MSC 35Q99 DOI: 10.14529/jcem250103

ALGORITHM FOR NUMERICAL STUDY OF DEGENERATE MODELS OF NONLINEAR DIFFUSION AND FILTRATION WITH A RANDOM INITIAL STATE

***N. A. Manakova*[1]**, manakovana@susu.ru,
***N. G. Nikolaeva*[1]**, nikolaevang@susu.ru
[1]South Ural State University, Chelyabinsk, Russian Federation

The article is devoted to the numerical study of one class of stochastic models of nonlinear diffusion and filtration with a random initial condition of Showalter–Sidorov. The nonlinear diffusion model describes the process of changing the concentration potential of a viscoelastic fluid filtering in a porous medium; the nonlinear filtration model describes the dependence of the pressure of a viscoelastic incompressible fluid filtering in a porous formation on the external load. The models under consideration study within an abstract semilinear equation of Sobolev type with p-coercive and s-monotone operator. An algorithm for the numerical solution method of one class of problems of mathematical physics is constructed. An example of applying the algorithm to the stochastic model of nonlinear diffusion under study is given.

Keywords: Sobolev type equations; stochastic model of nonlinear diffusion; stochastic model of nonlinear filtration; projection method.

Introduction

With the development of modern technologies, it has become possible to study and construct numerical methods for solving initial-boundary value problems for degenerate nonlinear equations and systems of partial differential equations. Obtaining an analytical solution to the initial boundary value problem for nonlinear equations is not always possible, which necessitates their numerical study. There are various approaches to the numerical solution of such problems. In the case of degenerate semilinear equations, the projection method has proven itself to be successful [1–3], as it allows one to take into account the possibility of degeneracy of the equation. The projection method was applied to the numerical study of a large class of degenerate deterministic [1–3] and stochastic [4, 5] models (Sobolev-type models). On its basis, approximate solutions of mathematical models are constructed, the coefficients of which satisfy a system of algebraic differential equations with the corresponding initial conditions. The existence of solutions to the initial value problem for a system of algebraic differential equations is proved using the phase space method and the existence theorem for a solution for a singular system of ordinary differential equations [6, 7].

The transition from the study of a deterministic model to a stochastic one is caused by the fact that measurement errors may occur in experiments, which leads to the need to consider a stochastic model. The study of stochastic partial differential equations became possible with the development of the modern theory of stochastic processes. The traditional approach to the study of stochastic models is the Ito–Stratonovich–Skorokhod method. This method allows one to move from differential equations to integral ones. At the moment, the method has been extended to the infinite-dimensional situation [8], and also various applications to classical [9] and non-classical [10, 11] models of mathematical physics are considered. Another approach, which has been widely used in recent years for studying stochastic Sobolev-type equations, is based on considering the equation in spaces of differentiable "noises" [12, 13], was first proposed for the study of Leontief-type stochastic systems [14].

Consider a complete probability space $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ and the set of real numbers $\mathbb{R}$, endowed with a σ-algebra. A measurable mapping $\xi : \Omega \to \mathbb{R}$ is called a *random variable.* The set of random variables having zero expectations (i.e. $\mathbf{E}\xi = 0$) and finite variance forms Hilbert space $\mathbf{L_2}$ (i.e. $\mathbf{D}\xi < +\infty$) with the inner product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\xi_2$, where $\mathbf{E}$, $\mathbf{D}$ are the expectation and variance of the random variable, respectively. Let $\mathfrak{D} \subset \mathbb{R}^n$ be a bounded domain with a boundary C^∞. Consider a *$\mathcal{H}$-valued differentiable stochastic K-process* η, satisfying stochastic models of nonlinear diffusion

$$(\lambda - \Delta) \overset{\circ}{\eta} - \mathrm{div}(|\nabla\eta|^{p-2}\nabla\eta) = 0,\ p \geq 2,\ \omega \in \Omega,\ (s,t) \in \mathfrak{D} \times (0,T), \tag{1}$$

$$\eta(\omega, s, t) = 0,\ \omega \in \Omega,\ (s,t) \in \partial\mathfrak{D} \times [0,T], \tag{2}$$

and nonlinear filtration

$$(\lambda - \Delta)) \overset{\circ}{\eta} - \alpha\Delta\eta + |\eta|^{p-2}\eta = 0,\ p \geq 2,\ \omega \in \Omega,\ (s,t) \in \mathfrak{D} \times (0,T), \tag{3}$$

$$\eta(\omega, s, t) = 0,\ \omega \in \Omega,\ (s,t) \in \partial\mathfrak{D} \times [0,T], \tag{4}$$

and initial Showalter–Sidorov condition

$$(\lambda - \Delta)(\eta(\omega, s, 0) - \eta_0(\omega, s)) = 0,\ \omega \in \Omega,\ s \in \mathfrak{D}. \tag{5}$$

Here $\overset{\circ}{\eta}$ is Nelson–Gliklikh derivative of a stochastic process [15–17]. Mathematical model (1), (2) with condition (5) describes the process of changing the concentration potential of a viscoelastic fluid filtered in a porous medium [18, 19], with the assumption of a randomly specified initial value $\eta_0(\omega, s)$ of the fluid concentration potential. Mathematical model (3), (4) describes the dependence of the pressure of a viscoelastic incompressible fluid (e.g. oil) filtering in a porous formation [20] on the external load (e.g. the pressure of water injected through wells into the formation). The parameter $\lambda \in \mathbb{R}$ and $\alpha \in \mathbb{R}_+$ characterize the viscous and elastic properties of the liquid, respectively.

