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ON THE INTERNAL INVERSE BOUNDARY PROBLEM IN THE STUDY OF CRITICAL CONDITIONS OF DISCRETE-CONTINUOUS HETEROGENEOUS JOINTS

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A system of plastic equilibrium equations is investigated, defined on a strip with an insert with a variable plasticity parameter along the strip, discontinuous at the boundary of the insert and the base material. This quasi-linear hyperbolic type system arises when studying the stress state of an inhomogeneous strip with a less durable interlayer during plane deformation under a tensile load. The Riemann invariants along the characteristics are approximately calculated. An analog of Hencky's first theorem is obtained. The conjugation problem for stresses at the contact boundary is approximately solved.

Keywords: discrete-continuous heterogeneous joints; stress state; plastic layer; weld joint; plane deformation; quasi-linear system of hyperbolic equations; Riemann invariants; Hencky integrals.

Introduction

In the works devoted to the strength of welded joints, the interlayer (insert) material was considered homogeneous in many cases [1–9]. Practically, the materials of welded joints are usually heterogeneous [10–13].

When studying the stress-strain state and bearing capacity of welded joints, in which the weld or interlayer in the zone of thermal influence can be a less durable layer, a boundary value problem arises for the system of equations of plastic equilibrium. It can be solved first in the vicinity of a free boundary by finding exactly or approximately Riemann invariants. They are used to solve the problem of coupling for stresses at the contact boundary. The found stresses are used in the sequel to calculate the critical load in the vicinity of the free boundary, as well as to redefine the problem in the middle part of the layer [14, 15]. In [14, pp. 105–124; 15, pp. 116–148; 16] (and others), the strength of the layer varied in thickness. Here, for the first time, the case is considered when the strength at a point depends on the distance between this point and the axis of symmetry of the strip.

The paper considers the case when the strength depends on the distance from the middle of the strip and varies in the layer and outside it.

The aim of the work is to obtain approximate first integrals on the characteristics in this case, and on this basis solve the stress coupling problem (internal inverse boundary problem) at the contact boundary between a less durable heterogeneous layer and a more durable base material.

The paper considers in the Cartesian coordinate system xOy a joint in the form of a strip $[-1; 1] \times (-\infty; \infty)$ with an insert (layer, interlayer) $[-1; 1] \times [-d; d]$, $0 < d < 1$. The layer material and the base joint material (BM) are assumed to be ideal rigid plastic media, isotropic but not homogeneous, the strength of which is characterized by a single parameter (for example, yield strength), depending on one variable: k=k(x). Note that if the plastic constant depends on the coordinates of the layer points, then the first integrals on the characteristics of this system are unknown. This does not allow you to use the characteristics method directly. If necessary, the values related to BM are given the superscript "+", and those related to layer "−". Everywhere $k^+ > k^-$. The results also apply to hardenable isotropic materials, assuming $k = \sigma_B$. Here σ_B denotes the ultimate strength. The critical moment of loading is considered, that is, the moment of loss of stability of the plastic deformation process [17, 18]. The case $k = k(y)$ is considered in [14–16].

Let's introduce the notation:

$$U(x) = k(x)/k_0, \;\; k_0 = k(0), \tag{1}$$

where the dimensionless function $U(x)$ characterizes the distribution of strength properties over the width of the strip (a function of layer inhomogeneity). It is obvious from (1) that $U(0) = 1$.

The stress state of the strip is determined by a system of equilibrium equations and the Mises plasticity condition:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \qquad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0, \tag{2}$$

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = 4U^2(x). \tag{3}$$

Here x and y are the dimensionless coordinates of the layer points, σ_x, σ_y and τ_{xy} – dimensionless stresses obtained by dividing the true stresses by k_0. Suppose that at the contact boundary $y = d$, the heterogeneity function has a jump, and

$$K(x) = \frac{U^+(x)}{U^-(x)}, \;\; K(x) > 1. \tag{4}$$

For most welded joints in steel structures, the mechanical heterogeneity is small: $K = 1.1; \ldots 1.4$; $1 < U(x) < 1,2$. Therefore, the tangential stresses on the contact surface are also small and have the order of $K - 1$. The function $U(x)$ is assumed to be even. Due to the symmetry, a quarter of the band can be considered: $[0; 1] \times [0; \infty]$.

