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STABILIZATION OF SOLUTIONS OF THE STOCHASTIC DZEKZER EQUATION

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The article considers the stochastic Dzekzer equation, which describes the evolution of the free surface of a filtered liquid. To study the stability and instability of solutions and the stabilization of unstable solutions, this equation in suitable functional stochastic spaces is considered as a linear stochastic equation of the Sobolev type. The solution to the stochastic equation is a stochastic process that is not differentiable by Newton – Leibniz at any point. Therefore, we use the derivative of the stochastic process in the sense of Nelson – Gliklikh. The question of stability and instability of solutions to the stochastic Dzekzer equation is solved in terms of stable and unstable invariant spaces. To solve the stabilization problem, we consider the stochastic equation of the Sobolev type as a system of three equations: one singular and two regular, defined on stable and unstable invariant spaces. With the help of a feedback loop, the problem of stabilizing unstable solutions is solved. A numerical experiment has been carried out. Graphs of the solution before stabilization and after stabilization are given.

Keywords: Sobolev type equations; stochastic Dzekzer equation; invariant spaces; stabilization problem.

Introduction

Groundwater is understood as groundwater with a free surface that accumulates on the first waterproof layer of rock from the surface of the earth (a water barrier). Let's consider a model of a free surface when a liquid moves in a porous soil. The Dzekzer equation

$$(\lambda - \Delta)u_t = \alpha \Delta u - \beta \Delta^2 u, \tag{1}$$

where $\alpha,\ \beta \in \mathbb{R}_+$ и $\lambda \in \mathbb{R}$, describes the shape of this surface [1]. Here the parameters $\alpha,\ \beta,\ \lambda$ characterize the environment. From the point of view of a deterministic approach, the solvability of the initial boundary value problem for the equation (1) was studied in [2], the existence of exponential dichotomies of solutions to the equation (1) is shown in [3].

In this article we will consider the equation (1) from the point of view of the stochastic approach. For this purpose, in suitable stochastic function spaces, we will consider equation (1) as a linear stochastic equation of Sobolev type

$$L \overset{\circ}{\eta} = M\eta, \tag{2}$$

where $\overset{o}{\eta}$ denotes the Nelson – Gliklikh derivative [4] of the stochastic process $\eta = \eta(t)$. The study of the existence of solutions of the stochastic equation (2) with a relatively bounded operator was started in [5]. In [6] and [7] it is shown the existence of solutions of a linear equation of Sobolev type in the relatively sectorial case and in the relatively radial case. In [8] studied high-order Sobolev equations, in [9] considered the initial-finite problem for the equation (2). In [10] studied the stability of the equation (2), works [11] — [12] are devoted to finding stable and unstable numerical solutions of non-classical stochastic equations that can be written in the form (2). In [13] the stabilization problem for a stochastic linear Sobolev equation with a relatively sectorial operator is considered for the first time.

The purpose of this article is to solve the problem of stabilization of the stochastic Dzekzer equation. For this, we consider the equation (1) as an equation (2) with a relatively sectorial operator. The existence of solutions to the Cauchy problem and the Showalter – Sidorov problem of the stochastic equation (1) is shown in [6]. The work consists of two paragraphs in addition to the introductory part. In the first paragraph, sufficient conditions for the existence of stable and unstable invariant spaces of the stochastic equation (1). The second paragraph is devoted to the solution of the problem of stabilization of unstable solutions of the stochastic Dzekzer equation based on the feedback principle. Here we present graphs of solutions of the stochastic equation (1) before stabilization and after stabilization.