The models under consideration belong to the class of semilinear Sobolev-type models [21], in which the nonlinear operator is p-coercive and s-monotone [22, 23]:

$$L \overset{o}{\eta} + M\eta + N(\eta) = 0,\ \ker L \neq \{0\}. \tag{6}$$

The Showalter–Sidorov initial condition [24] in the general case will take the form

$$L(\eta(0) - \eta_0) = 0. \tag{7}$$

To study model (1), (2) let's consider functional spaces $\mathcal{H} = L_2(\mathfrak{D})$, $\mathfrak{H} = \overset{0}{W}{}^1_2(\mathfrak{D})$, $\mathfrak{B} = \overset{0}{W}{}^1_p(\mathfrak{D})$, $\mathfrak{H}^* = W_2^{-1}(\mathfrak{D})$, $\mathfrak{B}^* = W_q^{-1}(\mathfrak{D}), p \geq 2, \frac{1}{p} + \frac{1}{q} = 1$. By $\mathfrak{B}^*$ and $\mathfrak{H}^*$ denote the conjugate spaces to $\mathfrak{B}$ and $\mathcal{H}$ relative to the scalar product $< \cdot, \cdot >$ в $\mathcal{H}$ respectively. Due to the choice of function spaces $\mathfrak{H}$ and $\mathfrak{B}$ there exists a dense and continuous embedding

$$\mathfrak{B} \hookrightarrow \mathfrak{H} \hookrightarrow \mathcal{H} \hookrightarrow \mathfrak{H}^* \hookrightarrow \mathfrak{B}^*, \tag{8}$$

which corresponds to the evolutionary case of embeddings and splitting of spaces for the problem under study [21].

Denote the $l \in \mathbb{N}$ is order of the Nelson–Gliklikh [15, 16] derivative of the one-dimensional stochastic process η. Consider space of differentiable "noises" $\mathbf{C}^l([0,T],\mathbf{L}_2)$, i.e. the space of stochastic processes from $\mathbf{C}([0,T],\mathbf{L}_2)$, which trajectories are almost surely differentiable by Nelson–Gliklikh. Choose a monotonely decreasing numerical sequence $K=\{\mu_k\}$ such that $\sum\limits_{k=1}^{\infty}\mu_k^2<+\infty$. Specify the Nelson–Gliklikh derivative of the stochastic K-process

$$\overset{\circ}{\eta}(t)=\sum_{k=1}^{\infty}\mu_k\overset{\circ}{\eta}_k(t)\varphi_k,$$

where the family of functions $\{\varphi_k\}$ forms a basis in the space $\mathcal{H}$, and series converges uniformly in the norm $\mathcal{H}_K\mathbf{L}_2$ on any compact set [17]. The space of differentiable K-"noises" $\mathbf{C}^l([0,T];\mathcal{H}_K\mathbf{L}_2)$ of continuous $\mathcal{H}$-valued stochastic K-process, which trajectories are almost surely continuously differentiable by Nelson–Gliklikh. Similar to the construction of space $\mathbf{C}^l([0,T];\mathcal{H}_K\mathbf{L}_2)$, let's construct spaces of differentiable K-"noises" $\mathbf{C}^l([0,T];\mathfrak{B}_K\mathbf{L}_2)$ and $\mathbf{C}^l([0,T];\mathfrak{H}_K\mathbf{L}_2)$ [25] and define the operators:

$$\langle L\eta,\zeta\rangle=\mathbf{E}\int_{\mathfrak{D}}(\lambda\eta\zeta+\nabla\eta\cdot\nabla\zeta)ds,\quad \eta,\ \zeta\in\mathfrak{H}_K\mathbf{L_2};\tag{9}$$

$$\langle N(\eta),\zeta\rangle=\mathbf{E}\int_{\mathfrak{D}}|\nabla\eta|^{p-2}\nabla\eta\cdot\nabla\zeta ds,\quad \eta,\zeta\in\mathfrak{B}_K\mathbf{L_2}.\tag{10}$$

Thus, model (1), (2) can be studied as part of the study of equation (6). To study model (3), (4) let's consider functional spaces $\mathcal{H}=L_2(\mathfrak{D})$, $\mathfrak{H}=\overset{0}{W}{}_2^1(\mathfrak{D})$, $\mathfrak{B}=L_p(\mathfrak{D})$, where $n=2, 2\le p<+\infty$ or $n\ge 3, 2\le p\le 2+\frac{4}{n-2}$. Due to the choice of function spaces $\mathfrak{H}$ и $\mathfrak{B}$ there exists a dense and continuous embedding

$$\mathfrak{H}\hookrightarrow\mathfrak{B}\hookrightarrow\mathcal{H}\hookrightarrow\mathfrak{B}^*\hookrightarrow\mathfrak{H}^*,\tag{11}$$

which corresponds to dynamic case of embeddings and splitting of spaces for the problem under study [21]. Let's construct operators for the nonlinear filtration model:

$$\langle L\eta,\zeta\rangle=\mathbf{E}\int_{\mathfrak{D}}(\lambda\eta\zeta+\nabla\eta\cdot\nabla\zeta)ds,\quad \eta,\zeta\in\mathfrak{H}_K\mathbf{L_2};\tag{12}$$

$$\langle M(\eta),\zeta\rangle=\mathbf{E}\int_{\mathfrak{D}}\alpha\nabla\eta\cdot\nabla\zeta ds,\quad \eta,\zeta\in\mathfrak{H}_K\mathbf{L_2};\tag{13}$$

$$\langle N(\eta),\zeta\rangle=\mathbf{E}\int_{\mathfrak{D}}|\eta|^{p-2}\eta\zeta ds,\quad \eta,\zeta\in\mathfrak{B}_K\mathbf{L_2}.\tag{14}$$

