to gray}(I_{color}, \cdot)$ is the procedural transformation of converting the color image I_{color} to the grayscale image I^1_{gray} without a variational parameter; $I^2_{gray} = O_T^{median}(I^1_{gray}, (n, n))$ is the median filtering of the grayscale image I^1_{gray} into the grayscale image I^2_{gray} with the variational aperture (n, n) according to (8); $I_{bin} = O_T^{gray\to bin}(255 - I^2_{gray}, \theta)$ is the procedural transformation of converting the grayscale image I^2_{gray} to the binary image I_{bin} with the conversion threshold θ; P-DAITS $S = R_P\left(I_{bin}, \cdot\right)$ calculates the aircraft area as the sum of pixels that are equal to 1 and represent all black pixels. In this mathematical model, the median filter is written as algorithm (8)

$$
\begin{array}{l}
I^2_{gray} = O_T^{median}\left(I^1_{gray}, (n, n)\right) \\
\{\ I^1_{gray} = \left\|x_{ij}\right\|,\ \ I^2_{gray} = \left\|y_{ij}\right\|,\ \ x_{ij}, y_{ij} \in \{0, ..., 255\}; \\
n\, mod\, 2 = 1; \\
\overline{v} = Sort(x_{kl});\ \ k = \overline{i-n, i+n},\ \ l = \overline{j-n, j+n}; \\
v_1 \leq v_2 \leq ... \leq v_{n^2-1} \leq v_{n^2}; \\
y_{ij} = v_p, p = \left\lceil \frac{n^2}{2} \right\rceil\ \},
\end{array} \tag{8}
$$

where the expression $n\, mod\, 2 = 1$ is a check of the aperture n for oddness; $\overline{v} = Sort(x_{kl})$ is a sorting of the vector $\overline{v}$ in non-decreasing order; $y_{ij} = v_p$ is a rule under which a new value of the pixel y_{ij} is the coordinate of the vector $\overline{v}$ with the index $p = \left\lceil \frac{n^2}{2} \right\rceil$.

The problem of parametric identification in the form (9) is solved using genetic algorithms [7].

$$
\begin{array}{l}
F(n, \theta) = \left|S^{\cdot} - R_P\left(R_T\left(I_{color}, \langle n, \theta\rangle\right), \cdot\right)\right| \to \min, \\
\left\{ \begin{array}{l} 3 \leq n \leq 9, \\ 0 \leq \theta \leq 1, \end{array} \right.
\end{array} \tag{9}
$$

where n is the aperture for the median filter $O_T^{median}\left(\cdot, (n, n)\right)$; θ is the threshold for the conversion transformation $O_T^{gray\to bin}(\cdot, \theta)$; $S^{\cdot}$ is the total area (for top view) of one of the aircrafts on the runway.

In the work [8], the problem of measuring a lake on satellite images is solved. In this problem, CV model (10) differs from model (7) in the procedural transformation of converting $O_T^{gray\to bin}(\cdot,\theta)$ and in the parametric transformation $R_P(\cdot,*)$.

$$\begin{array}{l} S = R_P\left(R_T\left(I_{color}, < n,\theta,q >\right),*\right) \\ \{\ I_{color} = \left\|\langle r_{ij}, g_{ij}, b_{ij}\rangle\right\|, \ \ r_{ij}, g_{ij}, b_{ij} \in \{0,...,255\}; \\ I^1_{gray} = \left\|x_{ij}\right\|, \ \ x_{ij} \in \{0,...,255\}; \\ I^2_{gray} = \left\|y_{ij}\right\|, \ \ y_{ij} \in \{0,...,255\}; \\ I_{bin} = \left\|z_{ij}\right\|, \ \ z_{ij} \in \{0,1\}; \\ I^1_{gray} = O_T^{color\to gray}(I_{color},*), \ \ x_{ij} = \left\lceil \frac{r_{ij}+g_{ij}+b_{ij}}{3} \right\rceil; \\ I^2_{gray} = O_T^{median}(I^1_{gray},(n,n)); \\ I_{bin} = O_T^{gray\to bin}(I^2_{gray},\theta), \ \ z_{ij} = \left\{ \begin{array}{l} 0, \ \ y_{ij} < \theta, \\ 1, \ \ y_{ij} \geq \theta; \end{array} \right. \\ m, S_k = O_P^{\text{bwlabeln}}(I_{bin},q), \ \ q \in \{4,8\}; \\ S = \{S_k | k = \overline{1,m}; |S_{\text{exp}} - S_k| \leq \varepsilon; \varepsilon \in [0,1]\}\ \}, \end{array} \qquad (10)$$