1. The Invariant Form of the System of Equations of Plastic Equilibrium and Its Integration Along the Characteristics

The method of characteristics is widely used in solving problems in solid mechanics, fluid and gas mechanics, and other branches of mechanics and mathematical physics. It is used in a number of engineering tasks related to structural strength. In the case of plane deformation, the method of characteristics (sliding line method) is used to study the bearing capacity of shells, rods, sheet structures [10–12] and others. It is a convenient tool for studying boundary value problems for quasi-linear equations of hyperbolic type [19].

His main idea is to find and use a constant value on the characteristic – the Riemann invariant. Riemann invariants do not always exist. In some cases, it is possible to find a value that varies little along the characteristic [14, pp. 105–124; 15, pp. 116–148; 16]. The same approach is used in this work.

When the plastic layer is stretched (compression is studied similarly) (3) has the form:

$$\sigma_y - \sigma_x = f, \quad f = 2\sqrt{U^2(x) - \tau_{xy}^2}. \tag{5}$$

Then the system (2), (3) is equivalent to the matrix equation

$$\frac{\partial \overline{\sigma}}{\partial x} + A\frac{\partial \overline{\sigma}}{\partial y} = B, \tag{6}$$

$$\overline{\sigma} = \begin{pmatrix} \sigma_x \\ \tau_{xy} \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 \\ 1 & \partial f/\partial \tau_{xy} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{7}$$

Note that if U depends on y, then B is not zero. The function f is defined in (5). The characteristic equation of the matrix A is the equation

$$\lambda^2 + \frac{2\tau_{xy}/U}{\sqrt{1 - (\tau_{xy}/U)^2}}\lambda - 1 = 0. \tag{8}$$

Integral curves of equations

$$\frac{dy}{dx} = \lambda_i, \quad i = 1, 2, \tag{9}$$

are called [19] the characteristics of the (6), (7) system, where the eigenvalues $\lambda_1 = \lambda_\xi$ and $\lambda_2 = \lambda_\eta$ are the roots of the equation (8). For more information, see [4; 5].

Let's write the system (6),(7) in invariant form, for which we multiply the equation (6) by the eigenvectors on the left:

$$\frac{\partial(\sigma_x + \nu_i)}{\partial x} + \lambda_i \frac{\partial(\sigma_x + \nu_i)}{\partial y} = \frac{\partial \nu_i}{\partial U} U'(x), \qquad i = 1, 2, \tag{10}$$

$$\nu_i(\tau_{xy}) = \sqrt{U^2 - \tau_{xy}^2} \pm U \arcsin\frac{\tau_{xy}}{U}. \tag{11}$$

In (11), $i = 1$ corresponds to the plus sign, and $i = 2$ corresponds to the minus sign. Functions ν_i is selected so that $\partial \nu_i/\partial \tau_{xy} = \lambda_i$, $i = 1, 2$. By virtue of the (9) equation (10) on the characteristics can be written as:

$$\frac{d(\sigma_x + \nu_i - U)}{dx} = \left(\frac{\partial \nu_i}{\partial U} - 1\right) U'(x), \qquad i = 1, 2. \tag{12}$$

Let's denote: $t = \tau_{xy}/U$. Then, using power series expansions of the functions $\arcsin(\cdot)$, $\sqrt{1 - (\cdot)^2}$ and $(\cdot)/\sqrt{1 - (\cdot)^2}$, we obtain from (11):

$$\nu_\xi(\tau_{xy}) = U\left(1 - \frac{t^2}{2} - \frac{t^2}{2} - \frac{t^4}{8} - \frac{t^6}{16} - ... + t + \frac{t^3}{3} + \frac{3t^5}{40} - ...\right), \tag{13}$$

$$\nu_\eta(\tau_{xy}) = U\left(1 - \frac{t^2}{2} - \frac{t^2}{2} - \frac{t^4}{8} - \frac{t^6}{16} - ... - t - \frac{t^3}{3} - \frac{3t^5}{40} - ...\right), \tag{14}$$

$$\frac{\partial(\nu_i)}{\partial U} = U'\left(1 + \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{3}{8}t^4 + ...\right) < U'\left(1 + \left(\frac{1}{2}t^2 + \frac{1}{3}t^3\right)\frac{1}{1-t^2}\right). \quad (15)$$

The latter inequality is based on decreasing (modulo) the coefficients of these series. Let

$$\Delta = \int_0^x U'\left(\frac{1}{2}t^2 \pm \frac{1}{3}t^3 + \frac{3}{8}t^4 + ...\right) dx. \quad (16)$$

Integration is carried out according to any characteristic. Then it follows from (12)–(16) that along each characteristic

$$\sigma_x + \nu_i - U + \text{const} = \Delta. \quad (17)$$

Here *const* is a constant on the characteristic.

