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ON THE PECULIARITIES OF THE MATHEMATICAL MODEL OF OPTIMAL DYNAMIC MEASUREMENT WHEN IMPLEMENTING THE SPLINE METHOD

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The article presents the results of computational experiments demonstrating the importance of initial conditions in modeling the states of a measuring device in the algorithm of the spline method. The discussed algorithm is one of the numerical methods used in the theory of optimal dynamic measurements, which allow to find the input signal from a known output signal (or observation) and a known transfer function of the measuring device. In all formulations of the problem, it is assumed that the inertia of the measuring device is taken into account, and the differences are due to the inclusion of interferences of various natures in the mathematical model. Consideration of interference as "white noise" led to the development of analytical and numerical methods for solving the problem under discussion. The article briefly provides theoretical information and an overview of numerical methods for using digital filters to process observation results with subsequent application of the spline method. However, new experimental data have shown that the standard initial conditions are insufficient to ensure connectivity conditions in the internal nodes of the spline. The initial conditions are proposed in the article, and the results of computational experiments are presented.

Keywords: optimal dynamic measurements; spline method; Leontief type system; initial condition.

Introduction

The theory of optimal dynamic measurements originates from a mathematical model of restoring a dynamically distorted signal from a known observed output signal and parameters of a measuring device (MD), which is based on the problem of optimal control for a Leontief type system [1]. The measuring device is modeled by a Leontief type system (or a description system)

$$\begin{cases} L\dot{x} = Ax + Bu, \\ y = Cx, \end{cases} \tag{1}$$

where L and A are matrices that characterize the structure of the MD, in some cases it is possible that $\det L = 0$ [2]; $x(t)$ and $\dot{x}(t)$ are vector-functions of the state of the MD and the velocity of the state change, respectively; $y(t)$ is a vector-function of observation; C is a rectangular matrix characterizing the interrelation between the system state and observation; $u(t)$ is a vector-function of measurements; B is a matrix characterizing interrelation between the system state and measurement. If L is not degenerate then system (1) can be reduced to

$$\begin{cases} \dot{x} = Mx + Fu, \\ y = Cx, \end{cases}$$

where $M = L^{-1}A$, $F = L^{-1}B$.

The initial Showalter – Sidorov condition

$$\left[(\alpha L - A)^{-1} L\right]^{p+1} (x(0) - x_0) = 0 \tag{2}$$

reflects initial state of the MD for some $x_0 \in R^n$, $\alpha \in \rho^L(M)$. The initial Showalter – Sidorov condition is equivalent to the initial Cauchy condition $x(0) = x_0$ in the case of $\det L \neq 0$.

The unknown input signal is found as a solution to the optimal control problem in which we minimize the penalty functional

$$J(v) = \min_{u \in U_\partial} J(x(u), u)$$

of the form

$$J(u) = J(x(u)) = \sum_{k=0}^{1} \int_0^\tau \left\| Cx^{(k)}(t) - y_0^{(k)}(t) \right\|^2 dt. \tag{3}$$

The form of functional (3) determines the main idea of the mathematical model of optimal dynamic measurements that is minimizing the discrepancy between the output signal $y(t) = Cx(t)$ modelled by system (1) and the observed output signal $y_0(t)$ (or observation) according to the readings of MD and their derivatives [3]. The function $v(t)$, at which the minimum of the penalty functional is reached, is called the optimal dynamic measurement.

Assuming that the input signal is distorted by interferences of the "white noise" type, it is necessary to consider the stochastic model of dynamic measurements, which is presented in Section 1 of the article. In Section 2, we give a brief overview of the proposed approaches to "purification" of observation [4] with the transition to a deterministic model of optimal dynamic measurements. In Section 3, we discuss the advantages of using the Kotelnikov sampling theorem for observations, and present the results of computational experiments.

