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NUMERICAL STUDY OF THE INFLUENCE OF INTERPHASE INTERACTION ON THE DYNAMICS OF THE GAS PHASE OF A GAS SUSPENSION IN OBLIQUE SHOCK WAVE

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This paper presents a numerical model of the dynamics of a gas suspension in a channel with varying geometry. To model the dynamics of a gas suspension, a continuum technique was used to describe the dynamics of a heterogeneous medium. The carrier medium was described as a viscous, compressible and thermally conductive gas. For the carrier medium and the dispersed component, a complete hydrodynamic system of dynamics equations was solved, including equations for the conservation of density, equations for the conservation of the spatial components of momentum and energy. The interfacial momentum exchange included the dynamic Archimedes force, the force of added masses and the aerodynamic drag force. The heat exchange between the carrier medium and the dispersed phase was also taken into account. The flow of an inhomogeneous medium and a homogeneous gas was described in a channel with expansion. To describe the dynamics of a continuous medium in a non-rectangular region, a transition was made to generalized coordinates. To integrate the system of equations, the finite-difference MacCormack method of second order accuracy was used. To suppress numerical oscillations, a nonlinear correction scheme for the numerical solution was used. A comparison was made of the results of calculations carried out for the continuum model of the dynamics of a gas suspension and the solution of a two-dimensional system of Navier-Stokes equations with similar boundary conditions. As a result of numerical calculations, it was revealed that interphase interaction has a significant effect on the dynamics of the carrier medium in a gas suspension. The dynamics of the carrier medium in a gas suspension differs significantly from the dynamics of a homogeneous gas. Due to interphase interaction, the intensity of the flow of the carrier medium in a gas suspension is lower than in a homogeneous gas.

Keywords: explicit finite difference scheme; continuum modeling technique; interphase interaction; Navier-Stokes equation; gassuspension.

Introduction

An application of mathematical methods is the modeling of processes in fluid and gas mechanics [1–3]. Many problems in fluid and gas mechanics are nonlinear in nature and are solved primarily not by analytical [1], but by numerical methods. In [2], a numerical analysis of various modifications of the large particle method was carried out in relation to problems of wave gas dynamics. In article [3], the process of propagation of a shock wave in a flat layer, that is, a homogeneous mixture of two gases having different densities, is numerically simulated. Unlike classical hydrodynamics [1], in the dynamics of inhomogeneous media [4–8] the nature of the flow depends on the interphase interaction. In [4] a general theory of the dynamics of multiphase media is presented. In [5] the problems of the movement of gas-liquid media at high speeds are studied, the theoretical foundations, calculation methods and applied problems are outlined. In [6], questions of mathematical modeling of shock wave processes in multiphase media are considered, equations are given and the structure of a shock wave in a mixture of gases and in a two-phase mixture is found. In a one-dimensional approximation, based on the ideal gas model, numerical calculations of the dynamics of dusty, gas-droplet and powder media were carried out in [7]. In [8], numerical algorithms were developed and the results of calculations of shock wave and detonation processes in gas suspensions of metal particles are presented. In [9], a numerical algorithm was developed for solving the Riemann problem for models of compressible two-phase flow containing non-conservative terms responsible for the interaction of phases. In [1], the interaction of homogeneous and heterogeneous detonation waves in mixtures containing aluminum particles and water droplets was studied using the methods of mechanics of multiphase media. Modeling of detonation suppression using clouds of inert particles was carried out. In [11], calculation formulas are given and the calculation methodology is described in detail in relation to a single-speed model of a heterogeneous medium in the presence of gravitational forces. In [12], particle flow was numerically simulated to study the process of mass transfer of microparticles in an air duct and in channels with different characteristic flow sizes. The process of formation of particle agglomerations was considered. The flow of single particles in a curved channel has been studied. In [13] were considered the issues of application of classical methods of hydromechanics based on the theory of functions of a complex variable for modeling the dynamics of multiphase media.

An analysis of works devoted to the dynamics of inhomogeneous media shows that the problem of describing the dynamics of gas suspensions has various practical applications. The goal of many studies of gas suspension flows is to take into account the interphase interaction on the overall flow of the mixture. To simulate flows of inhomogeneous media, equilibrium models are used that describe the flow of an inhomogeneous medium with mathematical models of the dynamics of a homogeneous medium, taking into account coefficients that take into account the inhomogeneity of the moving medium. Also, the dynamics of the medium can be described taking into account the differences in the concentrations of the components but without taking into account the interphase exchange of momentum and heat transfer – the diffusion technique [4]. When modeling flows in which the components of the mixture have similar mass fractions and different states of aggregation, continuum mathematical models are the most effective [4–8]. The continuum technique for modeling the dynamics of inhomogeneous media takes into account interphase interaction (exchange of momentum and heat) and for each phase a complete hydrodynamic system of equations is integrated. In this work, the purpose of the study is to study the influence of interphase interaction on the dynamics of the carrier medium of a gas suspension. A gas suspension with a large volumetric content of the dispersed phase was considered. The flow was described in a channel with a narrowing.

