# ENGINEERING MATHEMATICS

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## NONLINEAR ANALYSIS OF BEAM ON AN ELASTIC POLYMERIC FOUNDATION: A STUDY ON TRANSIENT AND FREQUENCY RESPONSES

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This study examines the dynamic behavior of a micro-beam resting on an elastic polymeric layer with finite depth, with a particular focus on the beam's nonlinear transient and steady-state response to a base excitation. The investigation considers both the beam's transverse motion and the elastic polymeric foundation's nonlinear squeezing motion within the coupled nonlinear governing equations. In order to examine the influence of different materials on the system's overall response, the nonlinear governing equations are discretized with respect to spatial coordinates and integrated over time, thereby obtaining transient solutions. In regard to frequency response, the coefficients of the harmonic Fourier-expanded response are obtained by balancing energy within a period. A series of numerical tests were performed, including the fast Fourier transform (FFT), the determination of time characteristics, and the phase portrait of the system. These tests encompassed a range of scenarios, including constant, stepwise, pulse, and harmonic accelerations. Furthermore, studies have been conducted to ascertain the frequency response of the system to single harmonic input signals with varying frequencies.

*Keywords: nonlinear analysis; vibrations; transient solutions; frequency response.*

## Introduction

The issue of unregulated vibrations poses a significant threat to the reliability and durability of mechanical systems, leading to the progressive deterioration of structural materials. In the absence of prompt mitigation, vibration loads can result in critical damage and complete equipment failure. In light of these challenges, specialists in engineering, mechanics, and related fields are undertaking endeavors to develop methodologies aimed at mitigating the deleterious effects of vibrations. Novel methods for detecting vibrations have the potential to enhance signal quality and reduce noise, as evidenced by recent studies [1, 2].

Furthermore, researchers have repeatedly underscored that vibrations can inflict detrimental effects not only on individuals but also on buildings and other structures. A detailed analysis of studies such as [3] reveals a variety of strategies for limiting vibro-dynamic impacts on building structures.

The proper selection of vibration isolation methods is of paramount importance, as it not only influences the efficiency of the equipment but also has implications for human safety. In recent decades, there has been a proliferation of various methods for vibration isolation, including passive [4], active [5], and semi-active techniques [6].

In pursuit of an optimal solution, the researchers propose a combination of technologies that integrate the simplicity of passive isolation with the dynamic control of active systems. This approach enables the attainment of substantial vibration protection while exhibiting minimal energy expenditure. One such method is the development of passive bioinspired insulators, which draw inspiration from bioinformatics [7, 8]. The objective of these insulators is to combine the stability and energy efficiency of passive designs with the high isolation capabilities of semi-active and active insulators [9]. The efficacy of this methodology is substantiated by the findings of the study conducted by [10].

Damping elastomers are frequently utilized as the foundation due to their exceptional elasticity and damping properties. Rubber has historically been a material of choice for vibration isolation, due to its softness and inherent damping properties. Nevertheless, polyurethane is emerging as a viable alternative, offering structural and cost benefits. Thermoplastic elastomers (TPEs) have emerged as potential alternatives to vulcanized rubber, offering comparable mechanical properties and the additional benefit of recyclability.

It is the authors' understanding that previous research on the vibration of a beam supported by an elastic polymeric bed has commonly employed models that represent the bed using equivalent or distributed springs and dampers based on the Winkler or Pasternak models. In general, these models do not take into account the effects of inertia. This is a reasonable assumption in the context of static analysis and low-frequency actuation scenarios, given the low density of the elastomer. Nevertheless, in scenarios involving relatively high-frequency actuations, the inertial characteristics of the system may become a notable factor, necessitating further examination. In the present study, we address this limitation by introducing a nonlinear dynamic model via elastodynamic equation of the bed to describe the behavior of a beam resting on an elastic polymeric foundation, accounting for the coupled motion of the foundation. The model accounts for mid-plane stretching of the beam through an averaging technique, resulting in an integro-differential equation [11, 12]. The foundation material is characterized by Hookean behavior, while the consideration of nonlinear strains is incorporated. Notably, shear effects are intentionally omitted in this formulation. The study investigates the influence of utilizing different elastic polymeric materials on the transient as well as steady-state response of the system.