Lemma 1. If $t < \alpha$, $\alpha < 1$, and $|U(x) - 1| < \beta$ then

$$|\Delta| < \left(\frac{1}{2} + \frac{1}{3}\alpha\right)\frac{\alpha^2\beta}{1-\alpha^2}.$$

Proof. It follows from (15) and the conditions of the lemma.

For example, if $\alpha = 0{,}3$, $\beta = 0{,}3$, then $|\Delta| < 0{,}018$; if $\alpha = 0{,}2$, $\beta = 0{,}2$, then $|\Delta| < 0{,}005$;

With a small mechanical inhomogeneity, the tangential stresses, and with them the α parameter, are small. It follows from the lemma that in practical problems, approximate equalities can be used instead of (17)

$$\sigma_x + \nu_i - U \approx \text{const} \quad (18)$$

along the characteristics with the error specified in the lemma. The *const* parameter is constant along any characteristic. The left part of (18) can be called the (approximate) Riemann invariant for the system of equations (6), (7).

Let the external pressure on the free boundary of be equal to p. Then on this border

$$\sigma_x = -p, \quad \tau_{xy} = 0. \quad (19)$$

The exception is the value of τ_{xy} at a special point, the exit point of the contact boundary to the free one (let's denote it A). Here, τ_{xy} takes a range of values and is the limit of τ_{xy}, depending on which characteristic this limit is calculated by. Taking into account (19), based on the formulas (11) and (18), we obtain that for each η characteristic that does not contain A,

$$\sigma_x = -\nu_\eta + U(x) + \Delta - p \approx U\left(\frac{t^2}{2} + t\right) - p, \quad (20)$$

$$\sigma_y = \sigma_x + 2U\left(1 - \frac{t^2}{2}\right) + \Delta \approx -\frac{\tau^2}{2U} + \tau + 2U - p, \quad (21)$$

and on each ξ-characteristic that does not contain A,

$$\sigma_x = -\nu_\xi + U(x) + \Delta - p \approx U\left(\frac{t^2}{2} - t\right) - p, \quad (22)$$

$$\sigma_y = \sigma_x + 2U\left(1 - \frac{t^2}{2}\right) + \Delta \approx -\frac{\tau^2}{2U} - \tau + 2U - p. \tag{23}$$

In general, it is necessary to put const (a constant value on the corresponding characteristic) in (20) – (23) instead of $-p$.

2. Calculation of Tangential and Normal Stresses at the Contact Boundary in the Vicinity of a Free Surface

In the (linear) conjugation problem for stresses, or the internal boundary value problem for stresses, it is required to find the stresses at the contact boundary, here a straight line $y = d$, by the values of the inhomogeneity functions on it U^+; U^-. The coupling conditions for stresses at the contact boundary are the equilibrium equations:

1) $\tau^-_{xy} = \tau^+_{xy}$ (we will denote $\tau^\pm_{xy} = \tau$ on the inner border) and
2) $\sigma^-_{xy} = \sigma^+_{xy}$.

We denote the $\tau(S)$ or simply τ tangential stresses at the point $S(x; d)$ of the contact surface. Using the formulas (21) and (23), we write the second conjugation condition as:

$$-\frac{\tau^2}{2U^+(S)} - \tau + 2U^+(S) \approx -\frac{\tau^2}{2U^-(S)} + \tau + 2U^-(S),$$

where does the quadratic equation for τ come from:

$$A\tau^2 - 2\tau + C \approx 0, \tag{24}$$

$$A = \frac{1}{2}\left(\frac{U^+(S) - U^-(S)}{U^-(S)U^+(S)}\right); \;\; C = 2U^+(S) - 2U^-(S). \tag{25}$$