Problems (1),(2) and (3),(4) relate to different cases of space embeddings, but the properties of the operators are preserved. The operator L is linear, continuous, self-adjoint, non-negative definite at $\lambda\ge-\lambda_1$ and Fredholm in both cases. The operator

N is smooth, s-monotone and p-coercive, the operator M is linear, continuous and 2-coercive [23]. Here $\{\lambda_k\}, \{\psi_k\}$ are the sequences of eigenfunctions and eigenvalues of the homogeneous Dirichlet problem for the Laplace operator $(-\Delta)$ in the domain $\mathfrak{D}$, numbered in non-decreasing order taking into account the multiplicity. Note that the orthonormal family of functions $\{\psi_k\}$ is total in space $\mathcal{H}$. In the future, instead of a family $\{\varphi_k\}$ will take a family $\{\psi_k\}$.

Investigation of problems (1), (2), (5) and (3) – (5) is based on the developed research method for an abstract stochastic equation (6) with s-monotone and p-coercive operator [25]. Let's consider stochastic K-processes $\eta = \eta(t)$ and $\zeta = \zeta(t)$ them equal if almost certainly each trajectory of one of the processes coincides with the trajectory of another process.

Definition 1. *A stochastic K-process η is called a solution to equation (6), if almost surely all trajectories of η satisfy equation (6) for all $t \in (0, T)$. A solution $\eta = \eta(t)$ to equation (6) is called a solution to Showalter–Sidorov problem (6), (7), if the solution satisfies condition (7) for some random K-variable η_0.*

Define $\eta_0 \in \mathfrak{H}_K\mathbf{L}_2$ in form

$$\eta_0 = \sum_{k=1}^{\infty} \mu_k \eta_{0k} \psi_k,$$

where $\{\eta_{0k}\} \subset \mathbf{L}_2$ is a sequence of random variables.

Theorem 1. *[26] Let the operator $L \in \mathcal{L}(\mathfrak{H}_K\mathbf{L}_2; \mathfrak{H}_K^*\mathbf{L}_2)$ is self-adjoint, non-negative definite at $\lambda \geq -\lambda_1$ and Fredholm, the operator $M \in \mathcal{L}(\mathfrak{H}_K\mathbf{L}_2; \mathfrak{H}_K^*\mathbf{L}_2)$ is 2-coercive, the operator $N \in C^\infty(\mathfrak{B}_K\mathbf{L}_2; \mathfrak{B}_K^*\mathbf{L}_2)$ is s-monotone and p-coercive and the embedding of spaces is performed (8). Then for any sequence of random variables $\{\eta_{0k}\} \subset \mathbf{L}_2$, for any $T \in \mathbb{R}_+$ there exists a solution $\eta \in \mathbf{C}^k([0, T]; \mathfrak{H}_K\mathbf{L}_2)$ to problem* (6), *(7).*

Define $\eta_0 \in \mathfrak{B}_K\mathbf{L}_2$ in form

$$\eta_0 = \sum_{k=1}^{\infty} \mu_k \eta_{0k} \psi_k,$$

where $\{\eta_{0k}\} \subset \mathbf{L}_2$ is a sequence of random variables.

Theorem 2. *[4] Let the conditions of Theorem 1 be satisfied and the embedding of spaces be valid (11). Then for any sequence of random variables$\{\eta_{0k}\} \subset \mathbf{L}_2$, for any $T \in \mathbb{R}_+$ there exists a solution $\eta \in \mathbf{C}^k([0, T]; \mathfrak{B}_K\mathbf{L}_2)$ to problem* (6), *(7).*

Thus, based on the abstract theory, we can conclude that there is a trajectory solution to Showalter–Sidorov problem (6), (7) $\eta \in \mathbf{C}^k([0, T]; \mathfrak{B}_K\mathbf{L}_2)$ и $\eta \in \mathbf{C}^k([0, T]; \mathfrak{H}_K\mathbf{L}_2)$ to problems (1), (2), (5) and (3), (4), (5) respectively.

1. Algorithm for a Numerical Method

Based on the modified projection method, we will construct an algorithm for the numerical solution of Showalter–Sidorov problem (7) for stochastic equation (6), which allows us to find approximate solutions for given initial values and model parameters.

0 Stage. Finding the sequences of eigenfunctions $\{\lambda_k\}$ and eigenvalues $\{\psi_k\}$ of the homogeneous Dirichlet problem for the operator L.

1 Stage. Choose a monotonically decreasing numerical sequence $K = \{\mu_k\}$ such that $\sum\limits_{k=1}^{\infty} \mu_k^2 < +\infty$, and the initial random variable in the form $\eta_0 = \sum\limits_{k=1}^{\infty} \mu_k \eta_{0k} \psi_k$ whose coefficients $\{\eta_{0k}\}$ are independent Gaussian random variables such that their variances are bounded ($\mathbf{D}\eta_{0k} \leq C$, $k \in \mathbb{N}$).

