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ON ONE INTERPRETATION OF THE NON-AUTONOMOUS LINEARIZED HOFF MODEL ON A GEOMETRIC GRAPH

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In the article, as an interpretation of the Hoff model on a geometric graph, it is proposed to consider the New York Twin Towers during the terrorist attack on September 11, 2001. Some of the design features of these towers can be well described within the framework of the Hoff model. Studies of the solution of the non-autonomous linearized Hoff model on a geometric graph have shown that such a structure has the property of stability and in the normal state the deflections of the beams from the zero position tend to zero. However, it is clear that as a result of the tragedy of September 11, 2001, conditions in the towers developed such that non-autonomy began to manifest itself in the model, due to which the critical load exceeded the thresholds and the structure lost stability. This was especially evident in the building of the World Trade Center 7, where there was no collision with an airplane and the building collapsed only as a result of a fire.

Keywords: stationary solution; asymptotic stability of solutions; I-beam construction.

Introduction

Consider the Hoff equation [1]

$$(\lambda - \lambda_0 + \Delta)u_t = \alpha u + \beta u^3 + f, \tag{1}$$

which, together with the boundary conditions, makes it possible to simulate the buckling of an I-beam, which is exposed to constant load, as well as high temperature. The set parameters α, $\beta \in \mathbb{R}$ reflect the characteristics of the properties of the I-beam material, whereas the parameters $\lambda, \lambda_0 \in \mathbb{R}_+$ are a characteristic of the load itself. The function $u = u(x,t)$ ($(x,t) \in \Omega \times \mathbb{R}$) helps to construct and describe the deviation of the I-beam from the vertical ($u = 0$), in which $\Omega \subset \mathbb{R}^m$ is a bounded domain within the boundaries of $\partial\Omega$ of the class C^∞. Note that the expression for the time derivative in the equation (1) can be zero and therefore (1) cannot be resolved relative to the time derivative. Such equations are called Sobolev type equations [2, 3, 4, 5, 6].

The dynamics of the I-beam structure is modeled by the Hoff equations

$$(\lambda_j - \lambda_0)u_{jt} + u_{jtxx} = \alpha_j u_j + \beta_j u_j^3 + f_j$$

given on a finite connected oriented graph $\mathbf{G} = \mathbf{G}(\mathfrak{V}, \mathfrak{E})$, where $\mathfrak{V} = \{V_i\}$ is a set of vertices and $\mathfrak{E} = \{E_i\}$ is a set of edges. Each edge E_j has a length $l_j \in \mathbb{R}_+$ and a cross-sectional area $d_j \in \mathbb{R}_+$. Function $u_j = u_j(x,t)$, $(x,t) \in (0, l_j) \times \mathbb{R}_+$ describes the deviation of the j-th beam from the equilibrium position. Parameters $\lambda_0, \lambda_j \in \mathbb{R}_+$ are the characteristics of the load on this beam, parameters $\alpha_j, \beta_j \in \mathbb{R}$ characterize the properties of the material of the j-th beam, and $f_j \equiv f_j(x,t)$ describes the external load.

Sobolev type equations on geometric graphs were first considered in [7]. The solvability of the Hoff model with constant coefficients on a graph within the framework of the Sobolev type equations theory was studied in [8, 9]. The optimal control problem for such models is considered, for example, in [10]. The solvability of non-autonomous Sobolev type equations was first considered in [13]. Using the proposed methods, various problems of [14, 15] were investigated, including a non-autonomous linearized Oskolkov model on a geometric graph [16]. When constructing a solution to a non-autonomous equation, we use the methodology proposed in [16, 17]. The solvability of the non-autonomous linearized Hoff model on geometric graphs was investigated in [18].

The solutions stability of Sobolev type equations with constant coefficients has been studied in many works (for more details, see [11]). Note that in such an analysis, information about the location of the relative spectrum of the problem is often used. However, this approach is not applicable for the Hoff model, since the relative spectrum of the model contains a point zero. Therefore, the second Lyapunov method [12] is used to analyze the stability of the solution. The stability of stationary solutions of the linearized Hoff model with constant coefficients on graphs is investigated in [12]. The stability of stationary solutions of linear non-autonomous Sobolev type equations was investigated in [19], and then these results were applied to the study of the stability of the zero solution of the linearized non-autonomous Hoff model on a geometric graph [20]. The purpose of this work is to correlate information about the buildings of the World Trade Center (WTC) in New York from the reports [21, 22] with the Hoff model.

