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## MODEL OF A RADAR SEMI-ACTIVE HOMING SYSTEM FOR A GUIDED MISSILE

*I. A. Chepurnov*<sup>1</sup>, chepurnov@bmstu.ru,

*V. O. Chervakov*<sup>1</sup>, vchervakov@bmstu.ru,

*D. S. Klygach*<sup>1</sup>, klygachds@susu.ru,

*D. V. Mavleeva*<sup>1</sup>, selezneva\_dv@mail.ru

<sup>1</sup>South Ural State University, Chelyabinsk, Russian Federation

The article updates the problem of improving mathematical models of computer simulators for training operators of anti-aircraft missile systems. A developed model of a semi-active radar homing system for a guided missile towards an air target is proposed. A description of the software application for simulation of a radar semi-active homing system for a guided missile at an air target is presented. The proposed simulation model and software application can be used both in the development and improvement of computer training models, and as part of separate automated training systems for training operators of anti-aircraft missile systems in military universities and military training centers.

*Keywords: simulator; anti-aircraft missile system; air target; radar; guided missile.*

### Introduction

The highest quality training of anti-aircraft missile weapon operators in military universities and military training centers is facilitated using simulators, which quickly reduces the gap between theory and practice [1].

Currently, computer simulators are widely used to train operators of anti-aircraft missile systems. Controls and alarms in such simulators are represented by graphic computer images with the possibility of using computer manipulators of various types. The main advantages of computer simulators are low cost, relatively short development time and ease of replication [2].

At the same time mathematical models of existing computer simulators for training anti-aircraft missile systems operators, as a rule, are not universal enough, can be approximate, and in some cases do not have a meaningful physical model and are only a “black box” with the corresponding input and output signals. In addition, such models are often not closed, i.e. require additional data obtained through physical experiment.

In some cases, missing data can be obtained using approximate (engineering) models and calculation methods. However, in many cases, these methods are semi-empirical and provide an assessment of the required parameters for schematized conditions and design solutions. In addition, engineering models are usually linear and allow only integral characteristics to be approximately predicted. One of the solutions to this problem is the development of effective and easily implemented in training complexes simulation models of the main processes characteristic of the anti-aircraft missile system functioning.

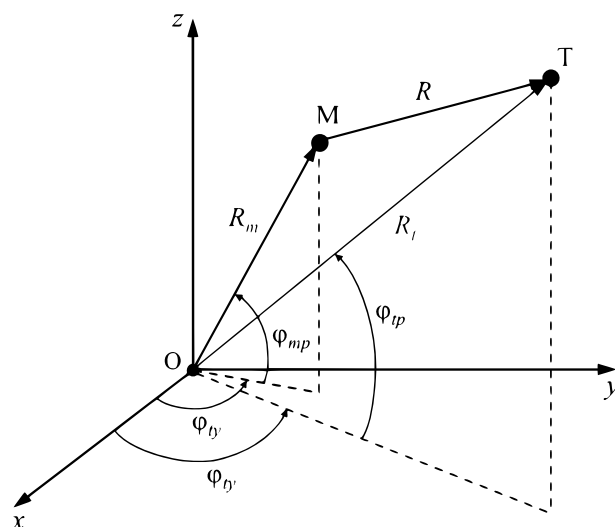
The purpose of this article is to present the results of a semi-active homing system for a guided missile at an air target simulation model development, as well as its software implementation.

## 1. Simulation of a Guided Missile Homing Process at an Air Target

As is known [3], the solution to the problem of missile flight control is to determine the mismatch, i.e. deviation of the missile from the position specified by the control law. A guided missile homing system uses a coordinator to determine the missile's location relative to the target throughout the flight. When using the proportional approach homing method, the main parameter measured in the coordinator is the angle of missile-target line rotation  $\varphi$ .

The angular rotation velocity of the missile-target line  $\dot{\varphi}$  is estimated by a coordinator filter, which can be a second order differentiating filter. Such a filter allows us to obtain an estimate of the filtered quantity (in our case, an estimate of the angle of missile-target line rotation  $\hat{\varphi}$ ), as well as its derivatives (estimates of the angular velocity  $\dot{\hat{\varphi}}$  and angular acceleration  $\ddot{\hat{\varphi}}$  of the missile-target line rotation).

The basic geometric relations for modeling of a guided missile homing process at an air target are presented in Fig. 1.