2 Stage. Representation of the required functions in the form of a Galerkin sum

$$\eta_N = \sum_{k=1}^{N} \mu_k \eta_k \psi_k,$$

where $\eta_k = \eta_k(w,t)$, $\psi_k = \psi_k(s)$, and substitution in(6):

$$L \overset{\circ}{\eta}_N + M\eta_N + N(\eta_N) = 0. \tag{15}$$

3 Stage. Scalarly multiplying the equation (15) by the eigenfunctions $\psi_k(s)$, $k = 1, ..., N$, we form a system of algebraic differential equations

$$\mathbf{E}\langle L\overset{\circ}{\eta}_N, \psi_k\rangle + \mathbf{E}\langle M\eta_N, \psi_k\rangle + \mathbf{E}\langle N(\eta_N), \psi_k\rangle = 0,\ k = 1, ..., N. \tag{16}$$

4 Stage. Generating a sequence of random variables $\{\eta_{0k}\} \subset \mathbf{L}_2$ and the composition of a random variable η_0.

5 Stage. Construction of the Showalter–Sidorov initial conditions for a system of algebraic differential equations (16)

$$\mathbf{E}\langle L\left(\eta_N(s,0) - \eta_0(s)\right), \psi_k(s)\rangle = 0,\ k = 1, ..., N. \tag{17}$$

6 Stage. We find the solution of the system of algebraic-differential equations (16) with initial conditions (17) by the Runge–Kutta method of 4-5 orders.

7 Stage. Plotting a trajectory of an approximate solution $\eta_N(s,t)$ to problem (6), (7).

8 Stage. For a statistical study of the solution to problem (6), (7) we repeat stages 4 through 7 m times with the generation of various $\{\eta_{0k}^l\}$ to construct trajectories of an approximate solution $\eta_N^l, l = 2, ..., m$ of problem (6), (7).

9 Stage. Finding the sample mean, sample average variance and standard deviation from the constructed sample $\eta_N^l, l = 1, ..., m$.

10 Stage. Evaluation of the obtained trajectories with a given probability $p = 0.997$ according to the 3σ rule.

Let us present the algorithm of the program implementing the developed numerical method using the example of studying the model of nonlinear diffusion with a random initial state (1), (2) on the segment. Let us describe its operation step by step and present the results of the computational experiment.

Step 1. We introduce the coefficients of nonlinear diffusion equation $\lambda = -1$, $p = 4$, length of the segment $l = \pi$, the number of Galerkin approximations $N = 5$, the parameter $T = 1$ of the time interval $[0,T]$, the parameters of the random influences – mathematical expectation and standard deviation, equal to 2. Choose a monotonely decreasing numerical sequence $\{K\} = \left\{\dfrac{1}{k}\right\}, k \in \mathbb{N}$. Thus, problem (1), (2), (5) will take the form

$$\overset{\circ}{\eta} + \overset{\circ}{\eta}_{ss} + 3(\eta_s)^2\eta_{ss} = 0,\ s \in (0,\pi),\ t \in (0,1], \tag{18}$$

$$\eta(0,t)=\eta(\pi,t)=0,\ t\in[0,1], \tag{19}$$

with the Showalter–Sidorov initial condition

$$\eta(s,0)-\eta_0(s)-\eta_{ss}(s,0)+\eta_{0ss}(s)=0,\ s\in[0,\pi]. \tag{20}$$

The generation of random variables included in the decomposition for the initial function $\eta_0(s)$ using the function **stats[random, normald[μ,σ]](1)**. The generation of random variables gave the following results:

$$\eta_0=0.648161051674\sqrt{2}\sin(s)+1.262983605797\sqrt{2}\sin(2s)-1.353800851640\sqrt{2}\sin(3s)+$$
$$+1.390132285285\sqrt{2}\sin(4s)-0.300173601945\sqrt{2}\sin(5s).$$

Step 2. A separate procedure finding the sequences of eigenfunctions and eigenvalues of the homogeneous Dirichlet problem for the operator $(-\Delta)$:

$$\psi_k(s)=\sqrt{\frac{2}{\pi}}\sin(ks),\ \lambda_k=k^2,\ k=1,2,\ldots.$$

Step 3. Using the **for k to 1 do N end do** loop, we form an approximate solution in the form of a Galerkin sum and substitute it into the (18) equation:

$$\eta_N^l=\sum_{k=1}^{N}\frac{1}{k}\eta_k(t)\sqrt{\frac{2}{\pi}}\sin(ks).$$

Step 4. A separate procedure, in the **for k to 1 do N end do** loop, forms a system of differential equations and a system of algebraic equations.

Step 5. The first equation of the system, which is algebraic, is solved at the initial time and $\eta_1(0)$ is found:

$$-3(\eta_{01}^3+3\eta_{01}^2\eta_{03}+8\eta_{01}\eta_{02}^2+16\eta_{01})\eta_{02}\eta_{04}+$$
$$+18\eta_{01}\eta_{03}^2+30\eta_{01}\eta_{03}\eta_{05}+32\eta_{01}\eta_{04}^2+50\eta_{01}\eta_{05}^2+12\eta_{02}^2\eta_{03}+$$
$$20\eta_{02}^2\eta_{05}+48\eta_{02}\eta_{03}\eta_{04}+80\eta_{02}\eta_{04}\eta_{05}+45\eta_{03}^2\eta_{05}=0,$$

$$\eta_1(0)=1.957738287440.$$

Step 6. A system consisting of algebraic and differential equations is solved, taking into account the initial function and the result obtained in step 5 η_{01}.The solution is found using the built-in procedure **dsolve**. As a result of the sixth step of the algorithm, we obtain a system of algebraic-differential equations of the form (16), which contains four differential equations and one algebraic equation – the first with Showalter–Sidorov conditions

$$\eta_2(0)=2.238580155726,\ \eta_3(0)=-2.399549532848,$$
$$\eta_4(0)=2.463945322320,\ \eta_5(0)=-0.532043856707.$$

Step 7. Functions for solving the problem are formed at times from 0 to T with a step frequency of $0,01T$.