1. Stability of the Null Solution of the Non-Autonomous Linearized Hoff Model On a geometric Graph

So let $\mathbf{G} = \mathbf{G}(\mathfrak{V}, \mathfrak{E})$ be a finite connected directed graph with a set of vertices $\mathfrak{V} = \{V_i\}$ and a set of edges $\mathfrak{E} = \{E_i\}$, where each edge E_j has a length of $l_j \in \mathbb{R}_+$ and a cross-sectional area of $d_j \in \mathbb{R}_+$. Consider on a geometric graph $\mathbf{G}$ the non-autonomous linearized Hoff equations

$$(\lambda - \lambda_0)u_{jt} + u_{jxxt} = \alpha_j(t)u_j \text{ for all } x \in (0, l_j), t \in \mathbb{R}, \tag{2}$$

where $u_j = u_j(x,t)$ $((x,t) \in (0,l_j) \times \mathbb{R}_+)$ characterizes the deviation of the j-th beam from the equilibrium position; parameters $\lambda_0, \lambda_j \in \mathbb{R}_+$ are the characteristics of the load on this beam, and the functions $\alpha_j(t)$ characterize the properties of the material of the j-th beam.

Denote by $E^{\alpha(\omega)}(V_i)$ a set of edges with the beginning (ending) at the vertex V_i, $t \in \mathbb{R}_+$, and set the “continuity” conditions

$$\begin{gathered} u_j(0,t) = u_k(0,t) = u_m(l_m,t) = u_n(l_n,t), \\ E_j, E_k \in E^{\alpha}(V_i), E_m, E_n \in E^{\omega}(V_i) \end{gathered} \tag{3}$$

at the vertices $\mathfrak{V}$ of the graph $\mathbf{G}$. For the equations (2) conditions (3) require continuity of solutions $u = (u_1, u_2, \ldots, u_j, \ldots)$ at the vertices $\mathfrak{V}$ of the graph $\mathbf{G}$ and mean that the beams are rigidly fixed at the nodes. In addition, we set the condition of "flow balance" at the vertices

$$\sum_{E_j \in E^\alpha(V_i)} d_j u_{jx}(0,t) - \sum_{E_k \in E^\omega(V_i)} d_k u_{kx}(l_k,t) = 0, \tag{4}$$

which is an analog of Kirchhoff's conditions for electric circuits. Note that the conditions (4) for equations (2) means that the nodes of the structure are stationary. So, the equations (2) with the conditions (3), (4) and some initial values

$$u_j(x,0) = u_{j0}(x), \tag{5}$$

are modeled in a linear approximation the buckling of I-beams in a structure, and (2) taking into account changes in material properties over time, which is described by scalar functions $\alpha_j(t)$.

The main result of [20] is to establish the asymptotic stability of the zero solution of the problem (2)–(5). That is means that small deviations from the equilibrium position tend to zero over time. So, the construction is described using such model is stable. Consequently, loss of stability may occur as a result of significant deviations (rupture), as well as a result of changes in essential characteristics of the model, for example, the appearance of creep of materials.

2. Interpretation of the Non-Autonomous Linearized Hoff Model on a Geometric Graph

2.1. General Information and the Concept of the Construction of the World Trade Center in New York

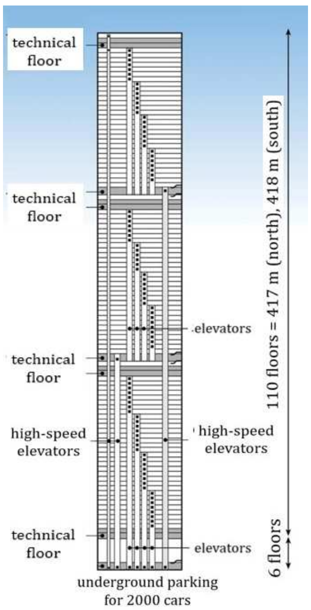

Fig. 1. The North and South towers of the WTC

The World Trade Center (WTC) consisted of several buildings (see Fig. 1) of different heights, located in lower Manhattan. The North and South Towers, built in 1970, were the tallest buildings in New York City and were a symbol of the city as well as an important tourist attraction.