**Fig. 1.** Basic geometric relations for modeling of a guided missile homing process of at an air target

In the figure, the vertical plane containing the missile (M) and target (T) defines the pitch plane, and the horizontal plane, coinciding with the  $xy$ , plane, defines the yaw plane. The target position is specified by the coordinates:

$$x_t = R_t \cos \varphi_{tp} \cos \varphi_{ty}, \quad y_t = R_t \cos \varphi_{tp} \sin \varphi_{ty}, \quad z_t = R_t \sin \varphi_{tp},$$

where  $R_t$  is a range to target;  $\varphi_{tp}$  is an angle of inclination of the target line of sight in the pitch plane;  $\varphi_{ty}$  is an angle of inclination of the target line of sight in the yaw plane.

Coordinates defining the missile position:

$$x_m = R_m \cos \varphi_{mp} \cos \varphi_{my}, \quad y_m = R_m \cos \varphi_{mp} \sin \varphi_{my}, \quad z_m = R_m \sin \varphi_{mp},$$

where  $R_m$  is a range to missile;  $\varphi_{mp}$  is an angle of inclination of the missile line of sight in the pitch plane;  $\varphi_{my}$  is an angle of inclination of the missile line of sight in the yaw plane.

To simulate the homing process of a guided missile at an air target, the authors chose the state space method, which, unlike classical methods, allows for clear formalization and automation of computational procedures [4, 5].

The movement of a system in state space is represented by a curve and shows the change in the position of the state vector in this space. State space models describe the behavior of a control object or a system in the time domain and allow working not only with linear systems and zero initial conditions. The state space is a matrix form of recording a system of differential equations of an automatic control system. This form is adapted for control theory by extracting from the Cauchy form algebraic equations that connect the internal coordinates of the system with the output (outputs).

Thus, the missile coordinator filter in state space can be described by the equation:

$$\dot{\mathbf{X}}_f = \mathbf{A}_f \mathbf{X}_f + \mathbf{B}_f \mathbf{U}_f,$$

where:  $\mathbf{X}_f$  is a filter state vector;  $\mathbf{U}_f$  is a filter input vector;  $\mathbf{A}_f$  and  $\mathbf{B}_f$  are filter matrices.

The implementation of the proportional approach method in the missile control loop is ensured by a command generation device described by the equation of state:

$$\mathbf{X}_g = \mathbf{N}_g \mathbf{U}_g,$$

where:  $\mathbf{N}_g$  is a matrix taking into account the proportionality coefficient of the guidance method;  $\mathbf{U}_g$  is a vector of input influences, including estimates of the angular velocity of the missile-target line rotation in the pitch and yaw planes.

The missile is stabilized and its flight is controlled by the autopilot. In state space, the missile autopilot equation has the form:

$$\dot{\mathbf{X}}_{ap} = \mathbf{A}_{ap} \mathbf{X}_{ap} + \mathbf{B}_{ap} \mathbf{U}_{ap},$$

where:  $\mathbf{X}_{ap}$  is a autopilot state vector, including the values of angular velocities of the missile velocity vector rotation in the pitch and yaw planes;  $\mathbf{U}_{ap}$  is a vector of input influences on the autopilot (commands from the command generation device);  $\mathbf{A}_{ap}$  and  $\mathbf{B}_{ap}$  are autopilot matrices.

Equation of target and missile in state space, respectively:

$$\dot{\mathbf{X}}_t = \mathbf{A}_t \mathbf{X}_t + \mathbf{B}_t \mathbf{U}_t,$$

$$\dot{\mathbf{X}}_m = \mathbf{A}_m \mathbf{X}_m + \mathbf{B}_m \mathbf{U}_m,$$

where:  $\mathbf{X}_t$  is a target state vector;  $\mathbf{X}_m$  is a missile state vector;  $\mathbf{U}_t$  is a vector of input influences on the target;  $\mathbf{U}_m$  is a vector of input influences on the missile;  $\mathbf{A}_t$  and  $\mathbf{B}_t$  are target matrices;  $\mathbf{A}_m$  and  $\mathbf{B}_m$  are missile matrices.

The trajectories of a guided missile and a maneuvering air target obtained as a result of modeling in the MATLAB environment are presented in Fig. 2.

The matrix form used in state space has an undeniable advantage in numerical solutions, and the clarity of mathematical formulations and the solutions themselves does not deteriorate even for multidimensional systems that describe the behavior of complex systems.