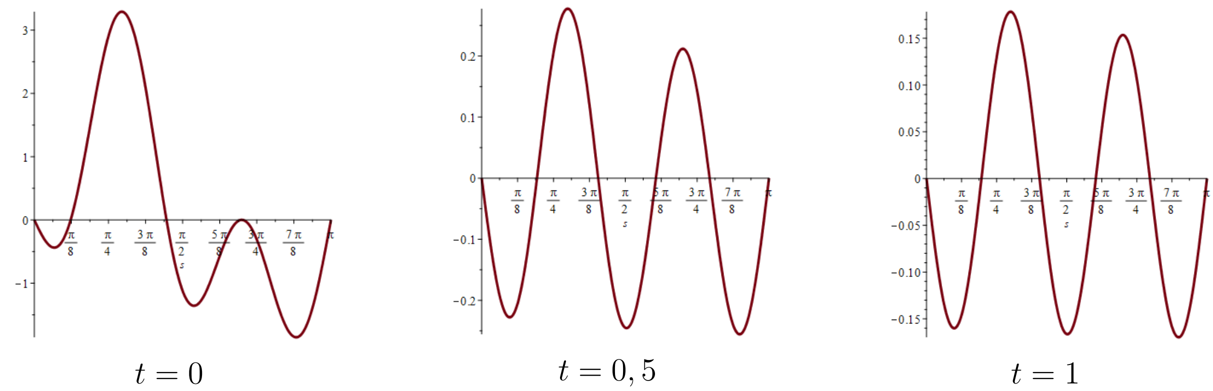

Fig. 1. Results of a computational experiment using the Galerkin projection method

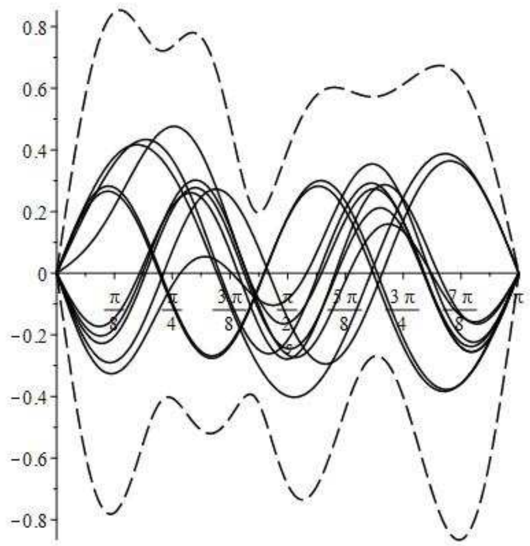

Fig. 2. Expectation $\mathbf{E}(\eta_N^l(s, 0, 5))$ and its estimate

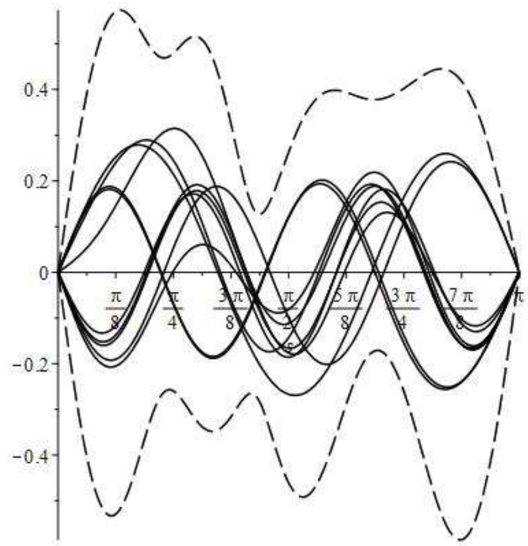

Fig. 3. Expectation $\mathbf{E}(\eta_N^l(s, 1))$ and its estimate

Step 8. Using the **plot, plot3d** (Fig. 1) built-in procedures, 2D and 3D plots of the functions obtained in step 7 are displayed.

The study of a stochastic model involves m computational experiments, each of which uses a generator of a random normally distributed variable η_{0k} with given parameters of mathematical expectation and variance for modeling. After the generation of random variables, the first three stages of the numerical solution of the Showalter–Sidorov–Dirichlet problem for the stochastic model. For subsequent processing of the results, a loop is run on i, which allows processing the results of m experiments in one program. Each cycle will allow you to get several implementations of the solution.

Step 9. For statistical study of the solution of problem (18) – (20) we repeat stages 5 through 7 $m = 10$ times with the generation of various $\{\eta_{0k}^l\}$ to construct trajectories of an approximate solution $\eta_N^l, l = 2, ..., m$.