## 1. Model Description

The structure is composed of two components as show in Figure 1. The first component is a thin micro-beam and is composed of metal. The second component is the foundation which exhibits a thickness greater than that of the top plate. The second layer is composed of an elastomer. It is hypothesized that the materials constituting the micro-beam and the foundation are homogeneous. The structure is exposed to external force with some acceleration $a_0(t)$. For this reason, that the structure will undergo vibrations in both the longitudinal and transverse directions. The model under consideration comprises the micro-beam, which is defined by the following dimensions: length $L$, width $b$, and thickness

$h_b$. The elastic polymeric layer possesses identical dimensions to the micro-beam, but its thickness, denoted $h_f$, differs.

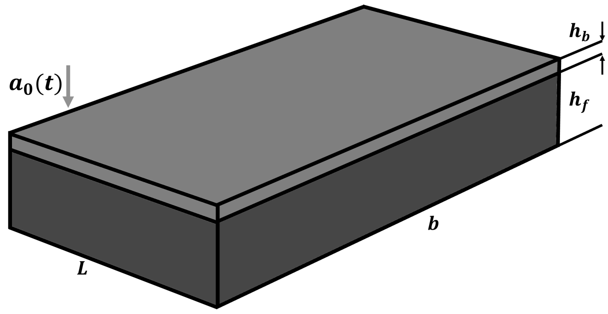


**Fig. 1**. Diagram of the micro-beam with an elastic polymeric layer before the force is applied

This allows us to present the governing equation for the transverse motion of the micro-beam as a nonlinear integro-differential equation:

$$N_1(W_b, a_0(t)) = E_b I \frac{\partial^4 W_b}{\partial x^4} - \frac{E_b A}{2L}\left[\int_0^L \left(\frac{\partial W_b}{\partial x}\right)^2 dx\right]\frac{\partial^2 W_b}{\partial x^2}$$
$$+\rho_b A\left(\frac{\partial^2 W_b}{\partial t^2} + a_0(t)\right) + c_b\frac{\partial W_b}{\partial t} + F(x,t) = 0, \qquad (1)$$

where the geometric characteristics of the micro-beam are defined by its length $L$, width $b$, and thickness $h_f$. The micro-beam's composition, i.e., the material from which it is made, is represented by the effective Young's modulus $E_b$, density $\rho_b$, and equivalent damping coefficients $c_b$. $A = bh$ is the cross section area, $I = \frac{1}{12}dh_b^3$ is the moment of inertia of the micro-beam. $a_0(t)$ is the external acceleration field. $F_f$ represents the force exerted by the foundation on the micro-beam.

$$F(x,t) = -bE_f\left[\frac{\partial W_f}{\partial z} + \frac{1}{2}\left(\frac{\partial W_f}{\partial z}\right)^2\right]|_{z=0}$$

The desired function $W_b(x,t)$ is the deflection of the micro-beam, defined to be positive in the direction to the right.

The nonlinear dynamic equation of motion of the elastic polymeric layer considering nonlinear strains in axial and transversal directions can be written as follows:

$$N_2(W_f) = \rho_f\frac{\partial^2 W_f}{\partial t^2} + c_f\frac{\partial W_f}{\partial t} - E_f\left(\frac{\partial^2 W_f}{\partial z^2} + \frac{\partial^2 W_f}{\partial z^2}\frac{\partial W_f}{\partial z}\right) = 0, \qquad (2)$$

where $W_f(x,z,t)$ and $E_f$ is the deflection and effective Young's modulus of the elastic polymeric layer respectively, $\rho_f$ and $c_f$ represent the density and equivalent damping coefficients of the elastic polymeric layer accordingly.

