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# SOLVABILITY OF INITIAL PROBLEMS FOR ONE CLASS OF DYNAMICAL EQUATIONS IN QUASI-SOBOLEV SPACES

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The equations, which are not solved with respect to the highest derivative, are now actively studied. Such equations are also called the Sobolev type equations. Note that these equations in Banach spaces are studied quite well. Quasi-Sobolev spaces are quasi normalized complete spaces of sequences. Recently these spaces began to be studied. The interest to such spaces and its equations is connected with a desire to fill up the theory more than with practical applications.

The paper is devoted to the study of solvability of the Cauchy problem and the Showalter – Sidorov problem for a class of equations considered in the quasi-Sobolev spases. To this end we use properties of the equation operators, namely the relative boundedness of the operators. To illustrate abstract results we consider an analogue of the Hoff equation in the quasi-Sobolev spaces.

*Keywords: Cauchy problem; Showalter – Sidorov problem; Sobolev type equation; Laplase quasi-operator; analogue of the Hoff equation.*

## Introduction

Consider a class of equations having the following form:

$$P_n(\Lambda)\dot{u} = Q_m(\Lambda)u, \tag{1}$$

where $P_n(x) = \sum\limits_{i=0}^{n} c_i x^i$, $c_i \in \mathbb{C}$, $c_n \neq 0$ and $Q_m(x) = \sum\limits_{j=0}^{m} d_j x^j$, $d_j \in \mathbb{C}$, $d_m \neq 0$ are polynomials such that $m \leq n$. Operator $\Lambda : \ell_q^{r+2} \to \ell_q^r$ is Laplace quasi-operator [1], operating in quasi-Sobolev sequence spaces [2]

$$\ell_q^r = \left\{ u = \{u_k\} \subset \mathbb{R} : \sum_{k=1}^{\infty} \left( \lambda_k^{\frac{r}{2}} |u_k| \right)^q < +\infty \right\},$$

where $r \in \mathbb{R}$ and $q \in (0,1)$, and sequence $\{\lambda_k\} \subset \mathbb{R}_+$ is such that $\lim\limits_{k\to\infty} \lambda_k = +\infty$. Equation (1) is the Sobolev type equation, because an operator in the right-hand side of (1) can be equivalent to 0, see [3]. Sobolev-type equations in quasi-Banach spaces began to be studied more recently [4]. The interest to such spaces and its equations is connected with a desire to fill up the theory, extending its results to these spaces, more than with practical applications.

The paper is devoted to the questions about solvability of the Cauchy problem

$$u(0) = u_0 \tag{2}$$

for dynamic equations of form (1) in quasi-Sobolev spaces. Note that the solutions of Cauchy problem (2) for equation (1) for all initial conditions do not always exist [3]. Therefore we also consider the Showalter – Sidorov problem [5]

$$P(u(0) - u_0) = 0 \tag{3}$$

for heterogeneous equation (1), which solutions exist for all initial conditions. Here $P$ is some spectral projector [6].

The paper is organized as follows. Section 1 gives preliminary information about the properties of operators in quasi Banach spaces, as well as about the relatively bounded operator. Section 2 gives main result of the paper about solvability of Cauchy problem (2) for equation (1) and Showalter – Sidorov problem (3) for heterogeneous equation (1). Section 3 considers an analogue of the linearized Hoff equation [7]

$$(\lambda + \Lambda)u_t = \alpha u, \qquad \lambda, \alpha \in \mathbb{R}$$

in quasi-Sobolev spaces. Note that reference list reflects the tastes of the author and can be supplemented.

## 1. Relatively Spectral Bounded Operators in Quasi-Sobolev Spaces

Recall that *quasi-Banach space* is a complete quasinormed lineal. Let $\{\lambda_k\} \subset \mathbb{R}_+$ be monotonic sequence such that $\lim\limits_{k\to\infty} \lambda_k = +\infty$, a $q \in \mathbb{R}_+$. Let

$$\ell_q^r = \left\{ u = \{u_k\} \subset \mathbb{R} : \sum_{k=1}^{\infty} \left( \lambda_k^{\frac{r}{2}} |u_k| \right)^q < +\infty \right\}.$$

Lineal $\ell_q^r$ for all $r \in \mathbb{R}$, $q \in \mathbb{R}_+$ with element quasinorm $u = \{u_k\} \in \ell_q^r$

$${}_q^r\|u\| = \left( \sum_{k=1}^{\infty} \left( \lambda_k^{\frac{r}{2}} |u_k| \right)^q \right)^{1/q}$$

is quasi-Banach space (for $q \in [1, +\infty)$ — Banach space). In [2] spaces $\ell_q^m$ are called *quasi-Sobolev* ones. Note that there are dense and continuous embeddings $\ell_q^l \hookrightarrow \ell_q^r$ for $r \le l$.