1. Invariant Spaces of the Stochastic Dzekzer Equation

Let $D \in \mathbb{R}^n$ be a bounded region, and its boundary $\partial D \in C^\infty$. Define the spaces $\mathfrak{U}$ and $\mathfrak{F}$:

$$\mathfrak{U} = \{u \in W_2^4 : u(x) = 0,\ (x) \in \partial D\},\ \mathfrak{F} = L_2(D).$$

Let $\{\varphi_k\}$ ($\{\psi_k\}$) be the eigenfunctions of the Laplace operator Δ orthonormalized relative scalar product in $\mathfrak{U}$ ($\mathfrak{F}$), the spectrum $\sigma(\Delta) = \{\nu_k\}$, the sequence $\mathbf{K} = \{\lambda_k\} \subset \mathbb{R}$ is such that $\sum\limits_{k=1}^{\infty} \lambda_k^2 < \infty$. Let denote $\{\xi_k\} \subset \mathbf{L_2}$ ($\{\zeta_k\} \subset \mathbf{L_2}$) is a sequence of uniformly bounded random variables with zero expectation and finite variance. Construct *a random* $\mathbf{K}$*-variable* $\xi = \sum\limits_{k=1}^{\infty} \lambda_k \xi_k \varphi_k$ $\left(\zeta = \sum\limits_{k=1}^{\infty} \lambda_k \zeta_k \psi_k\right)$. The spaces $\mathbf{U_K L_2}$ ($\mathbf{F_K L_2}$) are the replenishment of the linear envelope of random $\mathbf{K}$-values by the norm $\|\xi\|^2_{\mathbf{U_K L_2}} = \sum\limits_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\xi_k$ $\left(\|\zeta\|^2_{\mathbf{F_K L_2}} = \sum\limits_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\zeta_k\right)$.

Suppose that at the initial moment of time the distribution of the main hydrodynamic elements in the flow can be considered in the form:

$$\eta(0) = \eta_0, \quad \eta_0 = \sum_{k=1}^{\infty} \lambda_k \xi_k \varphi_k. \tag{3}$$

To describe the free surface system of the filtering fluid in porous soil, we will consider the stochastic Dzekzer equation. The operators $L,\ M : \mathbf{U_K L}_2 \to \mathbf{F_K L}_2$ are given by the formulas $L = \lambda - \Delta,\ \ M = \alpha\Delta - \beta\Delta^2$. Then the stochastic equation (1) can be viewed in the form (2).

Lemma 1. [6] *Let* $\alpha,\ \beta,\ \lambda \in \mathbb{R} \setminus \{0\}$, *then*

(i) operators $L, M : \mathbf{U_K L}_2 \to \mathbf{F_K L}_2$ *– linear and continuous operators;*

(ii) operator M *is strongly* $(L, 0)$*-sectorial.*

Theorem 1. [6] *Let $\alpha, \beta, \lambda \in \mathbb{R} \setminus \{0, \frac{\alpha}{\beta}\}$, then there exists a solution $\eta = \eta(t)$ to Cauchy problem (3) for equation (2), which has the form*

$$\eta(t) = \sum_{l=1}^{\infty}{}' \left[exp\left(\frac{\alpha\nu_l - \beta\nu_l^2}{\lambda - \nu_l} t\right) \left(\sum_{k=1}^{\infty} \lambda_k \xi_k < \varphi_k, \varphi_l >_0 \varphi_l\right)\right].$$

Definition 1. *A subspace $\mathbf{I} \subset \mathbf{U_K L}_2$ is called an invariant equation space (2), if for any $\eta_0 \in \mathbf{I}$ the solution to problem (2), (3) $\eta \in \mathbf{C}^1(\mathbb{R}; \mathbf{I})$.*

Definition 2. *An invariant subspace $\mathbf{I}^{s(u)} \subset \mathbf{P}$ is called a stable (unstable) invariant space of equation (2), if there exist such constants $N \in \mathbb{R}_+$ and $\nu_k \in \mathbb{R}_+$, that*

$$\|\eta^{s(u)}(t)\|_{\mathbf{U_K L}_2} \leq N_1 e^{-\nu_1(s-t)} \|\eta^1(s)\|_{\mathbf{U_K L}_2} \qquad \texttt{for } s \geq t (t \geq s),$$

where $\eta^{s(u)} = \eta^{s(u)}(t) \in \mathbf{I}^{s(u)}$ for all $t \in \mathbb{R}_+$. If the phase space splits into a direct sum $\mathbf{P} = \mathbf{I}^s \oplus \mathbf{I}^u$, then the solutions $\eta = \eta(t)$ of equation (2) have an exponential dichotomy.