1. Stochastic Model of Optimal Dynamic Measurements

Let $\Omega \equiv (\Omega, \mathrm{A}, P)$ be a complete probability space, $\mathbb{R}$ be a set of real numbers endowed with the Boreal σ-algebra. The measurable mapping $\xi : \Omega \to \mathbb{R}$ is called a random variable. The set of random variables with $E\xi = 0$ and finite variance forms a Hilbert space $\mathbf{L_2}$ with an inner product $< \xi_1, \xi_2 >= E(\xi_1 \xi_2)$. Let $I \subset \mathbb{R}$ be some interval. The mapping $\eta : I \times \Omega \to \mathbb{R}$ of the form $\eta = \eta(t, \omega)$ is called an *(one-dimensional) stochastic process*, therefore the value of the mapping $\eta = \eta(t, \cdot)$ is a random variable for every fixed $t \in I$, i.e. $\eta = \eta(t, \cdot) \in \mathbf{L}_2$ and the value of a stochastic process $\eta = \eta(\cdot, \omega)$ is called a *(sample) trajectory* for every fixed $\omega \in \Omega$. The random process η is called *continuous*, if almost surely all its trajectories are continuous. Denote by $C\mathbf{L}_2$ the space of continuous random processes. A continuous random process, which independent random variables are Gaussian, is called *Gaussian*. Denote by $\overset{o}{\eta}{}^{(\ell)}$ the ℓ-th Nelson – Gliklikh derivative of the stochastic process η [5]. The set of continuous stochastic processes having continuous

Nelson – Gliklikh derivatives up to order $k \in \mathbb{N}$ at each point of the set I forms a space, which is denoted by $C^k\mathbf{L}_2$.

Consider the stochastic model of the MD

$$\begin{cases} L\overset{\circ}{\xi} = A\xi + B(u+\phi), \\ \eta = C\xi + \nu, \end{cases} \tag{4}$$

$$\left[(\alpha L - A)^{-1} L\right]^{p+1} (\xi(0) - \xi_0) = 0. \tag{5}$$

Here the matrices L, A, B, C have the same sense as in (1). Random processes ϕ and ν determine noises in the circuits and at the output of the MD, respectively.

Similarly to the deterministic case, when investigating the problem on restoration of a dynamically distorted signal by random interference in the circuits and at the output of the MD, we consider the control problem

$$J(v) = \min_{u \in U_\partial} J(u), \tag{6}$$

where the functional

$$J(u) = J(\eta(u)) = \sum_{k=0}^{1} \int_0^\tau E \left\| \overset{\circ}{\eta}{}^{(k)}(t) - \eta_0^{(k)}(t) \right\|^2 dt \tag{7}$$

reflects the closeness of the real observation $\eta_0(t)$ and the virtual observation $\eta(t)$ obtained on the basis of a mathematical model of the MD.

The minimum point $v(t)$ of functional on the set U_∂ that is a solution to optimal control problem (4) – (7) is called an optimal dynamic measurement. In practice, there is only indirect information about $v(t)$.

2. Digital Filters and Spline Method

One of the developed directions in the theory of optimal measurements is the application of various methods to filtering the observation in order to obtain a smoothed observation function $\overline{y}_0(t)$ with a subsequent transition from stochastic model of optimal dynamic measurements (4) – (7) to the deterministic model

$$\begin{cases} L\dot{\overline{x}} = A\overline{x} + B\overline{u}, \\ \overline{y} = C\overline{x}, \end{cases} \tag{8}$$

$$\left[(\alpha L - A)^{-1} L\right]^{p+1} (\overline{x}(0) - x_0) = 0, \tag{9}$$

$$J(\overline{v}) = \min_{\overline{u} \in U_\partial} J(\overline{x}(\overline{u}), \overline{u}), \tag{10}$$

$$J(\overline{u}) = J(\overline{x}(\overline{u})) = \sum_{k=0}^{1} \int_0^\tau \left\| C\overline{x}^{(k)}(t) - \overline{y}_0^{(k)}(t) \right\|^2 dt. \tag{11}$$

Note that the solution $\overline{v}$ to problem (8) – (11) is an approximate solution to problem (4) – (7).

To obtain a smoothed observation, the work [6] uses an algorithm for constructing a smoothed one-dimensional observation signal under the condition that the signal shape is convex upwards and has a single maximum point. To accept the assumption of similarity of an observation and a smoothed observation function, we test a statistical hypothesis of normal distribution of the parameters for the cross sections of the process η_0. In addition, in combination with the numerical algorithm described in [7], this approach allows to take into account the condition of degradation of the MD.