Boundary conditions of equations (1), (2) are as follows:

$$W_b(0,t) = 0, \qquad \frac{\partial W_b}{\partial t}|_{x=0} = 0, \qquad W_b(L,t) = 0, \qquad \frac{\partial W_b}{\partial t}|_{x=L} = 0,$$
$$W_f(x,0,t) = W_b(x,t), \qquad W_f(x,h_f,t) = 0. \qquad (3)$$

## 2. Numerical Solution

Given the mobile nature of the boundary condition, a transformation was implemented to accommodate this motion:

$$W_f\,(x, z, t) \;= U\,(x, z, t) + \left(1 - \frac{z}{h_f}\right) W_b. \tag{4}$$

The concept of dimensionless quantities is introduced:

$$w_b = \frac{W}{h_f}, u = \frac{U}{h_f}, \xi = \frac{x}{L}, \zeta = \frac{z}{h_b}, \tau = \frac{t}{t^*}, \Omega = \omega t^* \tag{5}$$

where $t^*$ is a time scale, $t^* = \sqrt{\frac{\rho A L^4}{E_b I}}$.

In order to study the behavior of a structure under the influence of an external force, the Galerkin method was applied. The solutions of system (1), (2) can be expressed as follows in terms of the basis function. The basic functions satisfy the boundary conditions (3).

$$w_b(\xi, \tau) = \sum_{n=1}^{N} a_n(\tau)\phi_n(\xi), \tag{6}$$

$$u(\xi, \zeta, \tau) = \sum_{n=1}^{N}\sum_{m=1}^{M} b_{nm}(\tau)\phi_n(\xi)\psi_m(\zeta), \quad n = 1, 2, 3, \ldots, N, m = 1, 2, 3, \ldots, M \tag{7}$$

Subsequent to the substitution of the solution (6), (7), the dimensionless quantities (5) and expression (4) into the system of differential equations (1), (2), we obtain nonlinear ordinary differential equations that delineate the structural deflection:

$$\begin{aligned}\sum_{n=1}^{N} M_{nk}^{(1)}\ddot{a}_n + \sum_{n=1}^{N} C_{nk}^{(1)}\dot{a}_n + \sum_{n=1}^{N} K_{nk}^{(1)} a_n + \sum_{m=1}^{M}\sum_{n=1}^{N} E_{nmk}^{(1)} b_{nm} + \sum_{m=1}^{N}\sum_{n=1}^{N} Z_{nmk}^{(1)} a_n a_m + \\ + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{p=1}^{N} L_{nmpk}^{(1)} b_{nm} a_p + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{p=1}^{M}\sum_{s=1}^{N} F_{nmpsk}^{(1)} b_{nm} b_{pq} + \\ + \sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{p=1}^{N} N_{nmpk}^{(1)} a_n a_m a_p + \sum_{k=1}^{N} D_1^{(1)} a_0(\tau) = 0\end{aligned} \tag{8}$$

$$\begin{aligned}\sum_{m=1}^{M}\sum_{n=1}^{N} M_{mnkq}^{(2)}\ddot{b}_{mn} + \sum_{n=1}^{N} M_{nkq}^{(3)}\ddot{a}_n + \sum_{m=1}^{M}\sum_{n=1}^{N} C_{mnkq}^{(2)}\dot{b}_{mn} + \sum_{m=1}^{M}\sum_{n=1}^{N} K_{mnkq}^{(2)} b_{mn} + \\ + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{p=1}^{M}\sum_{s=1}^{N} N_{mnpskq}^{(2)} b_{mn} b_{ps} + \sum_{n=1}^{N} E_{nkq}^{(2)}\dot{a}_n + \\ + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{p=1}^{N} G_{mnpkq}^{(2)} b_{mn} a_p = 0,\end{aligned} \tag{9}$$

where the coefficients $M_{nk}^{(1)}, C_{nk}^{(1)}, K_{nk}^{(1)}$ and so on, are constructed by scalarly multiplying the error equations in $L_2(G)$.