Using (4), we write (24), (25) in the form (here $U = U^-(S)$):

$$(K-1)\tau^2 - 4KU\tau + 4K(K-1)U^2 \approx 0, \tag{26}$$

where from:

$$\tau \approx \frac{2KU}{K-1}\left(1 - \sqrt{1 - \frac{(K-1)^2}{K}}\right) \approx (K-1)\left(1 + \frac{(K-1)^2}{4K}\right)U \approx (K-1)U. \tag{27}$$

Here, the first expression is the exact solution of the equation (26). For practical purposes, the precision of the last expression is sufficient. It gives a relative error of about 0.0173 at $K = 1.3$, and about 0.0083 at $K = 1.2$.

Substitute $\tau \approx (K-1)U$ in (21) and (23), counting in (21) $U = U^-$, and in (23) $U = U^+ = KU^-$ (we write U instead of U^-). We get:

$$\sigma_y \approx \sigma^-_y \approx -\frac{(K-1)^2U}{2} + (K-1)U + 2U - p, \tag{28}$$

$$\sigma_y \approx \sigma^+_y \approx -\frac{(K-1)^2U}{2} - (K-1)U + 2KU - p, \tag{29}$$

Despite the external differences, the deviation between the values of (28) and (29) is insignificant. It is equal to $0{,}5(K-1)^3 U/K$. For example, for $K = 1{,}2$ $\sigma_y^+ - \sigma_y^- = 0.0033$. When using the penultimate expression in (27)

$$\sigma_y^+ - \sigma_y^- = \frac{U(K-1)^5}{4K^2}\left(1 + \frac{(K-1)^2}{8K}\right).$$

In this case, for $K = 1{,}2$ $\sigma_y^+ - \sigma_y^- = 0.00007U$.

3. Generalization of Hencky's Theorem

In [14–16], the case of dependence on y of the plasticity parameter was considered. The equation (18) has a different form there:

$$\sigma_x + \nu_i + U(y) \approx \text{const}.$$

This does not affect the generalization of Hencky's theorem; it has the same formulation as in the case of $U = U(x)$, discussed in the article. Let's show it. Let γ be the acute angle of inclination of ξ-characteristics to the OX axis. Then for the characteristic $y = y(x)$

$$\operatorname{tg}\gamma = \frac{dy}{dx} = \lambda_\xi = (1-t)/\sqrt{1-t^2}; \quad t = \cos 2\gamma; \quad \arcsin t = \frac{\pi}{2} - 2\gamma. \tag{30}$$

Expressing the functions ν_i $(i = 1, 2)$ in terms of γ, we get from (11) and (30):

$$\nu_\xi = U\left(\sin 2\gamma + \pi/2 - 2\gamma\right), \quad \nu_\eta = U\left(\sin 2\gamma + 2\gamma - \pi/2\right). \tag{31}$$

Along any ξ-characteristics of

$$\sigma_x = U(2\gamma - \pi/2 - \sin 2\gamma + 1) + \text{const},$$

$$\sigma_y = U(2\gamma - \pi/2 + \sin 2\gamma + 1) + \text{const},$$

$$\sigma = (\sigma_x + \sigma_y)/2 = U(2\gamma - \pi/2 + 1) + \text{const} \tag{32}$$

by virtue of the formulas (18), (30) and (31). Here, the term const is constant along each ξ-characteristic. Similar formulas take place for η-characteristics:

$$\sigma_x = U(-2\gamma + \pi/2 - \sin 2\gamma + 1) + \text{const},$$

$$\sigma_y = U(-2\gamma + \pi/2 + \sin 2\gamma + 1) + \text{const},$$

$$\sigma = (\sigma_x + \sigma_y)/2 = U(-2\gamma + \pi/2 + 1) + \text{const}. \tag{33}$$

by virtue of the formulas (18), (30) and (31). Here, the term const is constant along each η characteristic.

The formulas (32) and (33) for a constant U have the well-known form [1, 20, 14, 15]. They are usually written in numerical values:

$$\widetilde{\sigma} - 2k\gamma = \text{const}, \quad \widetilde{\sigma} + 2k\gamma = \text{const}$$

based on ξ-characteristics and η-characteristics, respectively. The const parameter is constant along any characteristic.