To obtain a smoothed observation, the works [8] and [9] use a digital moving average filter and the Savitsky – Golay digital filter, respectively. In both cases, for each experiment, it is necessary to select the parameters of digital filters that are the value and shift of the time window, data weights, which is a disadvantage of such methods. The advantage of these methods is their simplicity and the insignificance of information about the numerical characteristics of the noise. The work [10] uses an one-dimensional Kalman filter to obtain a smoothed observation under the assumption that "white noise" takes place only at the output of the MD. Note that its application requires information about the noise variance.

Note that all numerical algorithms for solving problem (8) – (11) use the approaches described in detail in [11, 12].

Let us describe the spline method for solving the problem of optimal dynamic measurement.

Suppose that the following components are given: the matrices included in system (4), the initial value $x_0 \in R^n$; the array of observed values Y_{0i} at the nodal points $t_i = 0, 1, \ldots, n$ of the output signal, and $t_{i+1} - t_i = \delta$, $t_0 = 0$, $t_n = \tau$.

Step 1. Divide the interval $[0, \tau]$into M intervals $[\tau_{m-1}, \tau_m]$, where $m = 1, 2, \ldots, M$, and $t_0 = \tau_0 = 0$, $t_n = \tau_M$.

Step 2. At each interval $[\tau_{m-1}, \tau_m]$, construct the interpolation function $y^{\ell}_{0m}(t)$ in the form of a polynomial of the degree $\ell \leq (n-1)/M$.

Step 3. For $m = 1, 2, \ldots, M$ at $[\tau_{m-1}, \tau_m]$, consecutively solve the optimal dynamic measurement problem.

$$\begin{cases} L\dot{\overline{x}}_m = A\overline{x}_m + B\overline{u}^{\ell}_m, \\ \overline{y}_m = C\overline{x}_m, \end{cases} \tag{12}$$

$$\left[(\alpha L - A)^{-1} L\right]^{p+1} (\overline{x}_m(0) - x_{m0}) = 0, \tag{13}$$

$$J(\overline{v}^{\ell}_m) = \min_{\overline{u}_m \in U_\partial} J(\overline{x}_m(\overline{u}), \overline{u}), \tag{14}$$

$$J(\overline{u}) = J(\overline{x}(\overline{u})) = \sum_{k=0}^{1} \int\limits_{\tau_{m-1}}^{\tau_m} \left\| C\overline{x}_m{}^{(k)}(t) - (\overline{y}^{\ell}_{0m}(t))^{(k)} \right\|^2 dt. \tag{15}$$

We find the approximate value of the optimal measurement $\overline{v}^{\ell}_m(t)$ in the form of a polynomial of the degree ℓ imposing the continuity condition

$$\overline{v}^{\ell}_m(\tau_m) = \overline{v}^{\ell}_{m+1}(\tau_m), \tag{16}$$

for $u \in \mathfrak{A}_{\partial m}$, where $\mathfrak{A}_{\partial m} \subset \mathfrak{A}_\partial$ is a closed convex subset of $\mathfrak{A}_\partial$.

Step 4. As a result, we get a spline function

$$\tilde{v}^{\ell}_k(t) = \bigcup_m v^{\ell}_{km}(t)$$

continuous on $[0, \tau]$.

3. About the Initial Conditions

We will conduct computational experiments using the results of the experiment with the sensor, which is based on the system

$$\begin{cases} L\dot{\overline{x}}_1 = \overline{x}_2, \\ L\dot{\overline{x}}_2 = -25\overline{x}_1 - 5\overline{x}_2 + 25\overline{u}, \\ \overline{y} = \overline{x}_2, \end{cases} \tag{17}$$

The test signal is one period of a sine wave with an amplitude of $0.48\,V$, a frequency of $2Hz$, with a phase shift of 270 degrees and a constant offset of $0.48\,V$

$$u = 0.48 \sin\left(4\pi t + \frac{3\pi}{2}\right) + 0.48; \qquad t = [0, 2\pi].$$

In all the figures, the test signal is displayed in a blue graph. In the first case, the observed signal (green graph) is distorted only by the inertia of the measuring device (Figure 1).