Let $\alpha,\ \beta > 0$ and $\lambda < 0$. Let $\lambda \neq \dfrac{\alpha}{\beta}$ or $h_0 \neq \dfrac{6k_p}{k_f}$. Then the relative spectrum $\sigma^L(M) = \sigma_l^L(M) \bigcup \sigma_r^L(M)$, where

$$\sigma_l^L(M) = \left\{\frac{(\alpha - \nu_k\beta)\nu_k}{\lambda - \nu_k} : \lambda > \nu_k\right\},\ \sigma_r^L(M) = \left\{\frac{(\alpha - \nu_k\beta)\nu_k}{\lambda - \nu_k} : \lambda < \nu_k\right\}.$$

Consider the spaces $\mathbf{I}^s = \{\eta \in \mathbf{U_K L}_2 : \langle \cdot, \varphi_l \rangle \varphi_l = 0,\ \nu_l > \lambda\}$, $\mathbf{I}^u = \{\eta \in \mathbf{U_K L}_2 : \langle \cdot, \varphi_l \rangle \varphi_l = 0,\ \nu_l < \lambda\}$. If $\lambda > \nu_1$, then $\sigma^L(M) = \sigma_l^L(M)$ and there exists only a stable invariant space.

Theorem 2. *Let $\lambda \in \mathbb{R}_-$, $\alpha,\ \beta \in \mathbb{R}_+$ and $\lambda \neq \dfrac{\alpha}{\beta}$. Then*

(i) If $\lambda < \nu_1$, then there exist an infinite-dimensional stable invariant space $\mathbf{I}^s$ and a finite-dimensional unstable invariant space $\mathbf{I}^u$ of equation (2).

(ii) If $\lambda > \nu_1$, then the phase space of equation (2) coincides with the stable invariant space $\mathbf{I}^s$.

Let $\alpha,\ \beta,\ \lambda > 0$. Then $\alpha - \nu_k\beta > 0$, $\lambda - \nu_k > 0$ and the relative spectrum

$$\sigma^L(M) = \left\{\frac{(\alpha - \nu_k\beta)\nu_k}{\lambda - \nu_k} < 0\right\}.$$

Theorem 3. *Let $\alpha,\ \beta,\ \lambda \in \mathbb{R}_+$, then the stable invariant space $\mathbf{I}^s$ coincides with the phase space $\mathbf{U}^1_\mathbf{K}\mathbf{L}_2$ of equation (2).*

2. Stabilization Problem

Let $\alpha,\ \beta > 0$ and $\lambda < \nu_1$. Then $\sigma_u^L(M)$ – bounded set, by Γ_r denote the contour bounding $\sigma_u^L(M)$ and lying lies in the left half-plane of the complex plane. The part of the spectrum $\sigma_s^L(M) = \sigma^L(M) \setminus \sigma_u^L(M)$ lies in the sector bounded by the contour $\Gamma_l = \{\mu \in \mathbb{C} : \text{Re}\mu < 0,\ |\text{arg}\mu| \in (\pi/2, \pi)\}$. By virtue of the theorem 2 the space $\mathbf{I}^s$ is a stable invariant space, and the space $\mathbf{I}^u$ is an unstable invariant space.

