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STABILIZATION OF SOLUTIONS TO A LINEAR SOBOLEV-TYPE EQUATION WITH A RELATIVELY SECTORIAL OPERATOR

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The article considers a linear equation of the Sobolev type with a relatively sectoral operator. This type of equation arises when modeling various processes: the evolution of the free surface of a liquid, the flow of a viscous incompressible liquid, plane-parallel thermoconvection of a viscoelastic incompressible liquid, etc. The paper considers the following stabilization problem: it is necessary to find a control action on the equation so that it becomes uniformly asymptotically stable. The solution to this problem is based on the theory of semigroups and groups of operators with kernels. In the case when the relative spectrum consists of two parts, one of which lies in the left half-plane of the complex plane, and the second in the right half-plane of the complex plane, it is possible to construct a resolving semigroup and a group of operators, and to carry out their exponential estimates. In this case, the solution of the equation can be represented as the sum of a stable and unstable solutions. The stabilization of an unstable solution is based on the feedback principle. An equation describing the evolution of the free surface of a liquid is considered as an application.

Keywords: Sobolev type equations; invariant spaces; the stabilization problem.

Introduction

Let us consider a linear Sobolev-type equation

$$L\dot{u} = Mu. \tag{1}$$

Here, the operators $L,\ M$ are linear and continuous operators acting from the Banach space $\mathfrak{U}$ to the Banach space $\mathfrak{F}$, the kernel of the operator L is nontrivial, and the operator M is a relatively sectorial operator. Equations of type (1) arise in modeling various physical processes.

Studies of equations unsolved with respect to the time derivative appeared in the works of A. Poincare. However, the systematic study of such equations began with the works of S.A. Sobolev. At present, the theory of Sobolev-type equations is actively developing, and various directions have been formed (see, for example, [1–4]). This study is based on the theory of operator semigroups with kernels. Papers [5–7] have been devoted to the study of the solvability of the Cauchy problem for equation (1) with respect to the sectorial operator. In [8], the stability of solutions of the equation (1) in terms of invariant spaces and dichotomies of solutions was studied for the first time. In [9], the method of Lyapunov functions was applied to study the stability of solutions. In [10], the solvability of a linear stochastic equation of Sobolev type is studied; in [11], stability results are obtained and numerical experiments are carried out.

The aim of the paper is to develop a general method for stabilizing solutions of the equation (1). The paper consists of two parts. The first part contains known results on the theory of semigroups and groups of operators with kernels and the phase space of a linear Sobolev equation. In the second part, following [8], we consider exponential estimates of semigroups and operator groups in the case when the relative spectrum does not intersect the imaginary axis, we solve the problem of stabilization of unstable solutions from the feedback principle. As an application, the Dzekzer equation is considered.

1. Resolvent Semigroups

Let $\mathfrak{U}$ and $\mathfrak{F}$ be Banach spaces, operators $L,\ M \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$. The set $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{F};\mathfrak{U})\}$ is the L-resolvent set, the set $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ is the L-spectrum of the operator M. Let $\mu_q \in \rho^L(M)$, $q = 0, 1, ..., p$. The operator-functions $R^L_\mu(M) = (\mu L - M)^{-1}L$ and $L^L_\mu(M) = L(\mu L - M)^{-1}$ are called respectively *right L-resolvent* and *left L-resolvent* of the operator M, and $R^L_{(\mu,p)}(M) = \prod\limits_{q=0}^{p}(\mu_q L - M)^{-1}L$ and $L^L_{(\mu,p)}(M) = \prod\limits_{q=0}^{p} L(\mu_q L - M)^{-1}$ — *right (L,p)-resolvent* and *left (L,p)-resolvent* of the operator M. The operator is called (L,p)-sectorial if for some real number a, some $K \in \mathbb{R}_+$ and some $\Theta \in (\pi/2, \pi)$ sector $S^L_a, \Theta(M) = \{\mu \in \mathbb{C} : |\arg(\mu - a)| < \Theta, \mu \neq a\}$ lies in the relatively resolvent set of the operator M and

$$\max\left\{||R^L_{(\mu,p)}(M)||_{\mathfrak{L}(\mathfrak{U})}, ||L^L_{(\mu,p)}(M)||_{\mathfrak{L}(\mathfrak{U})}\right\} \leq \frac{K}{\Pi^p_{q=0}|\mu_q - a|}$$

for all $\mu_q \in S^L_{a,\Theta}(M),\ q = 0, 1, ..., p.$

Definition 1. *Mapping $V^\bullet \in C(\mathbb{R}_+;\mathcal{L}(\mathfrak{V})$ ($V^\bullet \in C(\mathbb{R};\mathfrak{V})$) is called a semigroup (group) in the Banach space $\mathfrak{V}$, if*

$$V^sV^t = V^{s+t} \quad \forall s, t \in \mathbb{R}_+\ (\forall s, t \in \mathbb{R}). \tag{$*$}$$