Frequency analysis constitutes a crucial methodology for the study of vibrating systems, as it enables the determination of their dynamic characteristics and behavior under the influence of external forces. This enables optimization and enhancement of their performance [13]. In a manner analogous to the method outlined in [14], the system of equations is presented in the subsequent form:

$$\mathbb{R}(\wedge(\tau)) = 0. \tag{10}$$

Accordingly, for the periodic frequency response of the given equation (10), a Fourier expansion may be written as follows:

$$\wedge^{(j)}(\tau) = \sum_{n=0}^{\infty} a_n^{(j)} \sin(n\Omega\tau) \ + b_n^{(j)} \cos(n\Omega\tau) \ , \quad j = \{1, 2\}. \tag{11}$$

The solution space, which is infinite-dimensional (equation 11), can be expressed in a reduced P-dimensional space as shown below:

$$\wedge_P^{(j)}(\tau) = \sum_{p=0}^{P} a_p^{(j)} \sin(p\Omega\tau) \ + b_p^{(j)} \cos(p\Omega\tau), \quad j = \{1, 2\}; \quad P = 1, 2, 3 \ldots \tag{12}$$

$$A_p^j = \sqrt{\left(a_p^{(j)}\right)^2 + \left(b_p^{(j)}\right)^2}$$

where $A_p^j$ is the dimensionless longitudinal ($j = 1$) and transversal ($j = 2$) deflection amplitude of system in the $p$ harmony.

## 3. Numerical Results and Discussion

This section offers an evaluation of the study's outcomes, accompanied by illustrative examples the underscore the significance of employing elastic polymer layers. According to the model outlined in Section 2, the micro-beam and the elastic polymer layer exhibit the characteristics enumerated in Table 1 and Table 2. The micro-beam is composed of steel, while the elastic polymer layer exhibits viscoelastic properties. The numerical experiments conducted involved the consideration of three distinct materials: rubber, The second material is thermoplastic elastomers with the addition of sodium [15] (TRE) and mastic-type material ADEM-M [16].

**Table 1**

Material and geometrical properties of micro-beam

| | |
|---|---|
| Length $L$ | 1000 $\mu m$ |
| Thickness $h_b$ | 10 $\mu m$ |
| Width $b$ | 500 $\mu m$ |
| Density $\rho_b$ | 2330 $kg/m^3$ |
| Effective Young's modulus $E_b$ | 169 $GPa$ |

The present study employs the Hilbert basis expansion in spatial coordinates is used to solve nonlinear partial differential equations. The basis functions in the current study are as follows:

$$\psi_n(\zeta) = \sin(n\pi\zeta); \qquad \psi_n(0) = 0; \quad \psi_n(1) = 0,$$

**Table 2**

Properties of the elastic polymeric layer

| | |
|---|---|
| Length $L$ | 1000 $\mu m$ |
| Thickness $h_f$ | 70 $\mu m$ |
| Width $b$ | 500 $\mu m$ |
| **Material 1 – Rubber** | |
| Density $\rho_f$ | 1333 $kg/m^3$ |
| Effective Young's modulus $E_f$ | 9.7 $MPa$ |
| **Material 2 – TPE** | |
| Density $\rho_f$ | 1000 $kg/m^3$ |
| Effective Young's modulus $E_f$ | 18.8 $MPa$ |
| **Material 3 – ADEM-M** | |
| Density $\rho_f$ | 1250 $kg/m^3$ |
| Effective Young's modulus $E_f$ | 12 $MPa$ |

$$\varphi_n(\xi) = 1 - \cos(2n\pi\xi); \qquad \varphi_n(0) = \varphi_n(1) = 0; \quad \varphi_n'(0) = \varphi_n'(1) = 0.$$

The structure is driven in the $z$-direction as a result of an applied acceleration. This section will present the findings of a series of computational experiments. The goal of the experiments is to determine the response of a structure consisting of a micro-beam and various elastic polymeric layers, the properties of which are described in Table 2, to the applied external force at different values of the acceleration amplitude. The applied external force takes the form of a constant value, Heaviside step function, pulse function, and harmonic varying acceleration. All graphs will illustrate the deviation at the central point of connection of the micro-beam and the elastic polymeric layer.