Let $(\mathfrak{U};\ {}_{\mathfrak{U}}\|\cdot\|)$ and $(\mathfrak{F};\ {}_{\mathfrak{F}}\|\cdot\|)$ be quasi-Sobolev spaces. A linear operator $L : \mathfrak{U} \to \mathfrak{F}$ with definitional domain $\mathrm{dom}L = \mathfrak{U}$ is called *continuous*, if $\lim\limits_{k\to\infty} Lu_k = L\left(\lim\limits_{k\to\infty} u_k\right)$ for any sequence $\{u_k\} \subset \mathfrak{U}$, which converges in $\mathfrak{U}$. Note that in this case linear operator $L : \mathfrak{U} \to \mathfrak{F}$ is continuous, if it is bounded (that is, it maps bounded sets in bounded ones). Denote by $\mathcal{L}(\mathfrak{U};\mathfrak{F})$ a lineal (over the field $\mathbb{R}$) of bounded linear operators — quasi-Banach space with quasinorm

$${}_{\mathcal{L}(\mathfrak{U};\mathfrak{F})}\|L\| = \sup_{{}_{\mathfrak{U}}\|u\|=1} {}_{\mathfrak{F}}\|Lu\|.$$

Now let operators $L, M \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$. Following [3, s. 2.1], we consider $L$-resolvent set $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{F};\mathfrak{U})\}$ and $L$-spectrum $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ of operator $M$. Similarly remark 2.1.2 [3], it is easy to show that a set $\rho^L(M)$ is always open, therefore $L$-spectrum $\sigma^L(M)$ of operator $M$ is always closed. Furthermore, if $\rho^L(M) \ne \oslash$, then $L$-resolvent $(\mu L - M)^{-1}$ of operator $M$ is analytic in $\rho^L(M)$ [3, theorem 2.1.1]. We call an operator $M$ *$(L,\sigma)$-bounded*, if

$$\exists a \in \mathbb{R}_+ \ \forall \mu \in \mathbb{C} \ (|\mu| > a) \ \Rightarrow \ (\mu \in \rho^L(M)).$$

So let operator $M$ be $(L,\sigma)$-bounded. Select the contour $\gamma = \{\mu \in \mathbb{C} : |\mu| = h > a\}$ and construct the operators

$$P = \frac{1}{2\pi i}\int\limits_{\gamma} R^L_\mu(M)d\mu \quad \text{and} \quad Q = \frac{1}{2\pi i}\int\limits_{\gamma} L^L_\mu(M)d\mu,$$

where the integrals are Riman ones and exist by Theorem 2 [8] by the analyticity of right $R^L_\mu(M) = (\mu L - M)^{-1}L$ and left $L^L_\mu(M) = L(\mu L - M)^{-1}$ $L$-resolvents of operator $M$. Also by the analyticity of $R^L_\mu(M)$ and $L^L_\mu(M)$, operators $P$ and $Q$ do not depend on the radius $h$ of contour $\gamma$. Similarly the proof of [3, Lemma 4.1.1], it is easy to see that the operators $P \in \mathcal{L}(\mathfrak{U})$ $(\equiv \mathcal{L}(\mathfrak{U};\mathfrak{U}))$ and $Q \in \mathcal{L}(\mathfrak{F})$ are projections. Let $\mathfrak{U}^0 = \ker P$, $\mathfrak{U}^1 = \mathrm{im} P$, $\mathfrak{F}^0 = \ker Q$, $\mathfrak{F}^1 = \mathrm{im} Q$; and denote by $L_k$ $(M_k)$ a restriction of the operator $L$ $(M)$ on $\mathfrak{U}^k$, $k = 0, 1$.