The 110-storey buildings were 417 m high (WTC 1) and 418 m high (WTC 2). The height of each floor was 3.66 m, the ceiling height was 2.62 m, and the thickness of the floor (floor/trusses/ceiling) was 1.04 m. Each of the floors was a large space, virtually unbroken by columns or walls. This required a design solution to minimize the total mass of 110 floors, but strong enough to support a huge building and withstand lateral loads and excessive rocking. To understand the damage, let's take a closer look at the overall structure of the towers. Both WTC 1 and WTC 2 had a similar design. The columns supporting the building were located both on the outer edges and inside the core (see Fig. 2). The core contained elevators, stairwells and engineering shafts. The dense array of columns around the perimeter of the building had to withstand the lateral load due to hurricane winds, as well as distribute gravitational loads approximately gravitational loads approximately equally with the main columns. The floor system was supposed to provide rigidity and stability of the frame and tube system in addition to the load-bearing load on the floor.

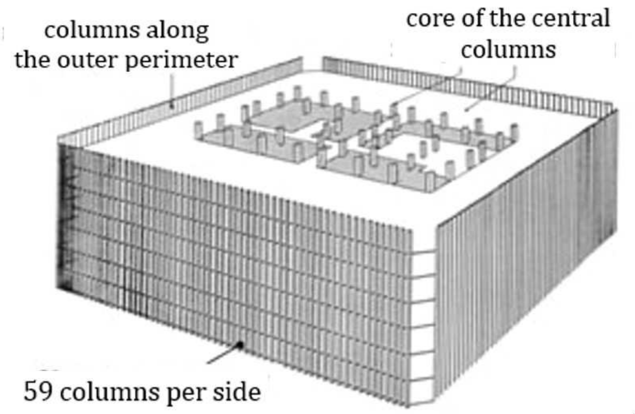

Fig. 2. The general structure of the floors of the twin towers

The outer wall consisted of 236 narrow facade columns, 59 on each side, all the outer columns and lintels were pre-made of welded panels three stories high and three columns wide (see Fig. 3). Inside each tower there was a core measuring 24 m by 42 m, consisting

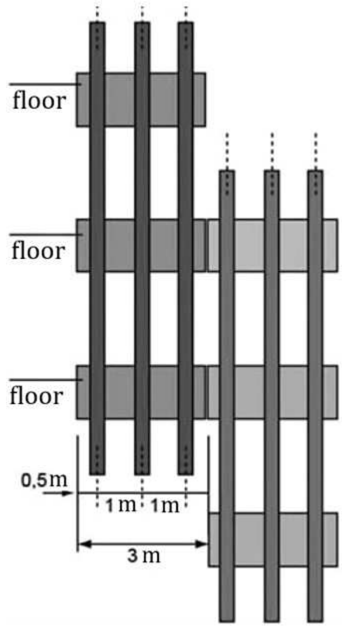

Fig. 3. Outer wall

of 47 supporting box-shaped columns, which decreased in size on the upper floors. Rectangular section up to the 85th floor and H-section from the 85th floor and above. The cross-section sizes ranged from 90 × 30cm for the inner core columns to 130 × 55 cm for the outer ones (see Fig. 4, 5).

The wall thickness of the columns on the lower floors reached 100 mm, on the upper floors ones is only 6 mm. Four massive corner columns carried almost a fifth of the total