Prior to examining the behavior of construction in response to external forces, it is imperative to illustrate the necessity of employing elastic polymeric layers. As illustrated in Fig.2, the amplitude of vibrations of a micro-beam with an elastic polymer layer of Material 1 is approximately two times less, than that of a micro-beam without it.

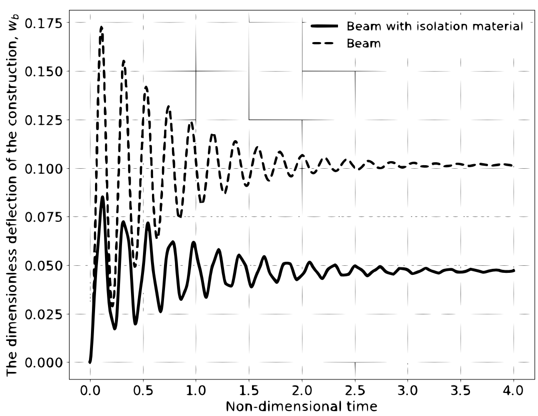


**Fig. 2**. The non-dimensional deflection of of the construction under the influence of acceleration $\overline{a}_0(\tau) = \overline{a}_0 H(\tau), \overline{a}_0 = 50$ with/without elastic polymeric layer

A series of experiments was conducted with the objective of determining the structural response to acceleration at varying values. Firstly, the case is considered when the acceleration is a constant value. In this case the time derivatives of $w_f$ and $w_b$ are zero. Applying the Galerkin weighted residual method results in a system of algebraic equations. Solving this system yield the static values of $w_f$ and $w_b$ will be obtained.