Let

$$\theta = \gamma - \pi/4.$$

Theorem 1. *Let the points K, L, M, and N form a (curved) rectangle of ξ-characteristics KL and NM and η-characteristics of NK and ML. Then*

$$U(K)\theta(K) - U(L)\theta(L) = U(N)\theta(N) - U(M)\theta(M). \tag{34}$$

When switching between two characteristics of one family along the characteristic of the other family, the values change $U \cdot (\gamma - \pi/4)$ does not depend on which characteristic of the other family makes the transition.

Proof. By virtue of (32) on ξ-characteristics

$$\sigma(K) - \sigma(L) = U(K)(2\theta(K) + 1) - U(L)(2\theta(L) + 1), \tag{35}$$

$$\sigma(N) - \sigma(M) = U(N)(2\theta(N) + 1) - U(M)(2\theta(M) + 1). \tag{36}$$

Similarly, for η characteristics, it follows from (33):

$$\sigma(K) - \sigma(N) = U(K)(-2\theta(K) + 1) - U(N)(-2\theta(N) + 1), \tag{37}$$

$$\sigma(L) - \sigma(M) = U(L)(-2\theta(L) + 1) - U(M)(-2\theta(M) + 1). \tag{38}$$

Substituting into the identity

$$\sigma(K) - \sigma(L) + \sigma(L) - \sigma(M) = \sigma(K) - \sigma(N) + \sigma(N) - \sigma(M)$$

right-hand sides of expressions (35)–(38), after simplifications, we obtain the equality (34), which completes the proof.

At a constant U, the equation (34) is Hencky's first theorem [1, 20].

Corollary 1. *In terms of the theorem*

$$\sigma(K) - U(K) - (\sigma(L) - U(L)) = \sigma(N) - U(N) - (\sigma(M) - U(M)), \tag{39}$$

Thus, with the accuracy determined by the lemma, when switching between two characteristics of one family along the characteristic of another family, the change in the values of $\sigma - U$ does not depend on this characteristic of the other family.

Proof. Let's write (35) and (36) as:

$$\sigma(K) - U(K) - (\sigma(L) - U(L)) = 2U(K)\theta(K) - 2U(L)\theta(L),$$

$$\sigma(N) - U(N) - (\sigma(M) - U(M)) = 2U(N)\theta(N) - 2U(M)\theta(M)$$

Subtracting the last two equalities and noting that on the right side, by virtue of (34), the sum is zero, we get (39).

If U is a constant, then (39) turns into the known ratio [1, 20]:

$$\sigma(K) - \sigma(L) = \sigma(N) - \sigma(M).$$

Conclusions

Riemann invariants are approximately found for a system of plastic equilibrium equations that determines the stress state of a joint – a strip with a less durable insert when the plasticity parameter is not constant along the strip, in the case of a small mechanical heterogeneity of the joint.

On this basis, the problem of coupling for stresses at the boundary between a less durable layer and the main part of the strip is solved, which is necessary to determine the critical load on an inhomogeneous strip containing a transverse less durable layer.

Approximate analogues of the Genki results are obtained when the heterogeneity of the material depends on x. They have the form (34) and (39).

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О ВНУТРЕННЕЙ ОБРАТНОЙ ГРАНИЧНОЙ ГРАНИЧНОЙ ЗАДАЧЕ ПРИ ИССЛЕДОВАНИИ КРИТИЧЕСКИХ СОСТОЯНИЙ ДИСКРЕТНО-НЕПРЕРЫВНЫХ НЕОДНОРОДНЫХ СОЕДИНЕНИЙ

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Исследуется система уравнений пластического равновесия, заданная на полосе со вставкой с переменным поперек полосы параметром пластичности, разрывным на границе вставки и основного материала. Эта квазилинейная система гиперболического типа возникает при изучении напряженного состояния неоднородной полосы с менее прочной прослойкой при плоской деформации под растягивающей нагрузкой. Приближенно вычисляются инварианты Римана вдоль характеристик. Получен аналог первой теоремы Генки. Приближенно решена задача сопряжения для напряжений на контактной границе.

Ключевые слова: напряженное состояние; неоднородный пластический слой; плоская деформация; квазилинейная система гиперболических уравнений; инварианты Римана; интегралы Генки.

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