1. Mathematical Model

The movement of the carrier medium is described by a system of Navier-Stokes equations taking into account interphase force interaction and heat transfer [14–18]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0, \tag{1}$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + p - \tau_{xx}) + \frac{\partial}{\partial y}(\rho uv - \tau_{xy}) = -F_x + \alpha \frac{\partial p}{\partial x}, \tag{2}$$

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial}{\partial x}(\rho uv+p-\tau_{yx})+\frac{\partial}{\partial y}(\rho v^2+p-\tau_{yy})=-F_y+\alpha\frac{\partial p}{\partial y}, \tag{3}$$

$$\frac{\partial e}{\partial t}+\frac{\partial}{\partial x}\left([e+p-\tau_{xx}]u-\tau_{xy}v-\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left([e+p-\tau_{yy}]v-\tau_{yx}u-\lambda\frac{\partial T}{\partial y}\right)= \tag{4}$$

$$=-Q-(|F_x|(u-u_1)+|F_y|(v-v_1))+\alpha\left(\frac{\partial(pu)}{\partial x}+\frac{\partial(pv)}{\partial y}\right).$$

Closing relations for equations (1)–(4):

$$p=(\gamma-1)(e-\rho(u^2+v^2)/2),\ e=\rho(I+(u^2+v^2)/2),\ D=\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right),$$

$$\tau_{xx}=\mu\left(2\frac{\partial u}{\partial x}-\frac{2}{3}D\right),\ \tau_{yy}=\mu\left(2\frac{\partial v}{\partial y}-\frac{2}{3}D\right),\ \tau_{xy}=\tau_{yx}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right).$$

The dynamics of the dispersed phase is described by the equation of conservation of the average density of the dispersed phase, the equations of conservation of momentum components and the equation of energy conservation [7]:

$$\frac{\partial\rho_1}{\partial t}+\frac{\partial(\rho_1u_1)}{\partial x}+\frac{\partial(\rho_1v_1)}{\partial y}=0, \tag{5}$$

$$\frac{\partial(\rho_1u_1)}{\partial t}+\frac{\partial}{\partial x}(\rho_1u_1^2)+\frac{\partial}{\partial y}(\rho_1u_1v_1)=F_x-\alpha\frac{\partial p}{\partial x}, \tag{6}$$

$$\frac{\partial(\rho_1v_1)}{\partial t}+\frac{\partial}{\partial x}(\rho_1u_1v_1)+\frac{\partial}{\partial y}(\rho_1v_1^2)=F_y-\alpha\frac{\partial p}{\partial y}, \tag{7}$$

$$\frac{\partial e_1}{\partial t}+\frac{\partial}{\partial x}(e_1u_1)+\frac{\partial}{\partial x}(e_1v_1)=Q. \tag{8}$$

The index "1" refers to the physical quantities of the dispersed phase; variables without an index describe changes in the physical parameters of the carrier medium. The following notations are used in the equations: ρ is gas density, u, v are components of the gas velocity vector $V=[u,v]$, e and T are energy and temperature of the carrier medium, p is gas pressure. Here λ, μ, γ are the coefficients of thermal conductivity, viscosity and adiabatic constant for the carrier medium, $I=RT/(\gamma-1)$ internal energy of the carrier medium (R is the gas constant), $\tau_{xx},\tau_{xy},\tau_{yy}$ are components of the viscous stress tensor of the carrier medium. For the dispersed phase, the following notations are used: α – volumetric content of the dispersed phase, $\rho_1=\alpha\rho_{10}$ – average density of the dispersed phase, ρ_{10} – physical density of the dispersed phase material, u_1, v_1 components of the dispersed phase velocity vector $V_i=[u_1,v_1]$, e_1 and T_1 thermal energy and temperature of the dispersed phase, $e_1=\rho_1C_pT_1$, C_p – heat capacity of the dispersed phase. The spatial components of the vector of interphase momentum exchange F_x, F_y are determined by expressions that include several forces of interaction between the carrier medium and particles $F_x=F_{xd}+F_{xA}+F_{xm}$, $F_y=F_{yd}+F_{yA}+F_{ym}$. As an interphase exchange of momentum, we took into account the aerodynamic drag force F_{xd}, F_{yd}, and the Archimedes dynamic force and F_{xA}, F_{yA}, and the force of the attached masses F_{xm}, F_{yim} [4]:

$$F_{xd}=\frac{3\alpha}{4d}C_d\rho\sqrt{(u-u_1)^2+(v-v_1)^2}\,(u-u_1)\,,$$

$$F_{xA} = \alpha\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right),$$

$$F_{xm} = 0.5\alpha\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\partial u_1}{\partial t} - u_1\frac{\partial u_1}{\partial x} - v_1\frac{\partial u_1}{\partial y}\right),$$

$$F_{yd} = \frac{3\alpha}{4d}C_d\rho\sqrt{(u_1-u_1)^2 + (v-v_1)^2}(v-v_1),$$

$$F_{yA} = \alpha\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right),$$

$$F_{ym} = 0.5\alpha\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} - \frac{\partial v_1}{\partial t} - u_1\frac{\partial v_1}{\partial x} - v_1\frac{\partial v_1}{\partial y}\right).$$

The heat exchange of the carrier medium with the dispersed phase is described by the following expression [7]:

$$Q = 6\alpha\lambda Nu_1(T - T_1)/d^2.$$

In the expressions for the aerodynamic drag force, C_d is the drag coefficient of the dispersed phase particle. All particles are assumed to be spherical in shape. Heat transfer and momentum exchange between the components of a gas suspension are specified by the following parameters [7]: relative Mach number M_1, relative Reynolds number Re_1, relative Nusselt number Nu_1 and the Prandtl number Pr:

$$C_d = C_d^0\phi(M_1)\varphi(\alpha), \quad C_d^0 = \frac{24}{Re_1} + \frac{4}{Re_1^{0.5}} + 0.4,$$

$$\phi(M_1) = 1 + \exp(-\frac{-0.427}{M_1^{0.63}}), \quad \varphi(\alpha) = (1-\alpha)^{-2.5},$$

$$Re_1 = d\rho|\mathbf{V} - \mathbf{V}_1|/\mu, \quad M_1 = |\mathbf{V} - \mathbf{V}_1|,$$

$$Pr = c_p\mu(\lambda)^{-1},$$

$$Nu_1 = 2\exp(-M_1) + 0.459Re_1^{0.55}Pr^{0.33}.$$

Here c_p is the heat capacity of the gas. When determining the drag coefficient C_d, the function $\varphi(\alpha)$ takes into account the multiplicity of particles [7]. This mathematical model does not take into account the collision of particles of the dispersed phase; the mathematical model of the dynamics of a polydisperse gas suspension described above describes the flows of inhomogeneous media in a wide range of relative Mach and Reynolds numbers [7]: $0 < M_1 \leq 2$, $0 < Re_1 \leq 2 \cdot 10^5$.

2. Numerical Method

When modeling flow in an area with complex geometry, the Thompson method was used [2,13,19]. To integrate the system of equations, a transition from physical coordinates (x, y) to generalized coordinates (ξ, η) was used $x = x(\xi, \eta), y = y(\xi, \eta)$. The system of equations was solved in generalized coordinates. The system of equations (1)–(8) was integrated using the explicit finite-difference MacCormack method of second order accuracy [2]. Let us consider a numerical algorithm using the example of a scalar nonlinear partial differential equation (9) for the function f, where $a(f)$, $b(f)$ $c(f)$ are nonlinear functions:

$$\frac{\partial f}{\partial t} + \frac{\partial a(f)}{\partial \xi} + \frac{\partial b(f)}{\partial \eta} = c(f) \tag{9}$$

For the nonlinear equation (9), the numerical solution by the explicit finite-difference MacCormack method on the nth time layer is written as follows [2]:

$$f^*_{jk} = f^{n-1}_{jk} - \frac{\Delta t}{\Delta \xi}(a^{n-1}_{j+1k} - a^{n-1}_{jk}) - \frac{\Delta t}{\Delta \eta}(b^{n-1}_{jk+1} - b^{n-1}_{jk}) + \Delta t c^{n-1}_{jk}, \tag{10}$$

$$f^n_{jk} = 0.5(f^*_{jk} + f^n_{jk}) - 0.5\frac{\Delta t}{\Delta \xi}(a^*_{jk} - a^*_{j-1k}) - 0.5\frac{\Delta t}{\Delta \eta}(b^*_{jk} - b^*_{jk-1}) + 0.5\Delta t c^*_{jk}. \tag{11}$$