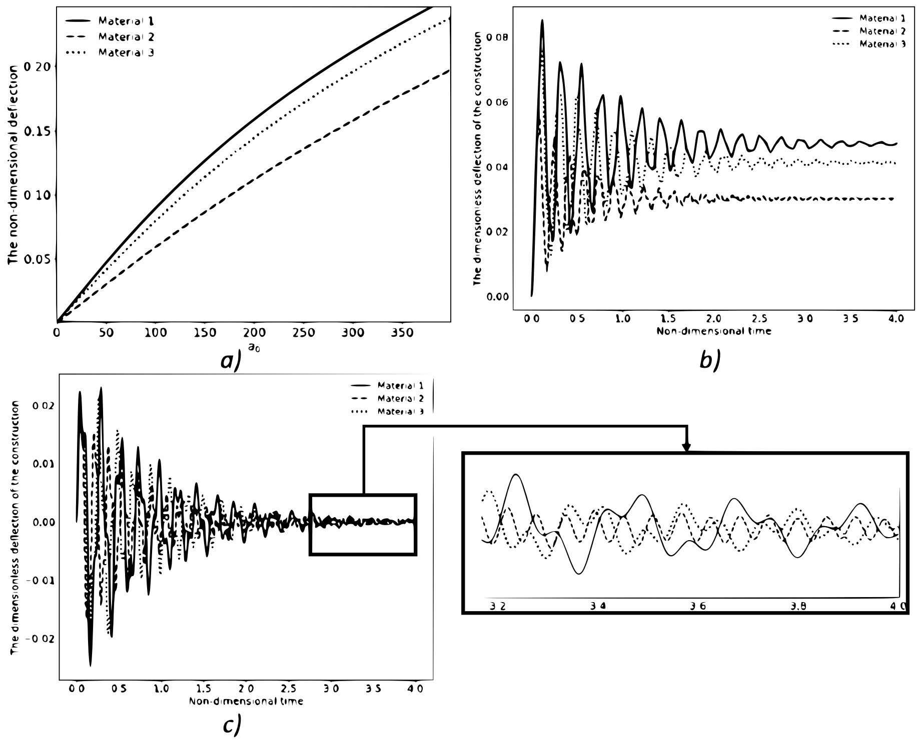


**Fig. 3**. The non-dimensional deflection of the construction at the central point of connection of the micro-beam and the damping layer under the influence of acceleration
a) $\overline{a}_0(\tau) = \overline{a}_0$
b) $\overline{a}_0(\tau) = \overline{a}_0 H(\tau)$, where $H(\tau)$ is the Heaviside function
c) $\overline{a}_0(\tau) = \overline{a}_0 \delta(\tau)$, where $\delta(\tau)$ is the pulse function

Fig. 3a depicts the impact of a constant acceleration $\overline{a}_0$ on the dimensionless deflection of a structural system comprising a micro-beam ($w_b$) and an elastic polymeric layer ($w_f$). As the constant acceleration increases, the deflection of the micro-beam also increases in accordance with the governing differential equation.

Fig. 3b and Fig. 3c show the effect of the applied acceleration $\overline{a}_0(\tau)$ on the non-dimensional deflection of construction, when the acceleration is in the form of a Heaviside step function and pulse function accordingly, where $\overline{a}_0 = 25$. The oscillation's amplitude reaches a maximum at the beginning of the time and then gradually decreases until it reaches a static position when acceleration is applied in the step form. In the case of applying a pulse acceleration, the amplitude reaches zero. The materials exhibit disparate vibration isolation properties. It can be observed that when utilizing Material 1, the system oscillates with the greatest amplitude. Material 2 exhibits the most minimal oscillation amplitude among all the materials tested, irrespective of the type of acceleration applied. It has been demonstrated that the utilization of this material will enhance the stability of the system. A more detailed examination of the properties of analogous materials may prove a fruitful avenue for future research.

The following experiments was conducted to ascertain the response of the structure to harmonic acceleration with varying amplitude, which is expressed in the following form:

$$\overline{a}_0(\tau) = \overline{a}_0 sin(\Omega\tau), \quad where\ \Omega = 30.$$

Material 1 was used for the experiment, the characteristics of this elastic polymeric layer are evaluated in accordance with the data presented in Table 2.