**Theorem 1.** [4] *Let operators $L, M \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$, and operator $M$ be $(L,\sigma)$-bounded. Then*
(i) *operators* $L_k, M_k \in \mathcal{L}(\mathfrak{U}^k;\mathfrak{F}^k)$, $k = 0, 1$;
(ii) *there exist operators* $L_1^{-1} \in \mathcal{L}(\mathfrak{F}^1;\mathfrak{U}^1)$ *and* $M_0^{-1} \in \mathcal{L}(\mathfrak{F}^0;\mathfrak{U}^0)$.

Suppose that $H = M_0^{-1}L_0$, $S = L_1^{-1}M_1$. Obviously, operators $H \in \mathcal{L}(\mathfrak{U}^0)$, $S \in \mathcal{L}(\mathfrak{U}^1)$.

**Definition 1.** An operator $M$ is called
(i) $(L,0)$-*bounded*, if $H \equiv \mathbb{O}$;
(ii) $(L,p)$-*bounded*, if $H^k \neq \mathbb{O}$ for $k = \overline{1,p}$, and $H^{p+1} \equiv \mathbb{O}$;
(iii) $(L,\infty)$-*bounded*, if $H^k \neq \mathbb{O}$ for $k \in \mathbb{N}$.

## 2. Solvability of Initial Problems

Let $\mathfrak{U}$ and $\mathfrak{F}$ be quasi-Banach spaces, operators $L, M \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$. Consider the linear Sobolev type equation

$$L\dot{u} = Mu. \tag{4}$$

Vector function $u \in C^\infty(\mathbb{R};\mathfrak{U})$ is called *a solution of equation* (4), if it satisfies (4). A solution $u = u(t)$ of equation (4) is called a solution of Cauchy problem (2) for equation (4) (briefly, problem (2), (4)), if in addition it satisfies Cauchy condition (2) for some $u_0 \in \mathfrak{U}$. Similarly, a solution $u = u(t)$ of equation (4) is called a solution of Showalter – Sidorov problem (3) for equation (4) (briefly, problem(3), (4)), if in addition it satisfies Showalter – Sidorov condition (3) for some $u_0 \in \mathfrak{U}$.

**Definition 2.** A set $\mathfrak{P} \subset \mathfrak{U}$ is called *phase space* of equation (4), if
(i) for any $u_0 \in \mathfrak{P}$ there exists a unique solution of problem (2), (4);
(ii) any solution $u = u(t)$ of equation (4) is in $\mathfrak{P}$ as a trajectory (that is, $u(t) \in \mathfrak{P}$ for all $t \in \mathbb{R}$).

**Theorem 2.** *Let operator $M$ be $(L,p)$-bounded, $p \in \{0\} \cup \mathbb{N}$. Then phase space of equation* (4) *is space* $\mathfrak{U}^1$.

*Proof.* By theorem 1, equation (4) equivalents to a system of two equations

$$H\dot{u}^0 = u^0, \qquad \dot{u}^1 = Su^1, \tag{5}$$

where $u^0 = u^0(t) \in \mathfrak{U}^0$ and $u^1 = u^1(t) \in \mathfrak{U}^1$ for all $t \in \mathbb{R}$. We differentiate the first equation by $t$ and apply operator $H$ on the left, consistently get

$$0 = H^{p+1}\frac{d^{p+1}}{dt^{p+1}}u^0(t) = H^p\frac{d^p}{dt^p}u^0(t) = \ldots = Hu^0(t) = u^0(t).$$

So all solutions of equation (4) are in $\mathfrak{U}^1$ as the trajectories. A unique solvability of the problem $u^1(0) = u^1_0$ for the second equation (5) for any $u^1_0 \in \mathfrak{U}^1$ is obvious because of a boundedness of the operator $S \in \mathcal{L}(\mathfrak{U}^1)$.

□

We return to equation (1). Consider degrees of Laplace quasi-operators [9] $\Lambda^n u = \{\lambda_k^{2n}u_k\}$, $n \in \mathbb{N}$. It is easy to see that $\Lambda^n : \ell_q^{r+2n} \to \ell_q^r$ is toplinear isomorphism, $r \in \mathbb{R}$.