Step 10. Finding the sample mean, sample mean variance and standard deviation for the constructed sample. Evaluation of the obtained trajectories with a given probability $p = 0,997$ according to the rule 3σ. As a result of the experiment with probability 0,997 we can use the estimate

$$\|\eta(s,t) - \mathbf{E}(\eta_N^l(s,t))\| < 3\sigma_\eta(t). \quad (21)$$

In Fig. 2 several function graphs are combined. Solid lines show the graphs of functions $\eta_N^l(s,0,5)$, $i = 1,\dots,10$, functions are represented by dotted lines $\mathbf{E}(\eta_N^l(s,0,5))+3\sigma_\eta(0,5)$ and $\mathbf{E}(\eta_N^l(s,0,5)) - 3\sigma_\eta(0,5)$, obtained numerically. In Fig. 3 shows the graphs of the functions $\eta_N^l(s,1)$, $i = 1,\dots,10$, $\mathbf{E}(\eta_N^l(s,1)) + 3\sigma_\eta(1)$ and $\mathbf{E}(\eta_N^l(s,1)) - 3\sigma_\eta(1)$. Graphs $\eta_N^l(s,1)$, $i = 1,\dots,10$ are shown by dotted lines, solid lines represent functions $\mathbf{E}(\eta_N^l(s,1)) + 3\sigma_\eta(1)$ and $\mathbf{E}(\eta_N^l(s,1)) - 3\sigma_\eta(1)$, obtained numerically, the estimate (21) is satisfied.

This work was funded by Russian Science Foundation 24-11-20037, https://rscf.ru/project/24-11-20037/.

References

1. Sviridyuk G.A., Sukacheva T.G. Galerkin Approximations of Singular Nonlinear Equations of Sobolev Type. *Soviet Mathematics (Izvestiya VUZ. Matematika)*, 1989, vol. 33 no. 10, pp. 56–59.

2. Bogatyreva E.A., Manakova N.A. Numerical Simulation of the Process of Nonequilibrium Counterflow Capillary Imbibition. *Computational Mathematics and Mathematical Physics*, 2016, vol. 56, no. 1, pp. 132–139. DOI: 10.1134/S0965542516010085

3. Bychkov E.V. Analytical Study of the Mathematical Model of Wave Propagation in Shallow Water by the Galerkin Method. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2021, vol. 14, no. 1, pp. 26–38. DOI: 10.14529/mmp210102

4. Perevozchikova K.V., Manakova N.A., Gavrilova O.V., Manakov I.M. Numerical Algorithm for Finding a Solution to a Nonlinear Filtration Mathematical Model with a Random Showalter–Sidorov Initial Condition. *Journal of Computational and Engineering Mathematics*, 2022, vol. 9, no. 2, pp. 39–51. DOI: 10.14529/jcem220204

5. Soldatova E.A., Keller A.V. Numerical Algorithm and Computational Experiments for One Linear Stochastic Hoff Model. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2024, vol. 17, no. 2, pp. 83–95. DOI: 10.14529/mmp240207
6. Sviridyuk G.A. On a Singular System of Ordinary Differential Equations. *Differential Equations*, 1987, vol. 23, no. 9, pp. 1637–1639.
7. Sviridyuk G.A., Sukacheva T.G. The Phase Space of a Class of Operator Equations of Sobolev Type. *Differential Equations*, 1990, vol. 26, no. 2, pp. 250–258.
8. Da Prato G., Zabczyk J. *Stochastic Equations in Infinite Dimensions.* Cambridge, Cambridge University Press, 1992. DOI: 10.1017/CBO9781107295513.
9. Kovacs M., Larsson S. Introduction to Stochastic Partial Differential Equations. *Proceedings of "New Directions in the Mathematical and Computer Sciences"*, Abuja, 2007, vol. 4, pp. 159–232.
10. Zamyshlyaeva A.A. Stochastic Incomplete Linear Sobolev Type High-Ordered Equations with Additive "White Noise". *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2012, no. 40(299), pp. 73–82.
11. Zagrebina S.A., Soldatova E.A. The Linear Sobolev-Type Equations with Relatively p-Bounded Operators and Additive "White Noise". *The Bulletin of Irkutsk State University. Series: Mathematic*, 2013, vol. 6, no. 1, pp. 20–34.
12. Favini A., Sviridyuk G.A., Zamyshlyaeva A.A. One Class of Sobolev Type Equations of Higher Order with Additive "White Noise". *Communications on Pure and Applied Analysis*, 2016, vol. 15, no. 1, pp. 185–196. DOI: 10.3934/cpaa.2016.15.185
13. Favini A., Zagrebina S.A., Sviridyuk G.A. Multipoint Initial-Final Value Problems for Dynamical Sobolev-Type Equations in the Space of Noises. *Electronic Journal of Differential Equations*, 2018, vol. 2018, no. 128, pp. 1–10.
14. Shestakov A.L., Sviridyuk G.A. On the Measurement of the "White Noise". *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2012, no. 13, pp. 99–108
15. Gliklikh Yu.E. *Global and Stochastic Analysis with Applications to Mathematical Physics.* Springer, London, 2011.
16. Nelson E. *Dynamical Theories of Brownian Motion.* Princeton, Princeton University Press, 1967.
17. Sviridyuk G.A., Manakova N.A. The Dynamical Models of Sobolev Type with Showalter–Sidorov Condition and Additive "Noise". *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2014, vol. 7, no. 1, pp. 90–103. DOI: 10.14529/mmp140108
18. Dzektser E.S. Generalization of the Equation of Motion of Ground Waters with Free Surface. *Doklady Akademii Nauk SSSR*, 1972, vol. 202, no. 5, pp. 1031–1033. (in Russian)
19. Polubarinova-Kochina P.Ya. *Theory of Groundwater Movement.* Moscow, Nauka, 1987. (in Russian)

20. Oskolkov A.P. Initial-Boundary Value Problems for Equations of Motion Nonlinear Viscoelastic Fluids. *Journal of Soviet Mathematics*, 1985, vol. 37, pp. 860–866. DOI: 10.1007/BF01387724

21. Manakova N.A. Mathematical Models and Optimal Control of the Filtration and Deformation Processes. *Bulletin of the South Ural State University. Series: Mathematical Modelling, Programming and Computer Software*, 2015, vol. 8, no. 3, pp. 5–24.