The stochastic equation (2) will be considered as a reduced system

$$H \overset{\circ}{\eta}{}^{0} = \eta^0, \tag{4}$$

$$L_s \overset{\circ}{\eta}{}^{s} = M_s \eta^s, \tag{5}$$

$$L_u \overset{\circ}{\eta}{}^{u} = M_u \eta^u, \tag{6}$$

which will be called *the Dzekzer system*. Here, by M_s (L_s) и M_u (L_u) denote the contractions of M (L) to $\mathbf{I}^s$ and $\mathbf{I}^u$. The operator $= M_0^{-1}L_0$, where M_0 (L_0) — the contractions of M (L) on the space $\mathbf{U}^0_{\mathbf{K}}\mathbf{L}_2$. The space $\mathbf{U}^0_{\mathbf{K}}\mathbf{L}_2$ has the form

$$\mathbf{U}^0_{\mathbf{K}}\mathbf{L}_2 = \left\{ \begin{array}{c} \{0\},\ \nu_k \neq \lambda, \\ \eta \in \mathbf{U}_{\mathbf{K}}\mathbf{L}_2 : \eta = \sum\limits_{k=1}^{\infty} \lambda_k \xi_k \varphi_k,\ \nu_k = \lambda. \end{array} \right\}$$

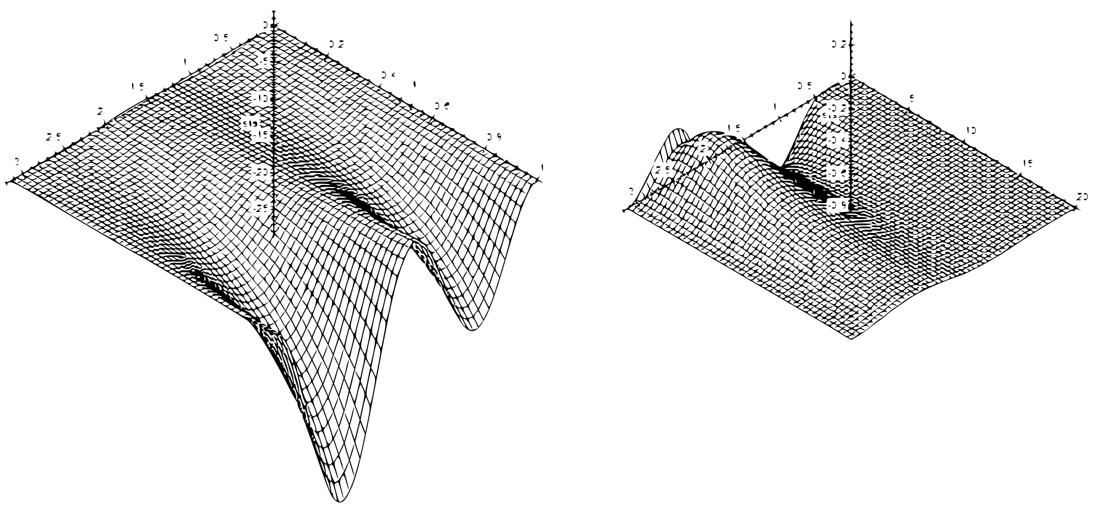

Fig. 1. Plots of the solution before stabilization and after stabilization in section $x = 1$

There exists a solving semigroup of the equation (5) $U^t_l = \frac{1}{2\pi i}\int\limits_{\Gamma_l}(\mu L_s - M_s)^{-1}L_s e^{\mu t}d\mu$ and the solving group of equation (6) $U^t_r = \frac{1}{2\pi i}\int\limits_{\Gamma_r}(\mu L_u - M_u)^{-1}L_u e^{\mu t}d\mu$. Due to the closed spectrum, there are constants $\alpha,\ \beta > 0$, such that $\mathrm{Re}\sigma^L_u(M) > \beta$ and $\mathrm{Re}\sigma^L_s(M) < -\alpha$. Then

$$\|U^t_l\|_{\mathcal{L}(\mathbf{U}_{\mathbf{K}}\mathbf{L}_2)} \leq Ce^{-\alpha t}, \|U^t_r\|_{\mathcal{L}(\mathbf{U}_{\mathbf{K}}\mathbf{L}_2)} \leq Ce^{\beta t},\ t \in \mathbb{R}_+. \tag{7}$$