where $I^1_{gray} = O_T^{color\to gray}(I_{color},*)$ is the procedural transformation of converting the color image I_{color} to the grayscale image I^1_{gray} without variational parameters; $I^2_{gray} = O_T^{median}(I^1_{gray},(n,n))$ is the procedural transformation of median filtering of the grayscale image I^1_{gray} into the grayscale image I^2_{gray} with the variational aperture (n,n) performed in accordance with (8); $I_{bin} = O_T^{gray\to bin}(I^2_{gray},\theta)$ is the procedural transformation of converting the grayscale image I^2_{gray} to the binary image I_{bin} with the variational conversion threshold θ; $m, S_k = O_P^{\text{bwlabeln}}(I_{bin},q)$ is the parametric transformation of segmentation of the binary image I_{bin} based on variational q-connection, which returns the areas S_k for individual observation objects and their total number m in the image; S is the estimate of the accuracy ε of calculating the areas S_k of individual observation objects.

It is allowed to form a sequence of N images of the object of observation with subsequent measurement of the desired feature p of the object of observation for a fixed set of values of the variational parameters $\overline{\alpha} = \langle\alpha_1^*, \alpha_2^*, ..., \alpha_n^*\rangle$ and $\overline{\beta} = \langle\beta_1^*, \beta_2^*, ..., \beta_n^*\rangle$. As a result of the experiment, the sample population $p = \left\{p_t^* \middle|\ t = \overline{1,N}\right\}$ of N images is formed. To estimate the desired feature p^* of the object of observation, it is necessary to solve the following statistical problems [9, 10]:

1) test the hypothesis of the selected law of distribution of the general population for the feature p based on the sample population;
2) estimate the confidence intervals for the parameters of the selected distribution law.

Conclusion

This method of structural synthesis of the software (algorithmic) part of VS undoubtedly has a high potential. Among its main advantages, we note: the presence of a single algorithm for the synthesis of VS models that allows to automate software development; mathematical statement of the problem of parametric identification of VS models that allows using existing mathematical optimization methods and algorithms for finding a set of its solutions; the recommendations formulated in the article allow statistical processing of the values of the variational parameters of the models to refine the values of the desired characteristics of the object of observation with a given accuracy.

The disadvantages of the described method of structural synthesis and parametric identification of the VS measurement function model are as follows: sensitivity of the method to the completeness and representativeness of sample data; source images must be of orthographic projection category; observed objects should have external contours in the form of closed polygons.

References

1. Gridin V.N., Titov V.S., Trufanov M.I. *Adaptive Technical Computer Vision.* Saint-Petersburg, Nauka, 2009, 441 p. (in Russian)
2. Syryamkin V.I., Syryamkin M.V., Titov D.V., Titov V.S., Trufanov M.I. *Adaptive Technical Computer Vision.* Moscow, Ru-Science, 2019, 448 p. (in Russian)
3. Iskhakov A.R., Malikov R.F. *Modeling of Technical Vision Systems in Modified Descriptive Image Algebras.* Ufa, Bashkir State Pedagogical University M. Akmulla, 2015, 159 p. (in Russian)
4. Iskhakov A.R. *Methods of Mathematical Modeling of Image Processing and Analysis in Modified Descriptive Image Algebras.* Ph.D. Thesis. Chelyabinsk, South Ural State University, 2017, 164 p. (in Russian)
5. Iskhakov A.R. Mathematical Methods of Modeling of Image Processing and Analysis in the Modified Descriptive Algebras of Images. *Journal of Computational and Engineering Mathematics,* 2016, vol. 3, no. 1, pp.3–9. DOI: 10.14529/jcem160101
6. Iskhakov A.R., Malikov R.F. Calculation of Aircraft Area on Satellite Images by Genetic Algorithm. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2016, vol. 9, no. 4, pp. 148–154. DOI: 10.14529/mmp160414
7. Kochurov B.I., Malikov R.F., Iskhakov A.R., Turikeshev G.T.G., Usmanova A.R., Kratynskaya E.B. Investigation of the Dynamics of the Area of Lake Aslykul (Southern Urals) by the Method of Image Processing of Satellite Images Based on an Algebraic Approach. *Teoreticheskaya i prikladnaya ekologiya*, 2021, no. 1, pp. 58–64. (in Russian) DOI: 10.25750/1995-4301-2021-1-058-064
8. Gatelyuk O.V. *Testing Statistical Hypotheses.* Saint-Peterburg, Lan', 2022, 112 p. (in Russian)