Here Δt,$\Delta \xi$, $\Delta \eta$ steps in time variable and spatial directions. In order to suppress numerical oscillations, a nonlinear correction scheme for the grid function (10), (11) was used [14,21]. Let $Z^n_{j,k}$ be an arbitrary independent function on the n-th time layer at node j,k. Then the correction algorithm would have the following form:

$$Z^{n*}_{j,k} = Z^n_{j,k} + \kappa(\delta Z^n_{j+1/2,k} - \delta Z^n_{j-1/2,k}),$$

where $Z^{n*}_{j,k}$ is the adjusted function. This algorithm is executed when

$$(\delta Z^n_{j-1/2,k}\delta Z^n_{j+1/2,k}) < 0 \text{ or } (\delta Z^n_{j+1/2,k}\delta Z^n_{j+3/2,k}) < 0.$$

Here $\delta Z^n_{j-1/2,k} = Z^n_j - Z^n_{j-1,k}, \delta Z^n_{j+1/2,k} = Z^n_{j+1,k} - Z^n_{j,k}, \delta Z^n_{j+3/2,k} = Z^n_{j+2,k} - Z^n_{j+1k}$, where κ is the correction coefficient. The size of the time step when implementing a numerical algorithm is selected based on the Courant–Friedrichs–Levy condition [2].

3. Computer Implementation

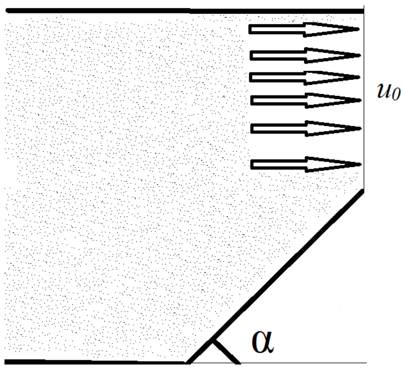


Fig. 1. Schematic representation of the simulated process

The implementation of the program code in the GNU–FORTRAN language has the following structure:
1) A finite-difference mesh is constructed.
2) The physical parameters of the phases are set.
3) The initial and boundary conditions for the carrier and dispersed phases are set.

4) The value of the interfacial heat and force exchange values is determined.
5) The main computational cycle is implemented, transferring the dependent gas-dynamic variables (gas, dispersed phase) to the next time step.
6) Nonlinear correction of grid functions is carried out. If necessary, the filtration mechanism is activated.
7) Boundary conditions are set on a new temporary layer. A comparison of the results of numerical calculations of the dynamics of a gas suspension using the described computational complex with analytical solutions of the dynamics of an ideal gas and an ideal equilibrium model of a gas suspension known from the literature is carried out in [18]. A study of the grid convergence of the numerical algorithm implemented in the software package was carried out in [15, 16]. On solid surfaces, homogeneous Dirichlet boundary conditions were specified for the velocity components, and homogeneous Neumann boundary conditions were specified for the remaining dynamic functions:

$$e(t,i,1) = e(t,i,2), e_1(t,i,1) = e_1(t,i,2),$$

$$e(t,i,N_2) = e(t,i,N_2-1), e_1(t,i,N_2) = e_1(t,i,N_2-1),$$

$$e(t,1,j) = e(t,2,j), e_1(t,1,j) = e_1(t,2,j),$$

$$e(t,N_1,j) = e(t,N_1-1,j), e_1(t,N_1,j) = e_1(t,N_1-1,j),$$

$$p(t,i,1) = p(t,i,2), p(t,i,N_2) = p(t,i,N_2-1),$$

$$p(t,1,j) = p(t,2,j), p(t,N_1,j) = p(t,N_1-1,j),$$

$$\rho(t,i,1) = \rho(t,i,2), \rho(t,i,N_2) = \rho(t,i,N_2-1),$$

$$\rho(t,1,j) = \rho(t,2,j), \rho(t,N_1,j) = \rho(t,N_1-1,j),$$

$$\rho_1(t,i,1) = \rho_1(t,i,2), \rho_1(t,i,N_2) = \rho_1(t,i,N_2-1),$$

$$\rho_1(t,1,j) = \rho_1(t,2,j), \rho_1(t,N_1,j) = \rho_1(t,N_1-1,j),$$

$$u(t,i,1) = 0, u_1(t,i,1) = 0, v(t,i,1) = 0, v_1(t,i,1) = 0,$$

$$u(t,i,N_2) = 0, u_1(t,i,N_2) = 0,$$

$$v(t,i,N_2) = 0, v_1(t,i,N_2) = 0,$$

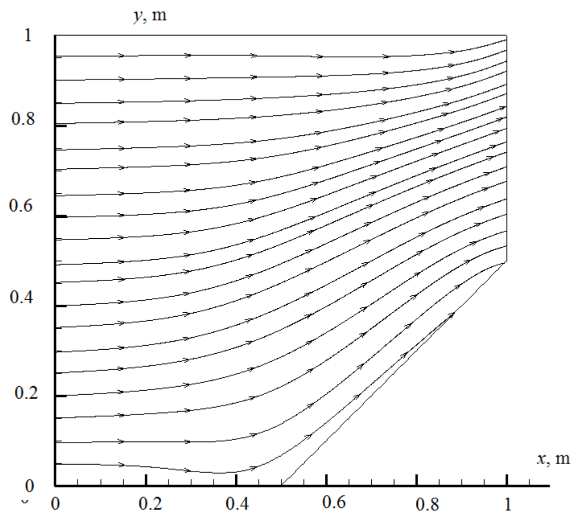
$$u(t,1,j) = u(t,2,j), u_1(t,1,j) = u_1(t,2,j),$$

$$v(t,1,j) = v(t,2,j), v_1(t,1,j) = v_1(t,2,j), u(t,N_1,j) = u_0,$$

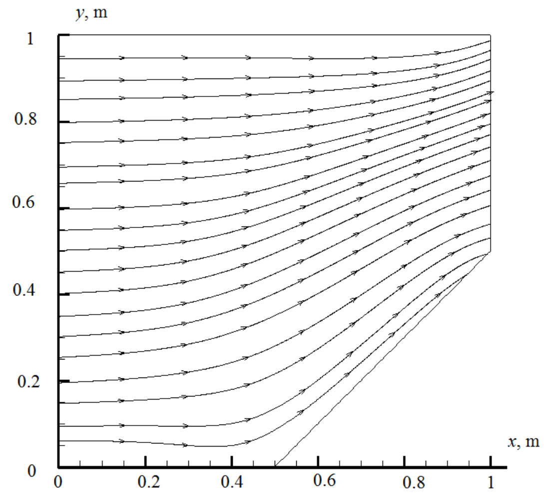
$$u_1(t,N_1,j) = u_1(t,N_1-1,j),$$

$$v(t,N_1,j) = v(t,N_1-1,j), v_1(t,N_1,j) = v_1(t,N_1-1,j).$$

Number of nodes in the longitudinal direction N_1, number of nodes in the transverse direction N_2.



a)



b)

Fig. 2. Flowlines: a) in the carrier medium; b) in the dispersed phase

4. Calculation Result

Air with water particles suspended in it was considered as a gas suspension. Physical density of the particle material ρ_{10}=1000 kg/m^3, heat capacity of the dispersed phase – c_d=4200 J/(kg·K), particle diameter d=2 μm, initial volumetric content of particles α_0=0.001. The initial density and temperature of the gas are $\rho = 1.29$ kg/m^3 and $T = 380$ K, respectively. Channel length L = 1m, channel width H = 1 m. Angle $\alpha = \pi/4$. At the initial moment of time the longitudinal component of the gas velocity vector is $u = u_0 = 136$ m/s, $v = 0$. At the exit from the channel the longitudinal gas velocity was set $u = u_0 = 136$ m/s. The grid included $N_1 = 200$ along the x coordinate and $N_2 = 200$ in the transverse direction, respectively (Fig. 1). When modeling, the following parameters of the carrier phase were set: $M = 29 \cdot 10^{-3}$ kg/mol is the molar mass of air, thermal conductivity of the carrier medium $\lambda = 0.02553$ W/(m·K), dynamic viscosity $\mu = 1.72 \cdot 10^{-5}$ Pa/s, $\gamma = 1.4$, $R = 8.31$ J/(mol·K).

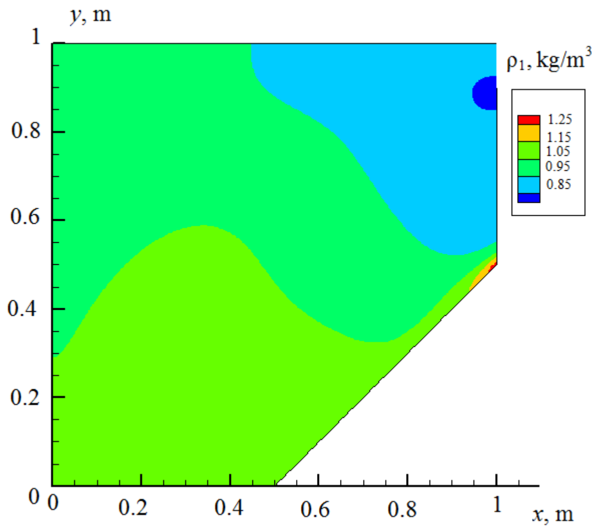


Fig. 3. Spatial distribution of the average density of the dispersed phase

Dispersed particles are set in motion by moving gas. Thus, the dynamics of the dispersed phase of the gas suspension is determined by the flow of the carrier medium. The flow lines of the gas and dispersed phases of the gas suspension are similar.