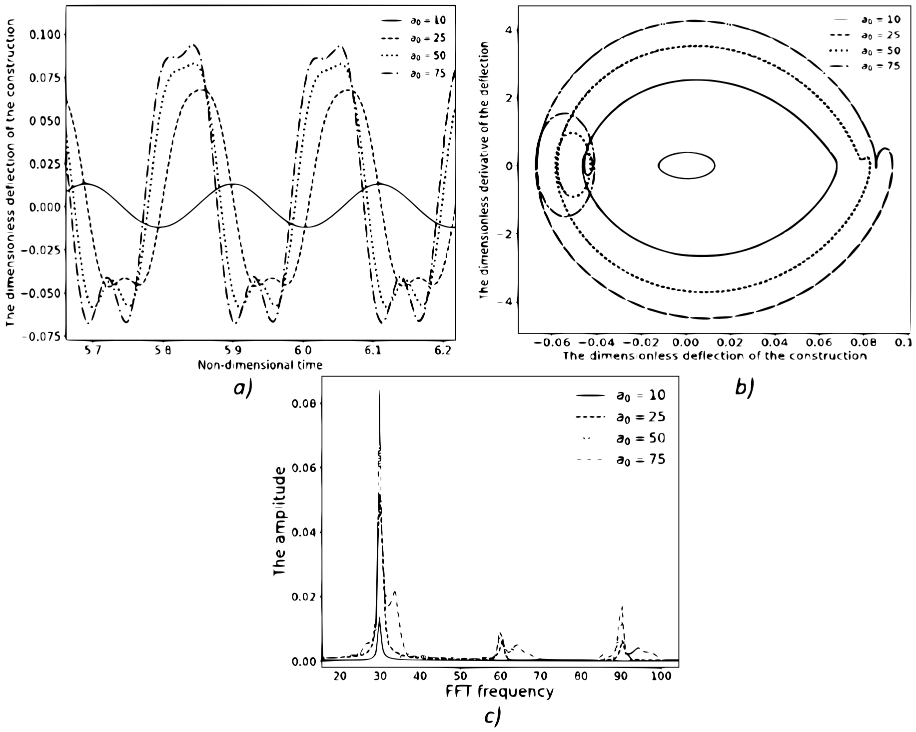


**Fig. 4**. The non-dimensional deflection of the structure under the influence of acceleration $a_0$ at the central point of connection of the micro-beam and various elastic polymeric layers a) Time history; b) Phase portrait; c) FFT analysis

The time history (Fig. 4a) demonstrates the amplitude and evolution of the system under dynamic load. Phase portraits (Fig. 4b) are a means of visualizing the trajectories of the system, thereby confirming its stability and possible nonlinear phenomena (i.e., bifurcations). Fig. 4c illustrates the results of the fast Fourier transform (FFT) analysis. The system exhibits nonlinear behavior, with primary resonance occurring at a frequency of $\Omega = 30$ and secondary resonance occurring at a frequency of $2\Omega$.

To comprehend how the structure or device responds to different input frequencies, a comprehensive understanding of frequency response analysis is required. This analysis provides insights into the stability, functionality, and potential issues of the structure by identifying its behavior, resonance sites, and frequency-dependent features. The design and optimization of structure in various domains, including engineering, electronics, and signal processing, rely on this study.

Fig. 5 displays the non-dimensional amplitude of the micro-beam's center deflection as a function of the non-dimensional excitation frequency under the applied harmonic acceleration, which can be written as:

$$\overline{a}_0(\tau) = \overline{a}_0 sin(\Omega\tau)$$

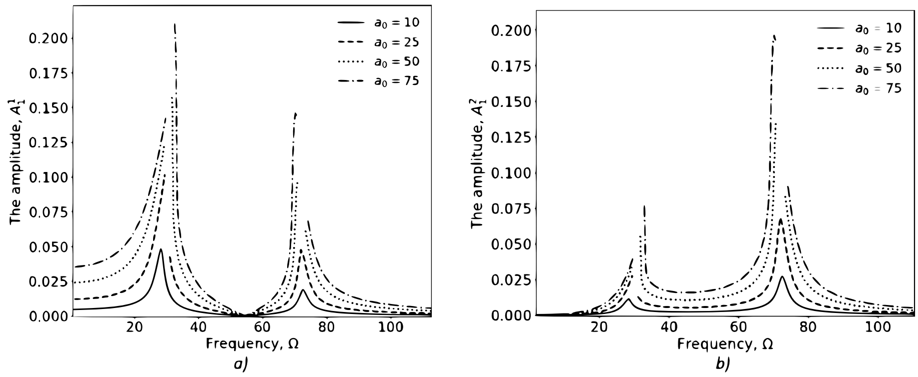


**Fig. 5**. The non-dimensional longitudinal (a) and transversal (b) amplitude of the micro-beam's center deflection versus properties of studied materials for different amplitudes of the applied harmonic acceleration

The structure under study exhibits quadratic and cubic nonlinearities, as well as coupling nonlinearities, which may result in the emergence of secondary resonances in addition to the primary resonances.