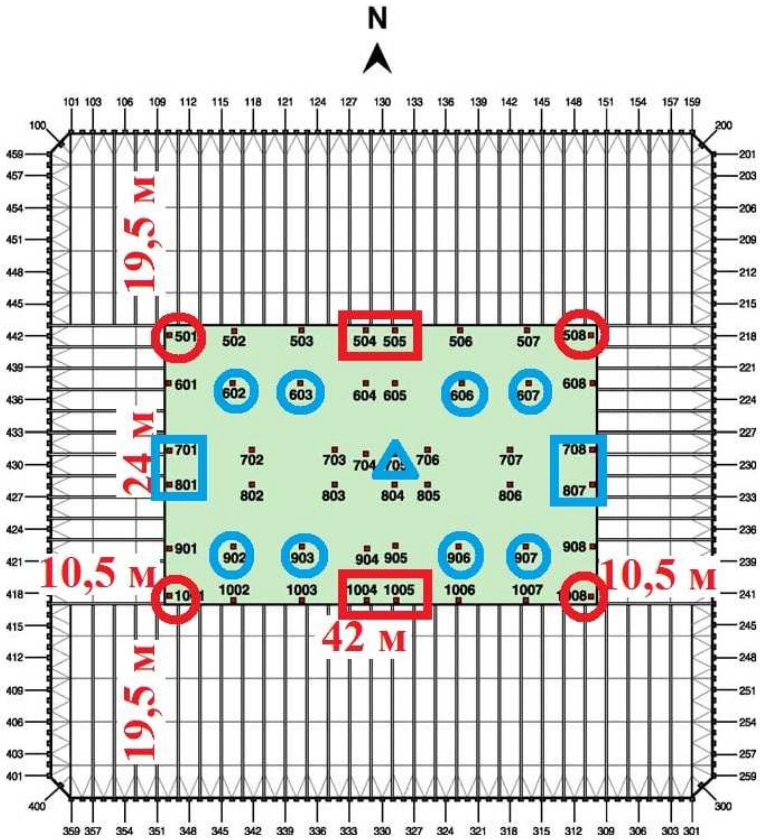

Fig. 4. Floor plan with indication of the types of column coupling

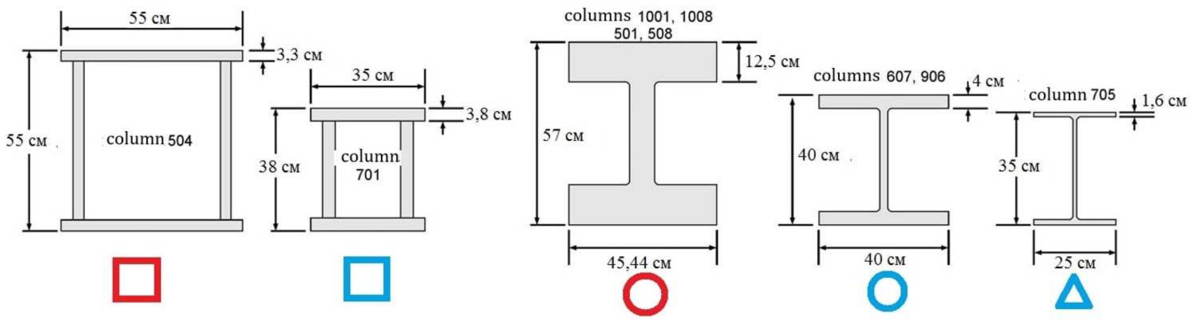

Fig. 5. Typical welded box elements and rolled wide shelves used for the main columns between the 83rd and 86th floors (in scale)

gravitational load on the main columns. The columns were interconnected by a grid of steel beams to support the floors.

The ceilings withstood their own weight, as well as temporary loads, provided lateral stability of the outer walls and distributed wind loads between the outer walls. The concrete slab was supported by a grid of lightweight steel rod trusses. The lower belts were connected to the jumpers using devices called viscoelastic dampers. When a strong wind hit the tower, these dampers absorbed energy, reducing the rocking and vibration expected from such a tall building. The use of such vibration damping devices in buildings was an innovation of that time.

The fourth main structural subsystem was located from the 107th floor to the roof of each tower. The hat truss (Fig. 6) provided additional connections between the main columns, as well as between the main and outer columns, ensuring load redistribution.

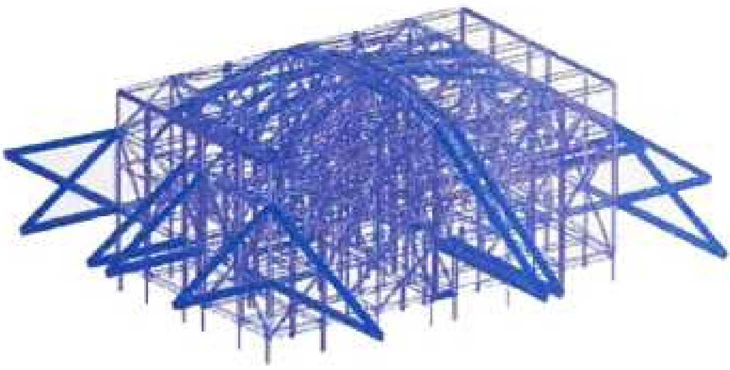

Fig. 6. The hat truss

General conclusions about the structural features of WTC buildings.

1. The main load is supported by columns (Fig. 3), which can be described using the edges of the graph **G**.

2. Fasteners and connections (Fig. 4, 5) satisfy the conditions (3), (4).

3. The hat truss (Fig. 6) is also a graph.

Thus, the construction of WTC buildings can be described within the framework of the Hoff model on the geometric graph (2)–(4). We emphasize that the non-autonomy of the model is manifested at high temperatures, so our model describes the dynamics of structures in case of fire.