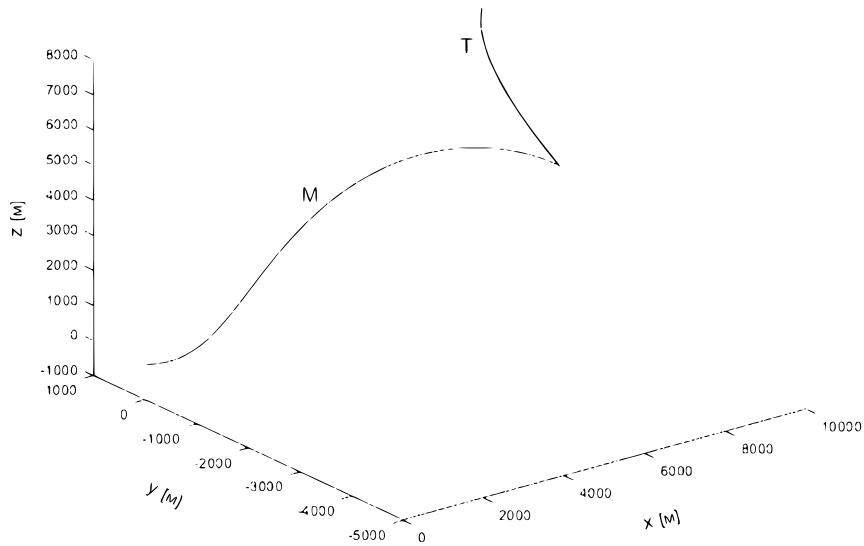


Fig. 2. Results of missile and maneuvering target trajectories modeling

## 2. Model of the Coverage Area of a Sectoral Anti-Aircraft Missile System Radar

Radars of modern anti-aircraft systems serve as means of detection, identification, tracking of air targets and guided missiles aimed at them, devices for transmitting control commands, as well as target illumination stations to ensure the operation of on-board radio direction finders of missiles. The use of phased arrays in radars in combination with high-speed computing systems makes it possible to simultaneously track several dozen targets and point more than ten guided missiles at them [6].

The radar coverage area is the area within which the station detects targets with a certain radar cross-section (RCS) with specified probabilities of correct detection and false alarm [7, 8, 9].

The coverage area of a sector radar is determined by the following parameters:

- The angular dimensions in azimuth  $\alpha$  and elevation  $\varepsilon$ ;
- The maximum and minimum radar range:

$$R_{\max}(\alpha, \varepsilon) = \sqrt[4]{\frac{P_i \cdot G_a^2(\alpha, \varepsilon) \cdot \lambda^2 \cdot \sigma_t}{(4\pi)^3 \cdot P_r \min}}, \quad R_{\min} = \frac{c \cdot \tau_i}{2},$$

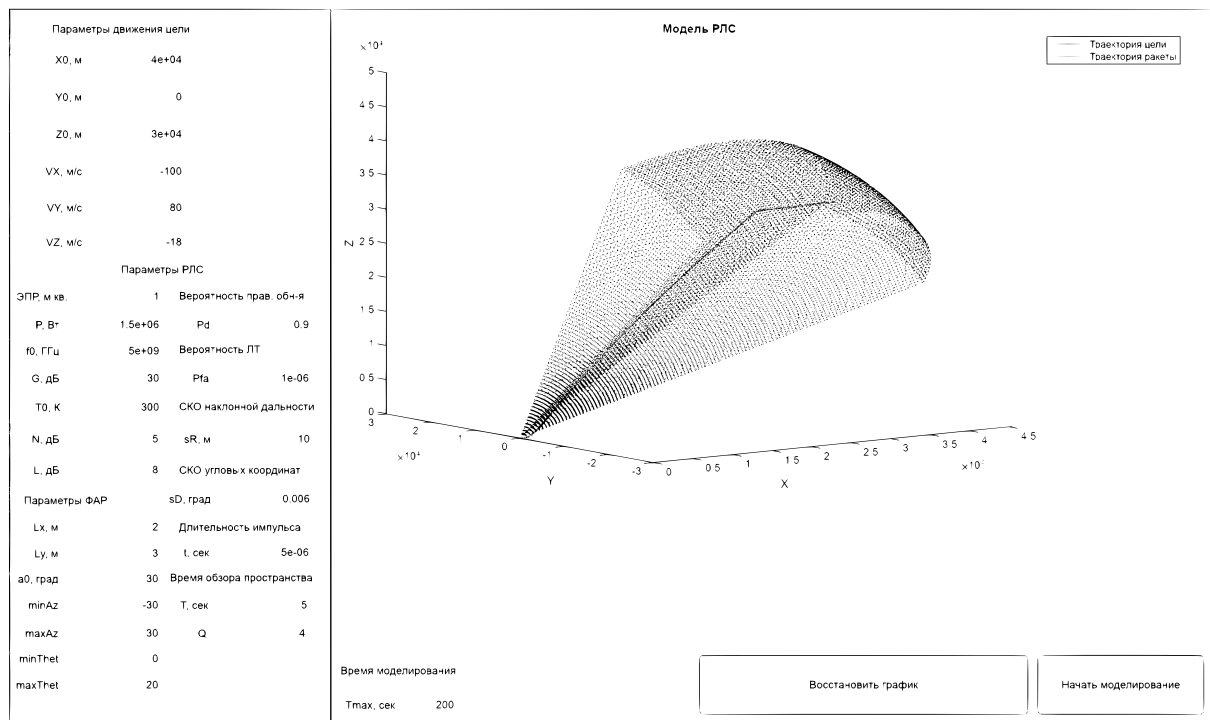
where  $P_i$  is a radar pulse power;  $G_a(\alpha, \varepsilon)$  is a phased array gain;  $\lambda$  is a radar operating wavelength;  $\sigma_t$  is RCS of the target;  $P_r \min$  is a radar receiver sensitivity;  $c = 3 \cdot 10^8$  m/s is a speed of light in free space;  $\tau_i$  is duration of the probing pulse.