In the second case, the observed signal (green graph) is distorted by the inertia of the measuring device and white noise η (Figure 2). In this case, the summand is added to the third equation of the system

$$\begin{cases} L\dot{\overline{x}}_1 = \overline{x}_2, \\ L\dot{\overline{x}}_2 = -25\overline{x}_1 - 5\overline{x}_2 + 25\overline{u}, \\ \overline{y} = \overline{x}_2 + \eta, \end{cases} \tag{18}$$

When restoring the input signal using the spline method algorithm, we assume the following initial conditions

$$\overline{x}_1(\tau_{m-1}) = \xi_{m-1}, \qquad \overline{x}_2(\tau_{m-1}) = p_{m-1}, \tag{19}$$

where $\xi_{m-1} = y_0(\tau_{m-1})$, $p_{m-1} = y_0'(\tau_{m-1})$. The implementation of the algorithm was stopped after step 20 due to an increase in error with each subsequent step (Figure 3).

A similar situation (Figure 4) was obtained using the following initial conditions

$$\overline{x}_1(\tau_m) = \xi_m, \qquad \overline{x}_2(\tau_{m-1}) = p_{m-1}, \tag{20}$$

where $\xi_m = y_0(\tau_m)$.

when using the initial conditions of the form

$$\overline{x}_1(\tau_m) = \xi_m, \qquad \overline{x}_2(\tau_m) = p_m. \tag{21}$$

or

$$\overline{x}_1(\tau_{m-1}) = \xi_{m-1}, \qquad \overline{x}_2(\tau_m) = p_m, \tag{22}$$

where $p_{m-1} = y_0'(\tau_{m-1})$. The results of restoring the input signal are shown in the Fig. 5 and Fig. 6. Note that incomplete testing of the inertia of the measuring device when solving the problem of restoring the input signal is quite common.

For the second case, in the presence of white noise, the initial conditions of four types were also used. The results of one cycle of the method described in [13] are shown in the Fig. 7 – 10.

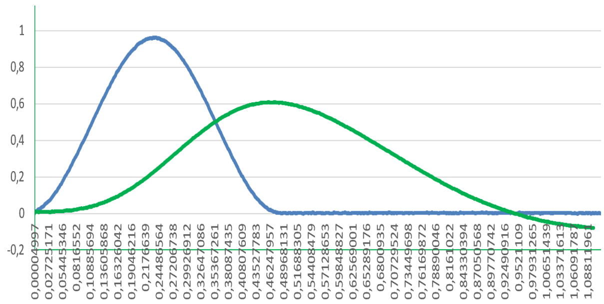

Fig. 1. The first case: blue color – $u(t)$, green color – $y_0(t)$

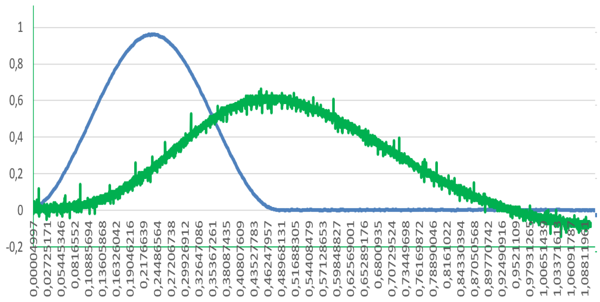

Fig. 2. The second case: blue color – $u(t)$, green color – $y_0(t)$

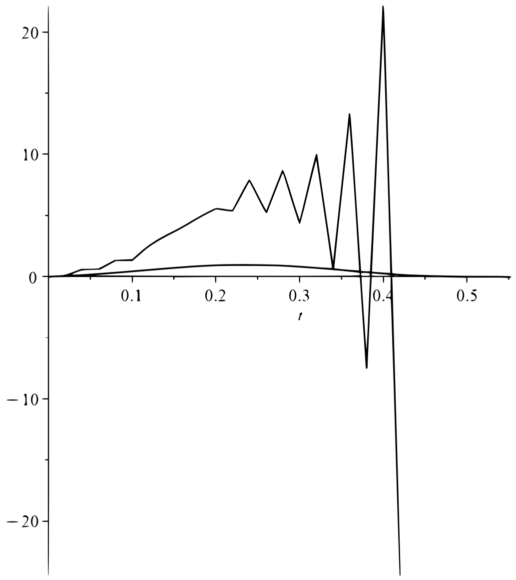

Fig. 3. The first case. Initial conditions (19)