The operator M is strongly $(L, 0)$-sectorial, then the solution $\eta^0 = \eta^0(t)$ of equation (4) is a zero random variable at $t \in \mathbb{R}_+$. Due to (7) for solving $\eta^s = \eta^s(t)$ of equation (5) it is satisfied $\lim\limits_{t\to+\infty} \|\eta^s(t)\|_{\mathbf{U}_{\mathbf{K}}\mathbf{L}_2} = 0$, for solving $\eta^u = \eta^u(t)$ of equation (5) is satisfied by $\lim\limits_{t\to+\infty} \|\eta^u(t)\|_{\mathbf{U}_{\mathbf{K}}\mathbf{L}_2} = +\infty$.

Therefore, consider the following stabilization problem. Need to find a random process χ, such that for the solution $\eta^u = \eta^u(t)$ of equation

$$L_u \overset{o}{\eta}{}^{u} = M_u \eta^u + \chi \tag{8}$$

executed

$$\lim_{t\to+\infty} \|\eta^u(t)\|_{\mathbf{U_K L_2}} = 0. \tag{9}$$

The random process χ will be looked up in the form $\chi = B\eta^u$. Let $B = -(\varepsilon + m)\mathbb{I}$, $m = \max_{\mu}\{\mu \in \sigma_r^L(M)\}$, $\varepsilon > 0$. Then

$$\sigma^{L_u}(M_u + B) = \left\{ \frac{(\alpha - \nu_k\beta)\nu_k}{\lambda - \nu_k} - (\varepsilon + m) < -\varepsilon \right\},$$

therefore, to solve $\eta^u = \eta^u(t)$ of equation (8), (9) is satisfied.

For the numerical experiment, assume $\lambda = -5$, $\alpha = 1$, $\beta = 0.2$, choose the square as the area D $[0,\pi]\times[0,\pi]$. With $\xi_1 = -0.68198$, $\xi_2 = 0.88769$, $\xi_3 = -0.73045$, $\xi_4 = 0.86707$ in Figure 1 are plots of the solution before stabilization (red) and after stabilization (blue) in section $x = 1$.

3. Conclusion

In the following we plan to consider the stabilization problem for semilinear stochastic equations of Sobolev type in the case of a relatively sectorial operator [14].

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СТАБИЛИЗАЦИЯ РЕШЕНИЙ СТОХАСТИЧЕСКОГО УРАВНЕНИЯ ДЗЕКЦЕРА

Е. Д. Ахмадеев, О. Г. Китаева

В работе рассматривается стохастическое уравнение Дзекцера, которое описывает эволюции свободной поверхности фильтрующейся жидкости. Для изучения устойчивости и неустойчивости решений и стабилизации неустойчивых решений данное уравнение в подходящих функционально-стохастических пространствах рассматривается в виде линейного стохастического уравнения соболевского типа. Решением стохастического уравнения является стохастический процесс, который не дифференцируем по Ньютону – Лейбницу ни в одной точке. Поэтому мы используем производную стохастического процесса в смысле Нельсона – Гликлиха. Вопрос об устойчивости и неустойчивости решений стохастического уравнения Дзекцера решается в терминах устойчивого и неустойчивого инвариантных пространств. Для решения задачи стабилизации стохастическое уравнение соболевского типа рассматриваем в виде системы трех уравнений: одного сингулярного и двух регулярных, определенных на устойчивом и неустойчивом инвариантных пространствах. С помощью контура обратной связи решена задача стабилизации неустойчивых решений. Проведен численный эксперимент. Приведены графики решения до стабилизации и после стабилизации.

Ключевые слова: уравнения соболевского типа; стохастическое уравнение Дзекцера; инвариантные пространства, задача стабилизации.

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