Modeling the process of radar detection and tracking of an air target is based on determining the fact of entry of its movement trajectory into the boundaries of the sector radar coverage area and their subsequent filtering.

### 3. Software Application for Simulation of a Semi-Active Guided Missile Homing System at a Target

An important advantage of the MATLAB environment is that in addition to the usual computational models, it allows you to create software applications for simulation modeling, which can be supplemented with the necessary programmable functionality, in particular predefined scenarios.

Based on the above-described partial models using the MATLAB package, the authors developed a software application for simulation of a radar semi-active homing system for a guided missile at an air target. The developed application has an intuitive graphical interface that allows you to set the required parameters of an air target and radar (Fig. 3).



**Fig. 3.** Application interface for simulation of a semi-active homing system for a guided missile at an air target

Estimation (filtering) of the coordinates of the target's trajectory that fall within the boundaries of the simulated coverage area of the sector radar is carried out using the Kalman filter implemented in software of the application.

### Conclusion

The results of simulation modeling using the developed application confirmed the adequacy of the developed models with the calculated data characterizing the process of homing a guided missile at a target.

The proposed simulation model and software application can be used both in the development and improvement of computer training models, and as part of individual automated training systems for training operators of anti-aircraft missile systems in military universities and military training centers.

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*Ilya A. Chepurnov, student, Department of Mathematical and Computational Modelling, South Ural State University (Chelyabinsk, Russian Federation), chepurnov@bmstu.ru*

*Vladimir O. Chervakov, student, Department of Mathematical and Computational Modelling, South Ural State University (Chelyabinsk, Russian Federation), vchervakov@bmstu.ru*

*Denis S. Klygach, PhD (Engineering), Associate Professor, Mathematical and Computer Modelling Department, South Ural State University (Chelyabinsk, Russian Federation), klygachds@susu.ru*

*Daria V. Mavleeva, student, Department of Mathematical and Computational Modelling, South Ural State University (Chelyabinsk, Russian Federation), seleznevadv@mail.ru*

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## МОДЕЛЬ ПОЛУАКТИВНОЙ СИСТЕМЫ САМОНАВЕДЕНИЯ УПРАВЛЯЕМОЙ РАКЕТЫ

*И. А. Чепурнов, В. О. Черваков, Д. С. Клыгач, Д. В. Мавлеева*

В статье актуализирована проблема совершенствования математических моделей компьютерных тренажеров для подготовки операторов зенитных ракетных комплексов. Предложена разработанная модель радиолокационной полуактивной системы самонаведения управляемой ракеты на воздушную цель. Представлено описание программного приложения для имитационного моделирования радиолокационной полуактивной системы самонаведения управляемой ракеты на воздушную цель. Предложенная имитационная модель и программное приложение могут быть использованы как при разработке и совершенствовании компьютерных тренажерных моделей, так и в составе отдельных автоматизированных обучающих систем для подготовки операторов зенитных ракетных комплексов в военных вузах и военных учебных центрах.

*Ключевые слова: тренажер; зенитный ракетный комплекс; воздушная цель; радиолокационная станция; управляемая ракета.*

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*Чепурнов Илья Александрович, студент, кафедра математического и компьютерного моделирования, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), [chepurnov@bmstu.ru](mailto:chepurnov@bmstu.ru)*

*Черваков Владимир Олегович, студент, кафедра математического и компьютерного моделирования, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), [vchervakov@bmstu.ru](mailto:vchervakov@bmstu.ru)*

*Клыгач Денис Сергеевич, кандидат технических наук, доцент, кафедра математического и компьютерного моделирования, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), [klygachds@susu.ru](mailto:klygachds@susu.ru)*

*Мавлеева Дарья Владимировна, студент, кафедра математического и компьютерного моделирования, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), [seleznevadv@mail.ru](mailto:seleznevadv@mail.ru)*

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