A semigroup $\{V^t : t \in \mathbb{R}_+\}$ is called *holomorphic*, if it is analytic in some sector containing the ray $\mathbb{R}_+$, and the condition (*) is satisfied. A group is called *holomorphic*, if it is analytic in the entire complex plane $\mathbb{C}$ and the condition (*) holds. A semigroup (group) is called *uniformly bounded*, if $\|V^t\|_{\mathfrak{U}} \leq$ const on any compact subset of $\mathbb{R}_+$ ($\mathbb{R}$). The set $\ker V^\bullet = \{v \in \mathfrak{V} : V^tv = 0 \exists t \in \mathbb{R}_+\}$ is called the *kernel*, and the set $\mathrm{im} V^\bullet = \{v \in \mathfrak{V} : v = V^0v\}$ is *image* of the analytic group $\{V^t : t \in \mathbb{R}_+\}$. Let $\mathfrak{U}^1$ ($\mathfrak{F}^1$) denote the closure of $\mathrm{im} R^L_{(\mu,p)}(M)$ ($\mathrm{im} L^L_{(\mu,p)}(M)$) in the norm of the space $\mathfrak{U}$ ($\mathfrak{F}$).

Theorem 1. [7] *Let the operator M be (L,p)-sectorial. Then there exists a holomorphic and uniformly bounded semigroup of operators*

$$U^t = \frac{1}{2\pi i}\int\limits_\Gamma R^L_\mu(M)e^{\mu t}d\mu \quad \left(F^t = \frac{1}{2\pi i}\int\limits_\Gamma L^L_\mu(M)e^{\mu t}d\mu\right), \tag{2}$$

where the contour $\Gamma \subset \rho^L(M)$ *is such that* $|arg\mu| \to \Theta$ *for* $\mu \to \infty$, $\mu \in \Gamma$, $t \in \mathbb{R}_+$.

Let the operator M be (L, p)-sectorial and the following conditions hold:

$$\mathfrak{U} = \mathfrak{U}^0 \oplus \mathfrak{U}^1, \ \mathfrak{F} = \mathfrak{F}^0 \oplus \mathfrak{F}^1, \tag{A1}$$

there exists an operator

$$L_1^{-1} \in \mathcal{L}(\mathfrak{F}; \mathfrak{U}). \tag{A2}$$

consider the linear equation (1). A vector-function $u \in C^\infty(\mathbb{R}; \mathfrak{U})$ is called a *solution* of the equation (1) if it satisfies this equation.

The set $\mathfrak{P} \subset \mathfrak{U}$ is called the *phase space* of the equation (1) if the solution of the equation (1) $u(t) \in \mathfrak{P}$ for all $t \in \mathbb{R}_+$, and for any $u_0 \in \mathfrak{P}$ there exists a unique solution to the problem

$$\lim_{t \to 0+} u(t) = u_0 \tag{3}$$

for the equation (1).

Theorem 2. [7] *Let the operator* M *be* (L, p)*-sectorial, and the conditions A1 and A2 hold. Then:*

(i) there exists a resolvent semigroup for the equation (1) of the form (2);

(ii) for any $u_0 \in \mathfrak{U}^1$*, the unique solution to the problem (1), (3) is given by* $u(t) = U^t u_0$.

2. Exponential Estimates and the Stabilization Problem

Let the operator M be (L, p)-sectorial, and the conditions A1 and A2 hold. Denote by $\mathfrak{U}^0 = \mathfrak{U} \ominus \mathfrak{U}^1$ and L_0, (M_0) the restriction of the operator L (M) to $\mathfrak{U}^0$, and let the operator $H = M_0^{-1} L_0$. Let $\sigma^L(M) \bigcap i\mathbb{R} = \emptyset$ and $\sigma^L_u(M) = \{\mu \in \sigma^L(M) : \mathrm{Re}\mu > 0\}$, $\sigma^L_u(M) \neq \emptyset$. Then $\sigma^L_u(M)$ is a bounded set, and let Γ_r be the contour that encloses $\sigma^L_u(M)$ and lies to the right of the imaginary axis. The part of the spectrum $\sigma^L_s(M) = \sigma^L(M) \setminus \sigma^L_u(M)$ lies in a sector bounded by the contour

$$\Gamma_l = \{\mu \in \mathbb{C} : \mathrm{Re}\mu < 0, \ |\mathrm{arg}\mu| \in (\pi/2, \pi)\}.$$