2.2. Fire and Collapse of WTC 1 and WTC 2 Buildings

Boeing aircraft (767-200 series) had a length of 48.5 m, a wingspan of 47.6 m, a fuselage diameter of 5.3 m and a tail height of 15.8 m. There were 34,000 liters of fuel on board and they were moving at a speed of about 900 km/h. The first plane crashed into the north wall of the WTC 1 at 8:46 a.m. on September 11, 2001, and the second 16 minutes later into the south wall of the WTC 2. The planes hit the outer columns, floors and the core of the towers. Subsequent fires weakened the structure of the core, floors and exterior walls. The core weakened, the ceilings sagged, and the southern outer wall bent inward. At 10:28 a.m., about 102 minutes after the plane hit, WTC 1 began to collapse.

The twin towers of WTC 1 and WTC 2 were very stable structures. Their collapse was caused by many simultaneous factors, not just the fire [21]. Some of the conclusions about the disaster of September 11, 2001 were the following.

1. The WTC towers most likely would not have collapsed under the combined effects of damage from aircraft strikes and the extensive multi-storey fires that occurred on September 11, 2001, if the fire insulation had not been severely displaced or only minimally displaced by the impact of the aircraft.

2. In the absence of structural and insulation damage, an ordinary fire similar to the fire of September 11, 2001 or less intense, probably would not have led to the collapse of the WTC tower.

2.3. Description of the WTC 7 Fire

WTC 7 (Fig. 7) differed in many ways from the WTC towers. According to its design system, it was a more typical high-rise building. He was not shot down by a plane. The fires in WTC 7 were very different from the fires in the towers, since WTC 7 was not flooded with tons of jet fuel, large areas of any floor did not catch fire at the same time. Instead, the fires at WTC 7 were similar to those that occurred in several tall buildings where automatic sprinklers did not work or were missing. These other buildings did not collapse, and WTC 7 burned down.

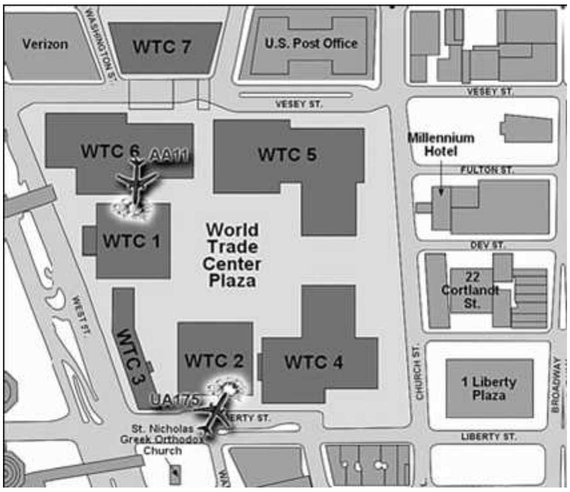

Fig. 7. The layout of the WTC buildings

WTC 7 was a 47-storey office building with an area of $200,000\text{m}^2$, located north of the WTC complex. It was built over the existing Con Edison electrical substation in the 1980s. Structurally, WTC 7 consisted of 4 tiers (Fig. 8).

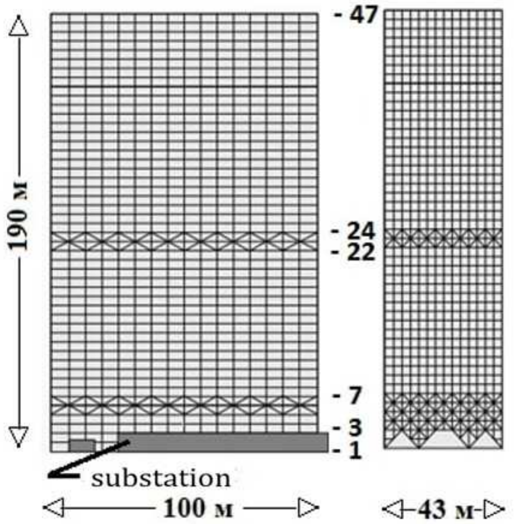

Fig. 8. The scheme of the WTC 7

1. Two two-storey lobbies are located on the lower four floors. The north side of the 1st and 2nd floors was a Con Edison substation.
2. Floors 5 and 6 were technical rooms.
3. Floors 7 to 45 were residential.
4. The 46th and 47th floors, mostly residential floors, have been structurally reinforced.