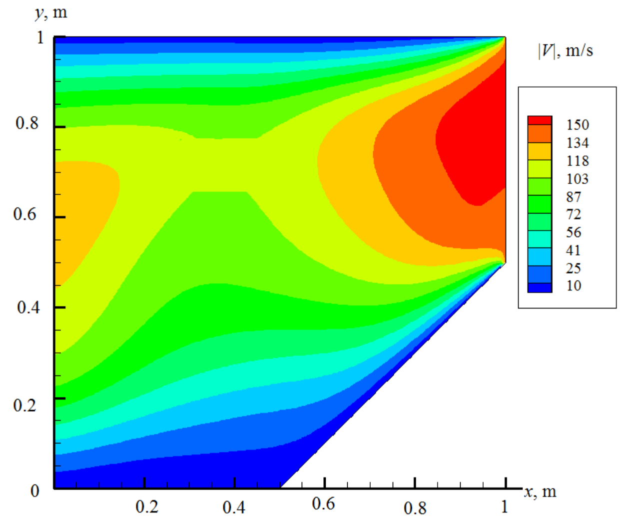
Fig. 3 shows the distribution of the average density of the dispersed phase at $t = 10$ ms. Near the channel outlet, a decrease in the average density of the dispersed phase is observed; in the lower part of the channel, near the ledge, an increase in the average density of the dispersed phase occurs.

Fig. 4 shows the spatial distribution of the modulus of velocity of a homogeneous gas and the carrier medium of a gas suspension. Due to the interphase interaction modulus of the flow velocity of the gas phase of the gas suspension reaches lower values in a heterogeneous medium in comparison.

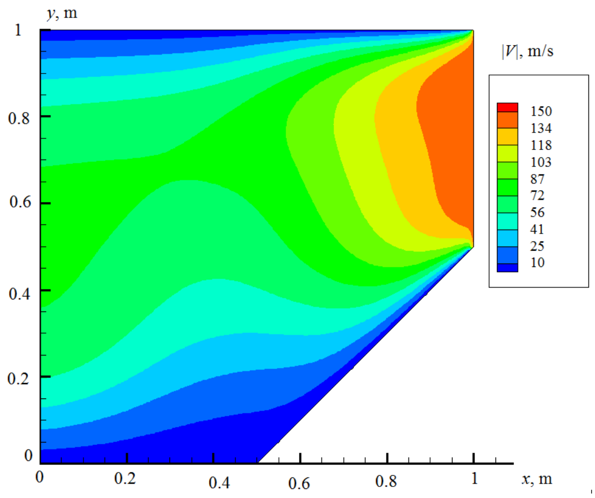
Fig. 5 shows the spatial distribution of pressure in a homogeneous gas, and the pressure of the gas phase of a gas suspension. In a homogeneous gas, the pressure reaches higher values than in a gas suspension. Both in a homogeneous gas and in the gas phase of a gas suspension, the highest pressure is achieved near the inclined surface of the channel.

In Fig. 6–8 show the spatial distributions of the gas velocity module, the longitudinal component of the velocity and the transverse component of the gas velocity. The quantitative value of the spatial components of the gas phase of a gas suspension is less important than in a homogeneous medium. Near the wedge (narrowing of the channel), the velocity of the homogeneous gas and the gas phase of the gas suspension reaches its lowest value.

In Fig. 9 and Fig. 10 there are spatial distributions of pressure and gas density for calculations of gas suspension and homogeneous gas flows. In numerical calculations, an increase in gas pressure and density is observed when flowing around a wedge. Which can be explained by the formation of an "oblique shock" of compaction. Analytical results for modeling an oblique shock wave for an ideal inviscid gas are described in the monograph [1]. In a gas suspension, the gas compaction reaches lower values than in a homogeneous medium, and therefore, in general, the pressure of the gas phase of the gas suspension reaches lower values than in a homogeneous gas. In a gas suspension, the pressure is



a)



b)

Fig. 4. Distribution of the modulus of velocity: a) of a homogeneous gas; b) of the carrier phase of the gas suspension

88.39% of the pressure in a homogeneous medium. The work [1] describes an analytical solution for changing the gas density (12), (13) when a flow of inviscid gas overcomes a step with an angle α at the top. Equation (12) was integrated by Newton's finite-difference method [22].