Select spaces $\mathfrak{U} = \ell_q^{r+2n}$ and $\mathfrak{F} = \ell_q^r$. Operators $L = P_n(x) = \sum\limits_{i=0}^{n} c_i x^i$, $c_i \in \mathbb{C}$, $c_n \neq 0$ and $M = Q_m(x) = \sum\limits_{j=0}^{m} d_j x^j$, $d_j \in \mathbb{C}$, $d_m \neq 0$, – polynomials such that $m \leq n$. Operators $L, M \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ by construction.

**Lemma 1.** *Let numbers $\lambda_k$ be roots of the polynomial $P_n(x)$ such that they are not roots of the polynomial $Q_m(x)$. Then an operator $M$ is $(L, 0)$-bounded.*

*Proof.* To construct the relative spectrum it is necessary that the condition $\ker L \cap \ker M = \oslash$ holds [3]. Therefore it is necessary that the numbers $\lambda_k$, which are roots of the polynomial $P_n(x)$, are not roots of the polynomial $Q_m(x)$. The relative spectrum is of the form

$$\sigma^L(M) = \left\{\mu \in \mathbb{C} : \mu_k = \frac{Q_m(\lambda_k)}{P_n(\lambda_k)}, \text{ при } k : P_n(\lambda_k) \neq 0\right\}.$$

Points of the relative spectrum $\sigma^L(M)$ tend to the end point, because $\lambda_k \to +\infty$ and $n \geq m$. Therefore, the set $\sigma^L(M)$ is bounded.

Space

$$\mathfrak{U}^0 = \begin{cases} \{0\}, & \text{if } P_n(\lambda_k) \neq 0 \text{ for all } k \in \mathbb{N}; \\ \{u \in \mathfrak{U} : u_k = 0, \ k \in \mathbb{N} \setminus \{l : P_n(\lambda_l) = 0\}\}, \end{cases}$$

therefore operator $H = M_0^{-1}L_0 = 0$. Consequently, the operator $M$ is $(L, 0)$-bounded.

□

Let $\{U^t : t \in \mathbb{R}\}$ be holomorphic degenerate group of operators, and $U^0$ be its unit. We consider an image $\mathrm{im}U^\bullet = \mathrm{im}U^0$ and a kernel $\ker U^\bullet = \ker U^0$ of this group. A group $\{U^t : t \in \mathbb{R}\}$ is called a resolving group of equation (4), if the following two conditions holds. First, vector function $u(t) = U^t u_0$ is a solution of equation (4) for any $u_0 \in \mathfrak{U}$. Second, an image $\mathrm{im}U^\bullet$ coincides with the phase space of equation (4).

**Theorem 3.** [4] *Let operator $M$ be $(L, p)$-bounded, $p \in \{0\} \cup \mathbb{N}$. Then there exists a unique group of solving operators for equation (4). This group is of the following form:*

$$U^t = \frac{1}{2\pi i}\int\limits_\gamma R^L_\mu(M)e^{\mu t}d\mu, \quad t \in \mathbb{R},$$

*where the contour $\gamma = \{\mu \in \mathbb{C} : |\mu| = h > a\}$.*

By Lemma 1 and Theorem 3 it is easy to show that holomorphic group of solving operators for equation (1) is of the following form:

$$U^t\cdot = \begin{cases} \sum\limits_{k=1}^{\infty} e^{\mu_k t}\langle \cdot, e_k\rangle e_k, & \text{если } P_n(\lambda_k) \neq 0,\ k \in \mathbb{N};\\ \sum\limits_{k\neq l} e^{\mu_k t}\langle \cdot, e_k\rangle e_k, & \text{если существует } l \in \mathbb{N}:\ P_n(\lambda_l) = 0. \end{cases}$$

The phase space of equation (1) by Lemma 1 and Theorem 2 is a set

$$\mathfrak{U}^1 = \begin{cases} \mathfrak{U}, & \text{если } P_n(\lambda_k) \neq 0,\ k \in \mathbb{N};\\ \{u \in \mathfrak{U}:\ u_l = 0,\ P_n(\lambda_l) = 0\}. \end{cases}$$

**Remark 1.** Under the hypotheses of Theorem 3, operators $L$ and $M$ on the space $\mathfrak{F}$ generate holomorphic degenerate group

$$F^t = \frac{1}{2\pi i}\int\limits_{\gamma} L^L_\mu(M)e^{\mu t}d\mu \quad -$$

group of solving operators for equation $L(\beta L - M)^{-1}\dot{f} = M(\beta L - M)^{-1}f$, where $\beta \in \rho^L(M)$.