WTC 7 generally complied with New York City building codes in force at the time.

The tower system was designed to distribute the weight of the building (gravitational loads) and counteract (lateral) wind loads. The frame included columns, ceilings, lintels, beams and stairs. The lateral loads were perceived by the outer frame. Gravitational loads were approximately equally perceived by 58 external columns and 24 internal columns. From the 7th to the 47th floor, WTC 7 was supported by 24 internal columns and 58 external columns. Columns 58 to 78 formed the core of the building. Columns 79, 80 and 81 were particularly large and supported long flights of floors on the east side of the building. On September 11, 2001, fires burned for almost seven hours, from the moment of the collapse of WTC 1 at 10:28:22 to 17:20:52, when WTC 7 collapsed. The collapse of WTC 7 was the first known case of a complete collapse of a high-rise building mainly due to fires.

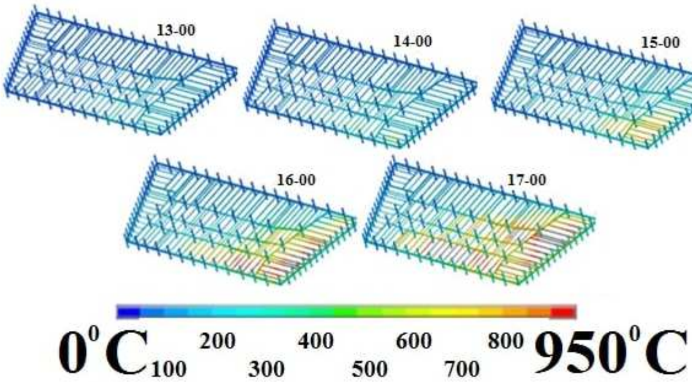

Fig. 9. Calculated temperature distribution (^{o}C) of the 13th floor steel frame at five different points in time

The collapse of WTC 1 damaged seven external columns between the 7th and 17th floors of the south and west sides of WTC 7. It also caused fires on at least 10 floors between the 7th and 30th floors, and fires got out of control from the 7th floor to the 9th and from the 11th to the 13th (Fig. 9). The fires on these six floors grew and spread because they were not extinguished by an automatic fire extinguishing system because there was no water in WTC 7. The fires were mainly concentrated on the eastern and northern sides of the house starting from about 15:00 to 16:00.

As the fire spread, some of the structural steel began to heat up. According to the generally accepted ASTM E-119 test standard, one of the criteria for determining the fire resistance class of a steel column or floor beam is determined by the time during which, under standard fire exposure, the average temperature of the column exceeds 538°C (1000°F) or the average temperature of the floor beam exceeds 593°C (1100°F).

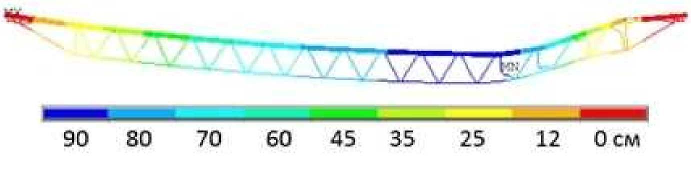

Fig. 10. Vertical displacement at 700^{o}C

These are temperatures at which there is a significant loss of strength and rigidity of steel. Due to the effectiveness of the fire protection coating, the highest temperature of the columns in WTC 7 reached about 300°C (570°F), and only on the eastern side of the building the floor beams reached or exceeded about 600°C (1100°F). The heat from these uncontrolled fires caused thermal expansion of steel beams on the lower floors of the east side of WTC 7, mainly at 400°C (750° F), damaging the floor frame on several floors (Fig. 10).

The damage to the columns was classified into four levels, as shown graphically in Fig. 11 (the colors represent the amount of plastic deformation, undamaged areas are shown in blue, and deformations at the level of 5% or higher are shown in red). Classification levels: light damage, medium damage, severe damage and rupture. The light level of damage had a low level of plastic deformations, but did not have significant structural deformations. The average level of damage had visible local deformations of the column cross-section (for example, local bending of the shelf), but without lateral displacements of the column centerline. The severe level of damage had significant deformations, which led to an irreversible deviation of the central line of the column. The out-of-order columns were completely torn apart and could not carry any load.