22. Sviridyuk G.A. A Problem for the Generalized Boussinesq Filtration Equation. *Soviet Mathematics (Izvestiya VUZ. Matematika)*, 1989, vol. 33, no. 2, pp. 62–73.

23. Manakova N.A., Sviridyuk G.A. Nonclassical Equations of Mathematical Physics. Phase Space of Semilinear Sobolev Type Equations. *Bulletin of the South Ural State University Series: Mathematics. Mechanics. Physics*, 2016, vol. 8, no. 3, pp. 31–51. DOI: 10.14529/mmph160304

24. Sviridyuk G.A., Zagrebina S.A. The Showalter–Sidorov Problem as a Phenomena of the Sobolev Type Equations. *The Bulletin of Irkutsk State University. Series: Mathematics*, 2010, vol. 3, no. 1, pp. 104–125.

25. Vasiuchkova K.V., Manakova N.A., Sviridyuk G.A. Degenerate Nonlinear Semigroups of Operators and Their Applications. *Springer Proceedings in Mathematics and Statistics. Semigroups of Operators – Theory and Applications*, 2020, pp. 363–378. DOI: 10.1007/978-3-030-46079-2_21

26. Manakova N.A., Nikolaeva N.G., Perevozchikova K.V. Investigation Solvability of the Stochastic Model of Nonlinear Diffusion with Random Initial Value. *Global and Stochastic Analysis*, 2024, vol. 11, no. 4, p. 27–33.

Natalia A. Manakova, DSc (Math), Professor, Head of the Department of Mathematical Physics Equations, South Ural State University (Chelyabinsk, Russian Federation), manakovana@susu.ru

Nadezhda G. Nikolaeva, assistant, Department of Mathematical Physics Equations, South Ural State University, (Chelyabinsk, Russian Federation), nikolaevang@susu.ru

Received January 14, 2025

УДК 517.9 **DOI: 10.14529/jcem250103**

АЛГОРИТМ ЧИСЛЕННОГО ИССЛЕДОВАНИЯ ВЫРОЖДЕННЫХ МОДЕЛЕЙ НЕЛИНЕЙНЫХ ДИФФУЗИИ И ФИЛЬТРАЦИИ СО СЛУЧАЙНЫМ НАЧАЛЬНЫМ СОСТОЯНИЕМ

Н. А. Манакова[1]***, Н. Г. Николаева***[1]
[1]Южно-Уральский государственный университет, г. Челябинск, Российская Федерация

Статья посвящена численному исследованию одного класса стохастических моделей нелинейной диффузии и фильтрации со случайным начальным условием Шоуолтера –Сидорова. Модель нелинейной диффузии описывает процесс изменения потенциала концентрации вязкоупругой жидкости, фильтрующейся в пористой среде, модель нелинейной фильтрации описывает зависимость давления вязкоупругой несжимаемой жидкости, фильтрующейся в пористом пласте, от внешней нагрузки. Рассматриваемые модели изучены в рамках абстрактного полулинейного уравнения соболевского типа с p-коэрцитивным и s-монотонным оператором. Построен алгоритм численного исследования одного класса задач математической физики. Приведен пример применения алгоритма к исследуемой стохастической модели нелинейной диффузии.

Ключевые слова: уравнения соболевского типа; стохастическая модель нелинейной диффузии; стохастическая модель нелинейной фильтрации; проекционный метод.

Литература

1. Свиридюк, Г.А. О галеркинских приближениях сингулярных нелинейных уравнений типа Соболева / Г.А. Свиридюка, Т.Г. Сукачева // Известия вузов. Математика. – 1989. – Т. 33, № 10. – С. 44–47.

2. Богатырева, Е.А. Численное моделирование процесса неравновесной противоточной капиллярной пропитки / Е.А. Богатырева, Н.А. Манакова // Журнал вычислительной математики и математической физики. – 2016. – Т. 56, № 1. – С. 125–132.

3. Бычков, Е.В. Аналитическое исследование математической модели распространения волн на мелкой воде методом Галеркина / Е.В. Бычков // Вестник ЮУрГУ. Серия: Математическое моделирование и программирование. – 2021. – Т. 14, № 1. – С. 26–38.

4. Perevozchikova, K.V. Numerical Algorithm for Finding a Solution to a Nonlinear Filtration Mathematical Model with a Random Showalter–Sidorov Initial Condition / K.V. Perevozchikova, N.A. Manakova, O.V. Gavrilova, I.M. Manakov // Journal of Computational and Engineering Mathematics. – 2022. – V. 9, № 2. – P. 39–51.

5. Солдатова, Е.А. Численный алгоритм и вычислительные эксперименты для одной линейной стохастической модели Хоффа / Е.А. Солдатова, А.В. Келлер // Вестник ЮУрГУ. Серия: Математическое моделирование и программирование. – 2024. – Т. 17, № 2. – С. 83–95

6. Свиридюк, Г.А. О разрешимости сингулярной системы обыкновенных дифференциальных уравнений / Г.А. Свиридюк // Дифференциальные уравнения. – 1987. – Т. 23, № 9. – С. 1637–1639.