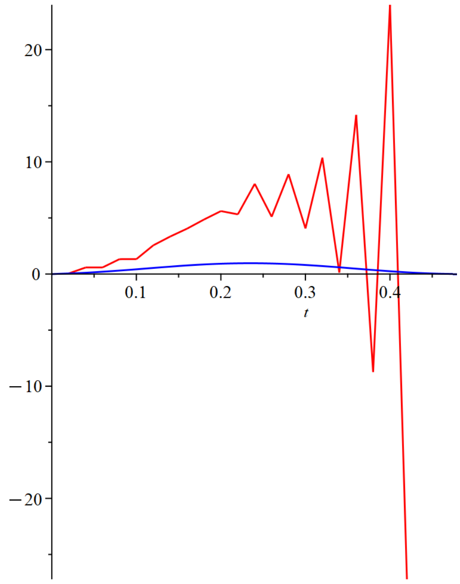

Fig. 4. The first case. Initial conditions (20)

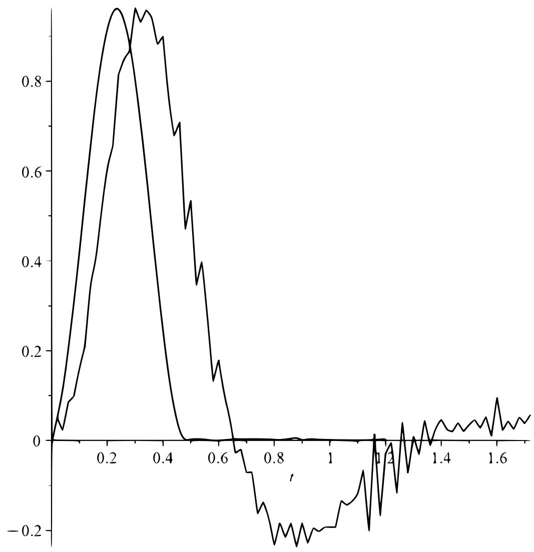

Fig. 5. The first case. Initial conditions (21)

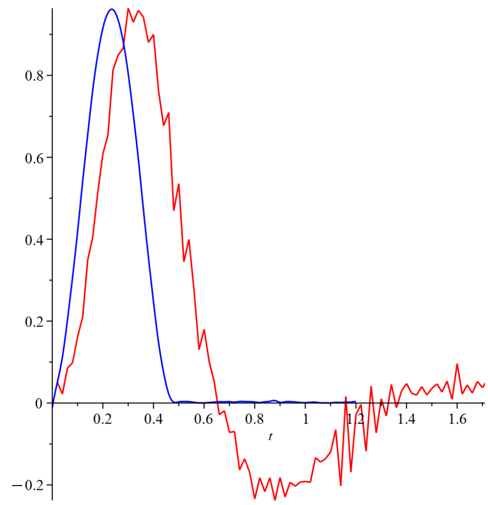

Fig. 6. The first case. Initial conditions (22)

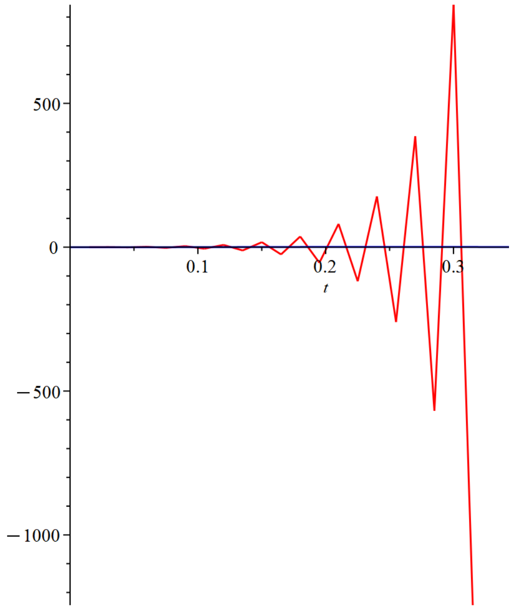

Fig. 7. The second case. Initial conditions (19)