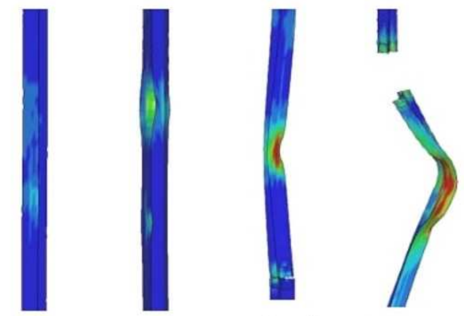

Fig. 11. Damage levels of the main columns

The initial local destruction, which marked the beginning of the probable sequence of the collapse of the WTC 7, was the warping of column 79. This warping occurred as a result of a process that occurred at temperatures of approximately 400°C (750°F), which is significantly lower than the temperatures considered in the existing practice of determining fire resistance indicators associated with a significant loss of steel strength. When steel (or any other metal) heats up, it expands. If columns or other steel elements counteract the thermal expansion of steel beams, forces arise in the structural elements that can lead to warping of the beams or failure of the joints.

The thermal expansion of the floor system around column 79 caused by the fire led to the collapse of floor 13, which caused a cascade of floor collapses. In this case, the floor beams on the east side of the building expanded so much that they moved the beam connecting the 79th and 44th columns to the west on the 13th floor (Fig. 12).

This movement was enough for the beam to come off the support on the 79th column. An unsupported beam and other damage caused by the fire led to the collapse of the 13th floor, which led to a cascade of floor collapses up to the 5th floor (which was much stronger). Many of these floors have already been at least partially weakened by fires in the area of Column 79. This left column 79 with insufficient lateral support, and as a result, the column buckled to the east, becoming the initial local failure to start the collapse.

Due to the warping of the 79th column between the 5th and 14th floors, the upper part of the 79th column began to descend (Fig. 13). The downward movement of column 79 led to the observed fracture of the east penthouse and its subsequent fall. Cascading collapses of the lower floors surrounding column 79 led to an increase in unsupported length, impacts from falling debris and a redistribution of loads on adjacent columns; and

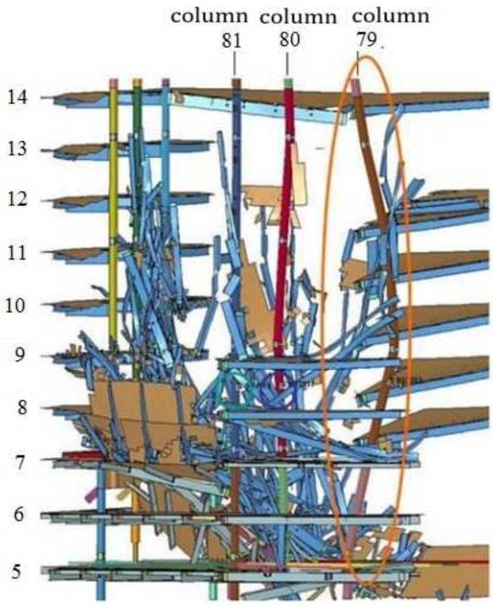

Fig. 12. The bend of column 79 to the east, when viewed from the southeast

column 80 and then column 81 also buckled. All floor connections to these three columns, as well as to the outer columns, failed, and the floors fell to the east side of the building.