Fig. 5 shows the non-dimensional amplitude of the micro-beam‘s center deflection versus various amplitudes of the applied harmonic acceleration $\overline{a}_0$. The frequency of the initial harmonic input is swept from a value of zero to a desired value that is greater than the system's natural frequencies $\omega_n^1 = 27.8$ and $\omega_n^2 = 72.3$. The amplitude of the center deflection of the micro-beam demonstrates an increase in accordance with the amplitude of the applied harmonic acceleration. When the applied acceleration reaches a relatively high value, the system's response exhibits nonlinear behavior. System clearly demonstrates a typical jumping phenomenon at the primary resonance $\Omega \approx \omega_n^1$, and there is also a secondary resonance when the excitation frequency is $\Omega \approx \omega_n^2$.

This study presents a comprehensive analysis of the factors necessary to ensure optimal system performance, minimize the amplitude of system vibrations, and guarantee structural integrity under diverse conditions. To comprehend the system's behavior, we employed sophisticated techniques such as frequency response analysis, numerical modeling, and nonlinear dynamic modeling. Our findings can inform the development of effective strategies to reduce vibrations, enhance stability, and create elastic systems capable of withstanding various influences.

## Conclusion

This study focuses on the nonlinear transient and steady-state behavior of a micro-beam situated on an elastic polymeric bed, which is comparable to those employed in isolation systems. In order to account for the inertial effects of the bed, the coupled governing equations obtained from the elasto-dynamic equation incorporate both the nonlinear squeezing motion of the elastic polymeric layer and the transverse motion of the micro-beam.

Subsequently, the nonlinear governing equations are discretized across spatial coordinates and then integrated in time to capture transient reactions. The experiments included cases with constant, step, and pulse excitation accelerations. Based on the observations, conclusions were drawn about the reactions of damping materials under external influence. The most common Material 1 (rubber) exhibited the largest deflection amplitude, while Material 2 (thermoplastic elastomers) demonstrated the smallest deflection amplitude.

The behavior of the system was analyzed under different materials and operating conditions using frequency response analysis. Frequency response analysis is employed to investigate the behavior of the structure under diverse material qualities and base excitation conditions. This method represents periodic responses by Fourier expansion and acquires coefficients by energy balancing within a period. This method was employed to predict the primary and secondary resonances in the harmonics of the response.

The present study contributes to our comprehension of the dynamics of beams supported by elastic polymer foundations. The findings offer a foundation for the development of resilient structures with enhanced isolation capabilities, particularly in the context of diverse material characteristics and operational scenarios. Future research could delve into more intricate models and an array of materials.

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# НЕЛИНЕЙНЫЙ АНАЛИЗ БАЛКИ НА УПРУГОМ ПОЛИМЕРНОМ ОСНОВАНИИ: ИССЛЕДОВАНИЕ ПЕРЕХОДНЫХ И ЧАСТОТНЫХ ХАРАКТЕРИСТИК

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В этом исследовании рассматривается динамическое поведение микро-балки, которая опирается на эластичное полимерное основание конечной толщины, особое внимание уделяется нелинейной переходной и устойчивой реакции микро-балки на базовое возбуждение. Рассматривается как поперечное движение балки, так и нелинейное сжимающее движение упругой полимерной основы в рамках связанных нелинейных дифференциальных уравнений. Чтобы изучить влияние различных материалов на общую характеристику системы, нелинейные управляющие уравнения дискретизируются по пространственным координатам и интегрируются по времени, тем самым получая решения для переходных процессов. Что касается частотной характеристики, то коэффициенты гармонической характеристики с расширением по Фурье получаются путем балансировки энергии в течение определенного периода. Был проведен ряд численных экспериментов, включая быстрое преобразование Фурье (FFT), определение временных характеристик и фазового портрета системы. Эти эксперименты охватывали постоянные, ступенчатые, импульсные и гармонические ускорения. Кроме того, были проведены исследования для определения частотной характеристики системы на гармонические входные сигналы с различной частотой.

*Ключевые слова: нелинейный анализ; колебания; переходные решения; частотная характеристика.*

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