Consider Showalter – Sidorov problem (3) for the linear inhomogeneous dynamical Sobolev-type equation

$$L\dot{u} = Mu + f, \tag{6}$$

where the vector function $f : [0, \tau] \to \mathfrak{U}$ with $\tau \in \mathbb{R}_+$ is defined below. Put $f = f^0 + f^1$, $f^1 = Qf$ and $f^0 = f - f^1$.

**Theorem 4.** [4] *Let $p \in \{0\}\cup\mathbb{N}$, $M$ be $(L, p)$-bounded operator, a vector function $f = f(t)$ with $f^0 \in C^{p+1}((0, \tau); \mathfrak{F}^0)$ and $f^1 \in C((0, \tau); \mathfrak{F}^1)$ as well as a vector $u_0 \in \mathfrak{U}$. Then there exists a unique solution $u \in C^1((0, \tau); \mathfrak{U})$ to problem (3) for equation (6), which in addition is of the form*

$$u(t) = -\sum_{k=0}^{p} H^k M_0^{-1} f^{0(k)}(t) + U^t u_0 + \int\limits_0^t U^{t-s}L_1^{-1}f^1(s)\,ds.$$

By Lemma 1 and Theorem 4 we have the following

**Theorem 5.** *Let numbers $\lambda_k$ be roots of the polynomial $P_n(x)$ such that they are not roots of the polynomial $Q_m(x)$, a vector function $f = f(t)$ with $f^0 \in C^{p+1}((0, \tau); \mathfrak{F}^0)$ and $f^1 \in C((0, \tau); \mathfrak{F}^1)$ as well as a vector $u_0 \in \mathfrak{U}$. Then there exists a unique solution $u \in C^1((0, \tau); \mathfrak{U})$ to problem (3) for equation $P_n(\Lambda)\dot{u} = Q_m(\Lambda)u + f$, which in addition is of the form*

$$u(t) = \begin{cases} \sum\limits_{k=1}^{\infty}\left(e^{\mu_k t}\langle u_0, e_k\rangle + \int\limits_0^t \dfrac{\langle f(s), e_k\rangle}{P_n(\lambda_k)}\, e^{\mu_k(t-s)}\right) e_k, & \text{если } P_n(\lambda_k) \neq 0,\ \forall k \in \mathbb{N};\\ -\sum\limits_{l\in\mathbb{N}:P_n(\lambda_l)=0} \dfrac{\langle f(t), e_l\rangle}{Q_m(\lambda_l)}\, e_l + \sum\limits_{k\neq l}\left(e^{\mu_k t}\langle u_0, e_k\rangle + \int\limits_0^t \dfrac{\langle f(s), e_k\rangle}{P_n(\lambda_k)}\, e^{\mu_k(t-s)}\right) e_k. \end{cases}$$

## 3. Hoff's Equation in Quasi-Sobolev Spaces

Consider the analogue of linear Hoff's equation [7]

$$(\lambda + \Lambda)u_t = \alpha u + f, \qquad \lambda, \alpha \in \mathbb{R}, \tag{7}$$

in the quasi-Sobolev spaces $\mathfrak{U} = \ell_q^{r+2}$ and $\mathfrak{F} = \ell_q^r$ with $r \in \mathbb{R}$ and $q \in \mathbb{R}_+$. In Banach spaces such equation was investigated widely, see [10, 11, 12] for example. Take the operators $L = P_1(\Lambda) = \lambda + \Lambda$ and $M = Q_0(\Lambda) = \alpha\mathbb{I}$ and reduce (7) to form (1). Moreover, by Lemma 1 the operator $M$ is an $(L, 0)$-bounded.

The phase spase of (7) has the form

$$\mathfrak{U}^1 = \begin{cases} \ell_q^{r+2}, & \text{if } \lambda_k \neq -\lambda \text{ for all } k \in \mathbb{N}; \\ \{u \in \ell_q^{r+2} : u_k = 0, \ \lambda_k = -\lambda\}. \end{cases}$$

In order to pose the Showalter – Sidorov problem, construct the projection $P$:

$$P = \begin{cases} \mathbb{I}, & \text{if } \lambda_k \neq -\lambda \text{ for all } k \in \mathbb{N}, \\ \mathbb{I} - \sum\limits_{k \in \mathbb{N}: k = l} e_k, & \text{if } \lambda_l = -\lambda \text{ for some } l \in \mathbb{N} \end{cases}$$

and similarly for the projection $Q$.