The operator $P_r = \dfrac{1}{2\pi i} \displaystyle\int_{\Gamma_r} R^L_\mu(M) d\mu \in \mathcal{L}(\mathfrak{U})$ is a projector. Denote by $\mathfrak{I}^u = \mathrm{im} P_r$, $\mathfrak{I}^s = \mathfrak{U}^1 \ominus \mathfrak{I}^u$, and let M_s (L_s) and M_u (L_u) denote the restrictions of M (L) to $\mathfrak{I}^s$ and $\mathfrak{I}^u$, respectively. Furthermore, the operator M_s is relatively bounded, while M_u is a relatively sectorial operator.

We will consider the equation (1) in the form of a reduced system:

$$H \dot{u}^0 = u^0, \tag{4}$$

$$L_s \dot{u}^s = M_s u^s, \tag{5}$$

$$L_u \dot{u}^u = M_u u^u. \tag{6}$$

The solution $u = u(t)$ of the equation (1) $u = u^0 + u^s + u^u$, where $u^0 = u^0(t)$ is the solution to equation (4), $u^s = u^s(t)$ is the solution to equation (5), $u^u = u^u(t)$ is the solution to equation (6).

Definition 2. *The equation (1) will be called uniformly exponentially stable if there exist constants $N > 0$ and $\gamma > 0$, such that for all $t \in \mathbb{R}_+$ and any $u_0 \in \mathfrak{P}$ the solution $u = u(t)$ of the problem (1), (3) satisfies the exponential estimate:*

$$\|u(t)\|_{\mathfrak{U}} \leq N e^{-\gamma t} \|u_0\|_{\mathfrak{U}}. \tag{7}$$

Let us solve the following stabilization problem. It is required to find such a control action on equation (1), that it becomes uniformly exponentially stable. If the operator M is (L, p)-sectorial, then the solution $u^0 = u^0(t)$ of equation (4) equals zero for any $t \in \mathbb{R}_+$. There exists a decaying semigroup for equation (5)

$$U_l^t = \frac{1}{2\pi i} \int\limits_{\Gamma_l} (\mu L_s - M_s)^{-1} L_s e^{\mu t} d\mu$$

and a resolving group for equation (6)

$$U_r^t = \frac{1}{2\pi i} \int\limits_{\Gamma_r} (\mu L_u - M_u)^{-1} L_u e^{\mu t} d\mu.$$

Due to the closure of the spectrum, there exist constants $\alpha,\ \beta > 0$, such that $\operatorname{Re} \sigma_u^L(M) > \beta$ и $\operatorname{Re} \sigma_s^L(M) < -\alpha$. Then

$$\|U_l^t\|_{\mathcal{L}(\mathfrak{U})} \leq C e^{-\alpha t},\ \|U_r^t\|_{\mathcal{L}(\mathfrak{U})} \leq C e^{\beta t},\ t \in \mathbb{R}_+. \tag{8}$$

Based on the estimates (8) it is clear that equation (5) is uniformly exponentially stable, whereas equation (6) is not uniformly exponentially stable.

So, the stabilization problem reduces to finding a vector function f , such that for the solution of the equation

$$L_u \dot{u}^u = M_u u^u + f. \tag{9}$$

the following condition holds:

$$\|u^u(t)\|_{\mathfrak{U}} \leq N_u e^{-\gamma_u t} \|u_0^u\|_{\mathfrak{U}}. \tag{10}$$

We will find f through feedback: $f = C u^u$, where C is a known linear bounded operator. Equation (9) then takes the form

$$L_u \dot{u}^u = M_u u_u + C u_u = (M_u + C) u_u. \tag{11}$$

Let $m = \max\limits_{\operatorname{Re} \mu} \mu \in \sigma(M_u)$. Let's put the operator $C = -(\varepsilon + m)\mathbb{I}$, where $\varepsilon > 0$ is arbitrarily small. Then the relative spectrum of the operator $M_u + C$ lies in the left half-plane of the complex plane, and for the solution of equation (11) condition (10) holds.