$$\mathrm{tg}(\alpha) = \frac{\sin^2(\beta) - \frac{c^2}{u_*^2}}{\sin^2(\beta) + \frac{c^2}{u_*^2} + \frac{\gamma+1}{2}}\mathrm{ctg}(\beta), \tag{12}$$

$$\rho_a = \rho_* \left(\frac{\frac{\gamma+1}{2}\sin^2(\beta)\frac{c^2}{u_*^2}}{1 + \frac{\gamma-1}{2}\sin^2(\beta)\frac{c^2}{u_*^2}} \right). \tag{13}$$

Here ρ_* gas density before flowing around the wedge, gas density after flowing around the wedge ρ_a, u_* is a longitudinal gas velocity before flowing around the wedge. The density

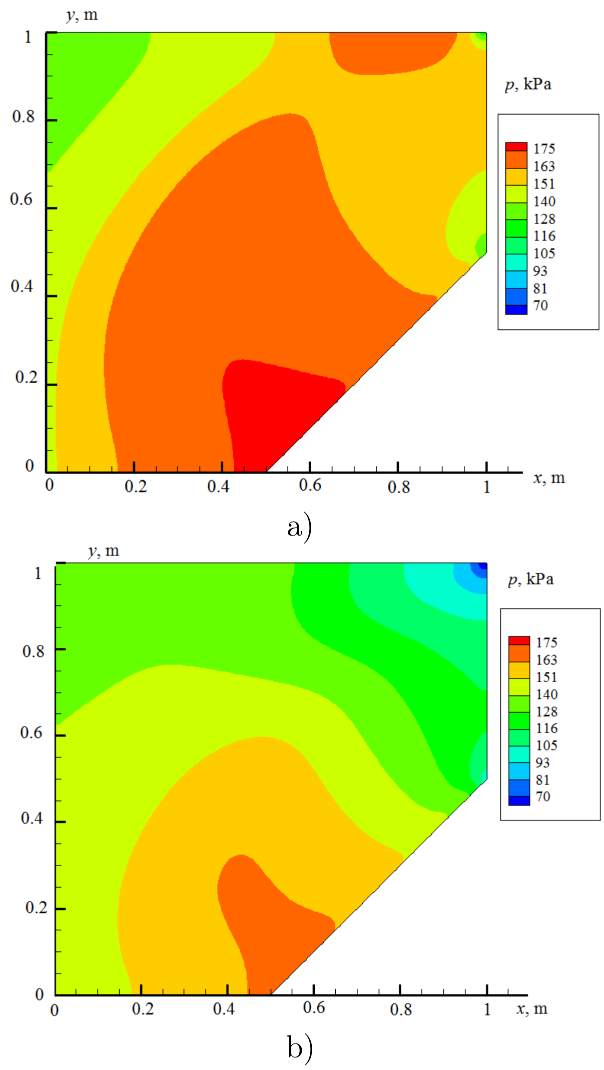


Fig. 5. Spatial distribution of pressure: a) of a homogeneous gas; b) of the carrier phase of the gas suspension

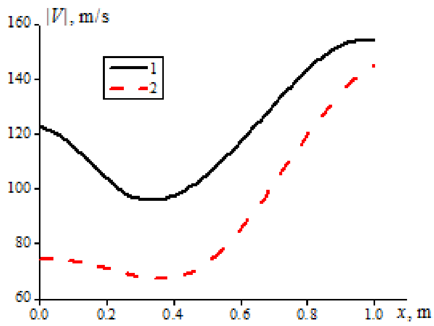


Fig. 6. Spatial distribution of the gas velocity modulus along the x axis ($y = H/2$), curve 1 is a homogeneous gas, curve 2 is a gas suspension

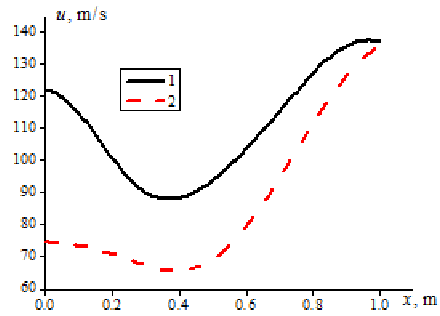


Fig. 7. Spatial distribution of the x component of gas velocity along the x axis ($y = H/2$), curve 1 is a homogeneous gas, curve 2 is a gas suspension