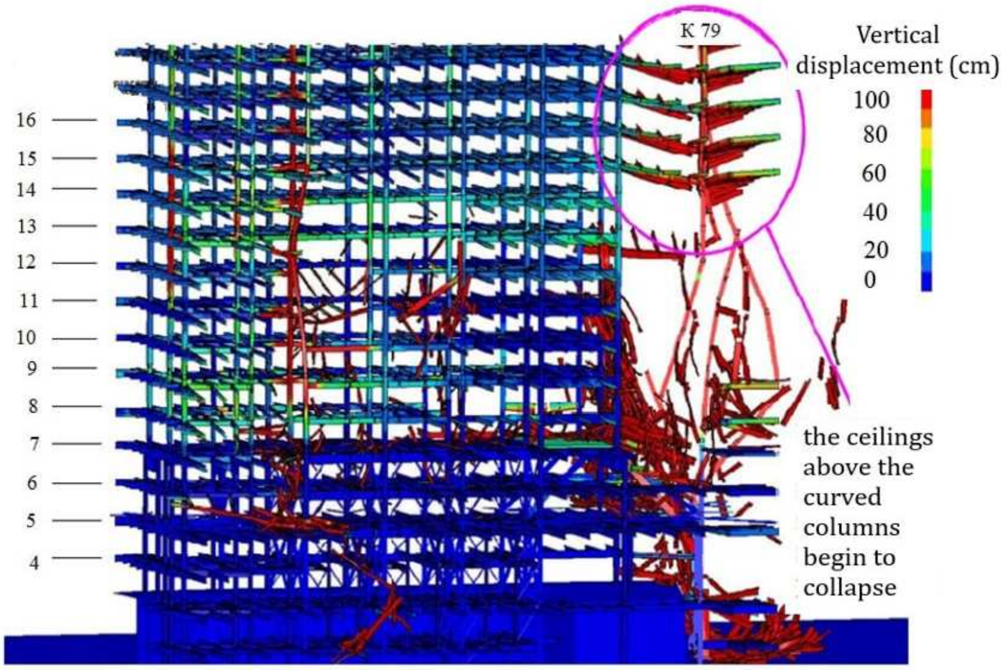

Fig. 13. The collapse of the WTC building 7

Then the destruction of the inner columns continued to the west. The farm collapsed due to the debris of falling floors. This led to the failure of columns 77 and 78, and soon columns 76. Then, each north-south line of the three main columns successively bent from east to west due to the loss of lateral support due to failures of the ceiling system under the action of forces arising from falling debris, which sought to push the columns to the west, and to the loads redistributed to them from the curved columns. Within seconds, the entire core of the building buckled.

The collapse of WTC 7 was in full swing. The shell of the outer columns bent between the 7th and 14th floors, as the loads were redistributed to these columns due to the downward movement of the building core and floors. Then the entire building above the area of the curved column moved down as a single unit, completing the collapse of the building. The highest temperature of the steel column in WTC 7 reached about 300°C and only on the eastern side of the building. The steel floor beams were heated to 600°C. However, the fire-induced warping of the floor beams and damage to the joints, which caused the warping of the critical column, initiating the collapse, occurred at temperatures below about 400°C (where thermal expansion prevails). At temperatures above 600°C, there is a significant loss of strength and rigidity of steel. During the collapse of WTC 7, the loss of strength or rigidity of steel was not as important as the thermal expansion of steel structures caused by heating.

Factors contributing to the collapse of WTC 7 included: thermal expansion of building elements such as floor beams and trusses, which occurred at temperatures below those usually taken into account in current practice to assess fire resistance; a significant increase in the effects of thermal expansion due to large floor spans in the building; connections between structural elements designed for resistance vertical gravity forces rather than thermal horizontal or lateral loads; and a general structural system not designed to prevent progressive collapse caused by fire.

The long-term impact of fire on the load-bearing structures of the WTC 7 core was comparable to the fire resistance of this structure. Through the system of connections between the core and the outer columns, the load on the core, with the loss of its bearing capacity, was redistributed to the outer walls. As a result, this additional load reduced the critical heating temperature of the external columns. This led to the loss of stability of the outer columns and the destruction of the entire building of the WTC 7.

Conclusion

Thus, the structures of the WTC buildings can be described in a linear approximation using a non-autonomous linearized Hoff model on a geometric graph. The complication of the (2)–(4) model will lead to a more subtle analysis of processes.

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ОБ ОДНОЙ ИНТЕРПРЕТАЦИИ НЕАВТОНОМНОЙ ЛИНЕАРИЗОВАННОЙ МОДЕЛИ ХОФФА НА ГЕОМЕТРИЧЕСКОМ ГРАФЕ

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В статье в качестве интерпретации модели Хоффа на геометрическом графе предлагается рассмотреть Нью-Йоркские башни близнецы во время теракта 11 сентября 2001 г. Некоторые конструктивные особенности этих башен можно хорошо описать в рамках модели Хоффа. Исследования решения неавтономной линеаризованной модели Хоффа на геометрическом графе показали, что такая конструкция обладает свойством устойчивости и в нормальном состоянии отклонения балок от нулевого положения стремятся к нулю. Однако ясно, что в результате трагедии 11 сентября 2001 года условия в башнях сложились такие, что в модели стала проявляться неавтономность в силу чего критическая нагрузка превысила пороговые значения и конструкция потеряла устойчивость. Особенно ярко это проявилось в здании Всемирного торгового центра 7, где не было столкновения с самолетом и здание обрушилось только в результате пожара.

Ключевые слова: стационарное решение; асимптотическая устойчивость решений; конструкция из двутавровых балок.

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