Let is consider the application of the obtained results to the Dzekzer equation:

$$(\lambda - \Delta) u_t = \alpha \Delta u - \beta \Delta^2 u. \tag{12}$$

where the parameters $\alpha,\ \beta \in \mathbb{R}_+$ and $\lambda \in \mathbb{R}$. Let $\Omega \in \mathbb{R}^n$ be a bounded region, and its boundary $\partial\Omega \in C^\infty$. Define the spaces $\mathfrak{U}$ and $\mathfrak{F}$:

$$\mathfrak{U} = \{u \in W_2^4 : u(x) = 0,\ (x) \in \partial\Omega\},\ \mathfrak{F} = L_2(\Omega).$$

Let $\{\varphi_k\}$ be the eigenfunctions of the Laplace operator Δ orthonormalized relative scalar product in $\mathfrak{U}$, the spectrum $\sigma(\Delta) = \{\nu_k\}$. In [7], it is shown that the operator M is strongly $(L, 0)$-sectorial, and for $\alpha, \beta, \lambda \in \mathbb{R} \setminus \left\{0, \frac{\alpha}{\beta}\right\}$, there exists a solution $\eta = \eta(t)$ to the Cauchy problem (1) for the equation (3) of the form:

$$u(t) = \sum_{l=1}^{\infty}{}' e^{\frac{\alpha\nu_l - \beta\nu_l^2}{\lambda - \nu_l}t} < u_0, \varphi_l > \varphi_l.$$

Let $\alpha,\ \beta > 0$ and $\lambda < \nu_1$. Then,

$$\sigma_u^L(M) = \left\{ \frac{(\alpha - \nu_k\beta)\nu_k}{\lambda - \nu_k} : \lambda < \nu_k \right\}.$$

for $\lambda < \nu_1$. For the stabilization problem, we take $B = -(\varepsilon + (\alpha - \nu_n\beta)\nu_n)\mathbb{I}$, where n is the index of the maximum value $\mu_k \in \sigma_u^L(M)$, $\varepsilon > 0$. Then the solution to the stabilized equation (12) is given by:

$$u(t) = \sum_{\lambda > \nu_k} e^{\frac{\alpha\nu_k - \beta\nu_k^2}{\lambda - \nu_k}t} < u_0, \varphi_k > \varphi_k + \sum_{\lambda < \nu_k} e^{\frac{\alpha\nu_k - \beta\nu_k^2 - (\varepsilon + \alpha\nu_n - \beta\nu_n^2)}{\lambda - \nu_k}t} < u_0, \varphi_k > \varphi_k.$$

Conclusion.

In the future, it is planned to develop a general approach to solving the problem of stabilizing solutions of semilinear Sobolev type equations with a relatively sectoral operator [12]. Following the work of [13–15], computational experiments are planned to find stable and unstable solutions and solve the stabilization problem.

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СТАБИЛИЗАЦИЯ РЕШЕНИЙ ЛИНЕЙНОГО УРАВНЕНИЯ СОБОЛЕВСКОГО ТИПА С ОТНОСИТЕЛЬНО СЕКТОРИАЛЬНЫМ ОПЕРАТОРОМ

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В статье рассматривается линейное уравнение соболевского типа с относительно секториальным оператором. Такого вида уравнения возникают при моделировании различных процессов: эволюции свободной поверхности жидкости, течения вязкой несжимаемой жидкости, плоскопараллельная термоконвекция вязкоупругой несжимаемой жидкости и т.п. В работе рассматривается следующая задача стабилизации: требуется найти управляющее воздействие на уравнение, чтобы оно стало равномерно асимптотически устойчивым. Решение данной задачи базируется на теории полугрупп и групп операторов с ядрами. В случае, когда относительный спектр состоит из двух частей, одна из которых лежит в левой полуплоскости комплексной плоскости, а вторая в правой полуплоскости комплексной плоскости, то можно построить разрешающие полугруппу и группу операторов, провести их экспоненциальные оценки. В этом случае решение уравнения можно представить в виде суммы устойчивого и неустойчивого решения. Стабилизация неустойчивого решения проводится на основе принципа обратной связи. В качестве приложения рассматривается уравнение, описывающее эволюцию свободной поверхности жидкости.

Ключевые слова: уравнения соболевского типа; инвариантные пространства; задача стабилизации.

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