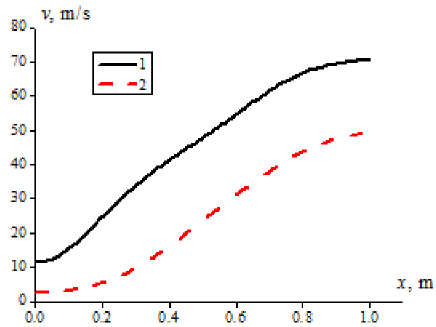


Fig. 8. Spatial distribution of the y-component of gas velocity along the x axis ($y = H/2$), curve 1 is a homogeneous gas, curve 2 is a gas suspension

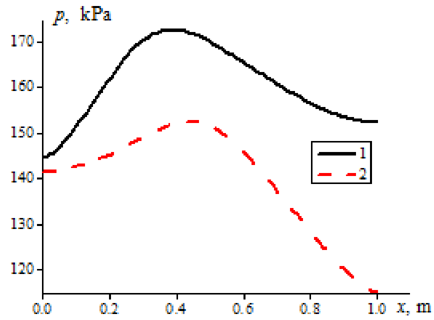


Fig. 9. Spatial distribution of gas pressure along the x axis ($y = H/2$), curve 1 is a homogeneous gas, curve 2 is a gas suspension

in the numerical solution for a gas suspension is ρ_{gs}/ρ_a=94.36% of the analytical solution, the gas density in the numerical solution for a homogeneous gas is ρ_{hg}/ρ_a=101.4%.

Conclusion

The work numerically simulated the flow of a gas suspension in a channel of varying cross-section. The dynamics of the gas suspension was simulated by a mathematical model that implements the continuum technique for modeling flows of inhomogeneous

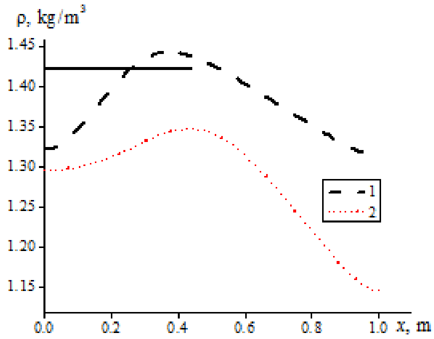


Fig. 10. Spatial distribution of gas density along the x axis ($y = H/2$), curve 1 is a homogeneous gas, curve 2is a gas suspension, straight line–analytical solution for inviscid gas

media. A comparison has been made of numerical calculations of the dynamics of a gas suspension and the flow described by the single-phase Navier-Stokes equation. Calculations demonstrate that the flow intensity in a gas suspension is significantly less than the flow intensity in a homogeneous medium. The revealed pattern can be explained by interfacial interaction. The calculation results can be used in the calculations of devices and industrial technologies in which flows of gas-dispersed media occur.

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ЧИСЛЕННОЕ ИССЛЕДОВАНИЕ ВЛИЯНИЯ МЕЖФАЗНОГО ВЗАИМОДЕЙСТВИЯ НА ДИНАМИКУ ГАЗОВОЙ ФАЗЫ ГАЗОВЗВЕСИ В КОСОМ СКАЧКЕ УПЛОТНЕНИЯ

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В данной работе представлена численная модель динамики газовзвеси в канале с изменяющейся геометрией. Для моделирования динамики газовзвеси применялась континуальная методика описания динамики неоднородной среды. Несущая среда описывалась как вязкий сжимаемый и теплопроводный газ. Для несущей среды и дисперсной компоненты решалась полная гидродинамическая система уравнений динамики включающая в себя уравнения сохранения плотности, уравнения сохранения пространственных составляющих импульса и энергии. Межфазный обмен импульса включал в себя динамическую силу Архимеда, силу присоединенных масс и силу аэродинамического сопротивления. Также учитывался теплообмен между несущей средой и дисперсной фазой. Течение неоднородной среды и однородного газа описывалось в канале с расширением. Для описания динамики сплошной среды в области с непрямоугольной формы осуществлялся переход к обобщенным координатам. Для интегрирования системы уравнений применялся конечно-разностным метод Мак-Кормака второго порядка точности. Для подавления численных осцилляций применялась схема нелинейной коррекции численного решения. Было проведено сопоставление результатов расчетов, проведенных для континуальной моделью динамики газовзвеси и решения двухмерной системы уравнений Навье–Стокса с аналогичным граничными условиями. В результате численных расчетов было выявлено, что межфазное взаимодействие оказывает существенное влияние на динамику несущей среды в газовзвеси. Динамика несущей среды в газовзвеси существенным образом отличается от динамики однородного газа. За счет межфазного взаимодействия интенсивность течения несущей среды в газовзвеси ниже, чем в однородном газе.

Ключевые слова: явная конечно-разностная схема; континуальная методика моделирования; межфазное взаимодействие; уравнение Навье–Стокса; газовзвесь.

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