7. Свиридюк, Г.А. Фазовые пространства одного класса операторных полулинейных уравнений типа Соболева / Г.А. Свиридюк, Т.Г. Сукачева // Дифференциальные уравнения. – 1990. – Т. 26, № 2. – С. 250–258.

8. Da Prato, G. Stochastic Equations in Infinite Dimensions / G. Da Prato, J. Zabczyk. – Cambridge: Cambridge University Press, 1992.

9. Kovacs, M. Introduction to Stochastic Partial Differential Equations / M. Kovacs, S. Larsson // Proceedings of "New Directions in the Mathematical and Computer Sciences", National Universities Commission, Abuja, 2007.

10. Замышляева, А.А. Стохастические неполные линейные уравнения соболевского типа высокого порядка с аддитивным "белым шумом" / А.А. Замышляева // Вестник ЮУрГУ. Серия: Математическое моделирование и программирование. – 2012. – № 40(299). – С. 73–82.

11. Загребина, С.А. Линейные уравнения соболевского типа с относительно p-ограниченными операторами и аддитивным "белым шумом" / С.А. Загребина, Е.А. Солдатова // Известия Иркутского государственного университета. Серия: Математика. – 2013. – Т. 6, № 1. – С. 20–34.

12. Favini, A. One Class of Sobolev Type Equations of Higher Order with Additive "White Noise" / A. Favini, .A. Sviridyuk, A.A. Zamyshlyaeva // Communications on Pure and Applied Analysis. – 2016. – V. 15, № 1. – P. 185–196.

13. Favini, A. Multipoint Initial-Final Value Problems for Dynamical Sobolev-Type Equations in the Space of "Noises" / A. Favini, S.A. Zagrebina, G.A. Sviridyuk // Electronic Journal of Differential Equations. – 2018. – V. 2018, № 128. – P. 1–10.

14. Шестаков, А.Л. Об измерении "белого шума" / А.Л. Шестаков, Г.А. Свиридюк // Вестник ЮрГУ. Серия: Математическое моделирование и программирование. – 2012. – № 27(286). – С. 99–108.

15. Gliklikh, Yu.E. Global and Stochastic Analysis with Applications to Mathematical Physics / Yu.E. Gliklikh. – London: Springer, 2011.

16. Nelson, E. Dynamical Theories of Brownian Motion / E. Nelson. – Princeton: Princeton University Press, 1967.

17. Свиридюк, Г.А. Динамические модели соболевского типа с условием Шоуолтера – Сидорова и аддитивными "шумами"/ Н.А. Манакова, Г.А. Свиридюк // Вестник ЮУрГУ. Серия: Математическое моделирование и программирование. – 2014. – Т. 7, № 1. – С. 90–103.

18. Дзекцер, Е.С. Обобщение уравнения движения грунтовых вод / Е.С. Дзекцер // Доклады академии наук СССР. – 1972. – Т. 5. – С. 1031–1033.

19. Полубаринова–Кочина, П.Я. Теория движения грунтовых вод / П.Я. Полубаринова–Кочина. – М.: Наука, 1987.

20. Осколов, А.П. Начально-краевые задачи для уравнений движения нелинейных вязкоупругих жидкостей / А.П. Осколов // Записки научных семинаров Санкт-Петербургского отделения математического института им. В.А. Стеклова РАН. – 1985. – Т. 147. – С. 110–119.

21. Манакова, Н.А. Математические модели и оптимальное управление процессами фильтрации и деформации / Н.А. Манакова // Вестник ЮУрГУ. Серия: Математическое моделирование и программирование. – 2015. – Т. 8, № 3. – С. 5–24.

22. Свиридюк, Г.А. Одна задача для обобщенного фильтрационного уравнения Буссинеска / Г.А. Свиридюк // Известия высших учебных заведений. Математика. – 1989. – Т. 33, № 2. – С. 62–73.

23. Манакова, Н.А. Неклассические уравнения математической физики. Фазовые пространства полулинейных уравнений соболевского типа / Н.А. Манакова, Г.А. Свиридюк // Вестник ЮУрГУ. Серия: Математика. Механика. Физика. – 2016. – Т. 8, № 3. – С. 31–51.

24. Свиридюк, Г.А. Задача Шоуолтера – Сидорова как феномен уравнений соболевского типа / Г.А. Свиридюк, С.А. Загребина // Известия Иркутского государственного университета. Серия: Математика. – 2010. – Т. 3, № 1. – С. 104–125.

25. Vasiuchkova, K.V. Degenerate Nonlinear Semigroups Of Operators And Their Applications / K.V. Vasiuchkova, N.A. Manakova, G.A. Sviridyuk // Springer Proceedings in Mathematics and Statistics. Semigroups of Operators – Theory and Applications. – 2020. – P. 363–378.

26. Manakova, N.A. Investigation Solvability of the Stochastic Model of Nonlinear Diffusion with Random Initial Value / N.A. Manakova, N.G. Nikolaeva, K.V. Perevozchikova // Global and Stochastic Analysis. – 2024. – V. 11, № 4. – P. 27–33.

Манакова Наталья Александровна, доктор физико-математических наук, профессор, заведующий кафедрой, кафедра уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), manakovana@susu.ru

Николаева Надежда Геннадьевна, ассистент, кафедра уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), niolaevang@susu.ru

Поступила в редакцию 14 января 2025 г.