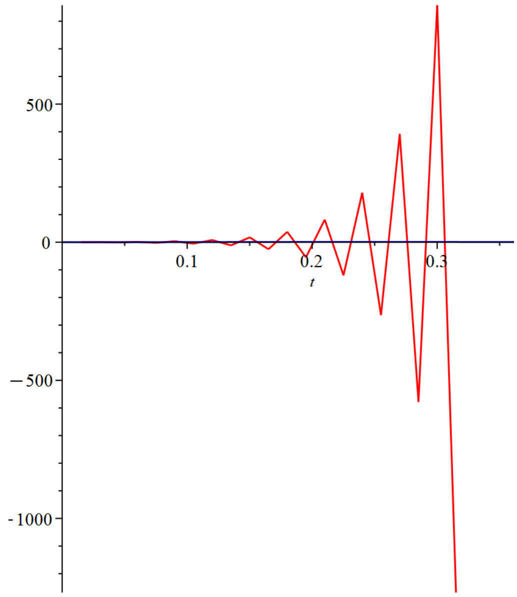

Fig. 8. The second case. Initial conditions (20)

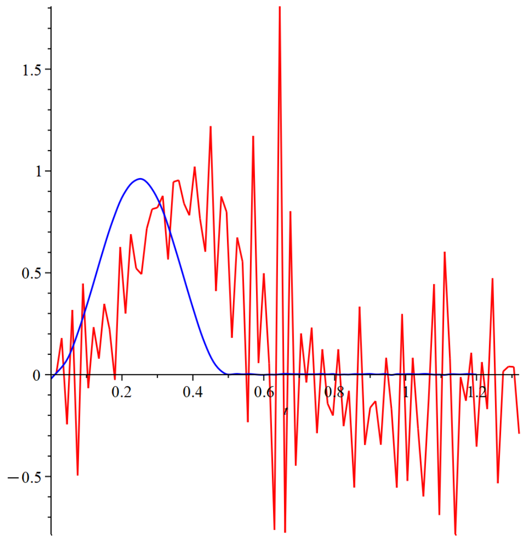

Fig. 9. The second case. Initial conditions (21)

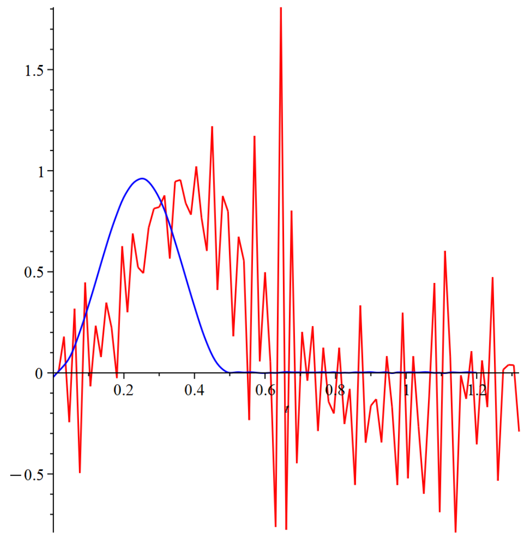

Fig. 10. The second case. Initial conditions (22)

Thus, when implementing the spline method, it is necessary to use the initial conditions of the form (21) or (22) that better ensure the connection of the spline links in the internal nodes.

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ОБ ОСОБЕННОСТЯХ МАТЕМАТИЧЕСКОЙ МОДЕЛИ ОПТИМАЛЬНОГО ДИНАМИЧЕСКОГО ИЗМЕРЕНИЯ ПРИ РЕАЛИЗАЦИИ СПЛАЙН-МЕТОДА

А. В. Келлер, И. А. Колесников

В статье представлены результаты вычислительных экспериментов, демонстрирующих важность начальных условий при моделировании состояний измерительного устройства в алгоритме сплайн-метода. Обсуждаемый алгоритм является одним из используемых в теории оптимальных динамических измерений численных методов, позволяющих по известному выходному сигналу (или наблюдению) и известной передаточной функции измерительного устройства находить входной сигнал. Во всех постановках задачи предполагается учет инерционности измерительного устройства, а различия обусловлены включением в математическую модель различных по природе помех. Рассмотрение помехи в качестве "белого шума" привело к развитию аналитических и численных методов решения обсуждаемой задачи. В статье кратко приведены теоретические сведения и обзор численных методов по использованию цифровых фильтров для обработки результатов наблюдения с последующим применением сплайн метода. Однако новые экспериментальные данные показали недостаточность стандартных начальных условий для обеспечения условий связанности во внутренних узлах сплайна. В статье предложены начальные условия, приведены результаты вычислительных экспериментов.

Ключевые слова: оптимальные динамические измерения; сплайн метод; система леонтьевского типа; начальные условия.

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