It is easy to construct the operator

$$L_1^{-1} = \begin{cases} \sum\limits_{k=1}^{\infty} (\lambda + \lambda_k)^{-1} e_k, & \text{if } \lambda_k \neq -\lambda \text{ for all } k \in \mathbb{N}, \\ \sum\limits_{k \in \mathbb{N}: k \neq l} (\lambda + \lambda_k)^{-1} e_k, & \text{if } \lambda_l = -\lambda \text{ for some } l \in \mathbb{N}. \end{cases}$$

By Theorem 5, for Showalter – Sidorov problem (3), (7) we have

**Corollary 1.** *Let $r, \lambda, \alpha \in \mathbb{R}$, $\tau, q \in \mathbb{R}_+$, $u_0 \in \mathfrak{U}$, $f^0 \in C^1((0,\tau); \mathfrak{F}^0)$, and $f^1 \in C((0,\tau); \mathfrak{F}^1)$. Then there exists a unique solution $u \in C^1((0,\tau); \mathfrak{U})$ to problem (3), (7); moreover, it is of the form*

$$u(t) = \begin{cases} \sum\limits_{k=1}^{\infty} \left( e^{\frac{\alpha t}{\lambda + \lambda_k}} \langle u_0, e_k \rangle + \int\limits_0^t \frac{\langle f(s), e_k \rangle}{\lambda + \lambda_k} e^{\frac{\alpha(t-s)}{\lambda + \lambda_k}} \right) e_k, & \text{if } \lambda_k \neq -\lambda, \ \forall k \in \mathbb{N}; \\ -\sum\limits_{l \in \mathbb{N}: \lambda_l = -\lambda} \frac{\langle f(t), e_l \rangle}{\alpha} e_l + \sum\limits_{k \neq l} \left( e^{\frac{\alpha t}{\lambda + \lambda_k}} \langle u_0, e_k \rangle + \int\limits_0^t \frac{\langle f(s), e_k \rangle}{\lambda + \lambda_k} e^{\frac{\alpha(t-s)}{\lambda + \lambda_k}} \right) e_k, \end{cases}$$

*here*

$$\mathfrak{F}^0 = \begin{cases} \{0\}, & \text{if } \lambda_k \neq -\lambda \text{ for all } k \in \mathbb{N}; \\ \{f \in \mathfrak{F} : f_k = 0, \ k \in \mathbb{N} \setminus \{l : \lambda_l = -\lambda\}\}; \end{cases}$$

$$\mathfrak{F}^1 = \begin{cases} \mathfrak{F}, & \text{if } \lambda_k \neq -\lambda \text{ for all } k \in \mathbb{N}; \\ \{f \in \mathfrak{F} : f_k = 0, \lambda_k = -\lambda\}. \end{cases}$$

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# РАЗРЕШИМОСТЬ НАЧАЛЬНЫХ ЗАДАЧ ДЛЯ ОДНОГО КЛАССА ДИНАМИЧЕСКИХ УРАВНЕНИЙ В КВАЗИСОБОЛЕВЫХ ПРОСТРАНСТВАХ

***Ф.Л. Хасан***

Уравнения, не разрешенные относительно старшей производной, в настоящее время являются активно изучаемой областью. Такие уравнения также называют уравнениями соболевского типа. В банаховых пространствах такие уравнения изучены довольно полно. Квазисоболевы пространства – это квазинормируемые полные пространства последовательностей. Эти пространства начали изучаться совсем недавно. Интерес к таким пространствам и уравнениям в них продиктован не столько практическими приложениями, сколько желанием пополнить теорию.

В данной работе изучается разрешимость задач Коши и Шоуолтера – Сидорова для одного класса уравнений, рассматриваемых в квазисоболевых пространствах. При этом использовались свойства операторов уравнения, а именно относительную ограниченность операторов. В качестве иллюстрации абстрактных результатов рассмотрен аналог уравнения Хоффа в квазисоболевых пространствах.

*Ключевые слова: задача Коши; задача Шоуолтера – Сидорова; уравнения соболевского типа; квазиоператор Лапласа; аналог уравнения Хоффа.*

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