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СТРУКТУРНЫЙ СИНТЕЗ СИСТЕМЫ ТЕХНИЧЕСКОГО ЗРЕНИЯ И ЕЕ ПАРАМЕТРИЧЕСКАЯ ИДЕНТИФИКАЦИЯ СО СТАТИСТИЧЕСКОЙ ОЦЕНКОЙ ВАРИАЦИОННЫХ ПАРАМЕТРОВ

А. Р. Исхаков, В. Р. Акбашев

В статье представлены результаты исследования проблемы структурного синтеза системы технического зрения и ее параметрической идентификации. Приведены основные классы процедурных преобразований изображений. Описан метод построения модели системы технического зрения для измерения детерминированного признака с использованием процедурных и параметрических преобразований модифицированных дескриптивных алгебр изображений. Функция измерения детерминированной количественной характеристики объекта наблюдения определена в виде суперпозиции процедурной и параметрической дескриптивной алгебраической схемы преобразования изображений. Задача параметрической идентификации сформулирована в виде задачи нелинейной оптимизации. Приведены примеры использования описываемого метода при измерении площади объектов различной природы. Сформулированы рекомендации по статистической обработке значений измеряемой характеристики с целью уточнения его значения.

Ключевые слова: преобразование изображений, измерение признаков, модель системы технического зрения, структурный синтез модели, параметрическая идентификация модели, нелинейная оптимизация.

Литература

1. Гридин, В.Н. Адаптивные системы технического зрения / В.Н. Гридин, В.С. Титов, М.И. Труфанов. – Санкт-Петербург: Наука, 2009. – 441 с.

2. Адаптивные системы технического зрения / В.И. Сырямкин, М.В. Сырямкин, Д.В. Титов [и др.]. – Москва: Русайнс, 2019. – 448 с.

3. Исхаков, А.Р. Моделирование систем технического зрения в модифицированных дескриптивных алгебрах изображений / А.Р. Исхаков, Р.Ф. Маликов. – Уфа: Башкирский государственный педагогический университет им. М. Акмуллы, 2015. – 159 с.

4. Исхаков, А.Р. Методы математического моделирования обработки и анализа изображений в модифицированных дескриптивных алгебрах изображений: диссертация на соискание ученой степени кандидата физико-математических наук / А.Р. Исхаков. – Челябинск, 2017. – 164 с.

5. Iskhakov, A.R. Mathematical Methods of Modeling of Image Processing and Analysis in the Modified Descriptive Algebras of Images / A.R. Iskhakov // Journal of Computational and Engineering Mathematics. – 2016. – V. 3, №1. – P. 3–9.

6. Iskhakov, A.R. Calculation of Aircraft Area on Satellite Images by Genetic Algorithm / A.R. Iskhakov, R.F. Malikov // Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software. – 2016. – V. 9, №4. – P. 148–154.

7. Исследование динамики площади озера Аслыкуль (Южное Предуралье) методом обработки изображений космических снимков на основе алгебраического подхода / Б.И. Кочуров, Р.Ф. Маликов, А.Р. Исхаков [и др.] // Теоретическая и прикладная экология. – 2021. – №1. – С. 58–64.
8. Гателюк, О.В. Проверка статистических гипотез / О.В. Гателюк, Н.В. Манюкова. – Санкт-Петербург: Лань, 2022. – 112 с.

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