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SOLUTION FOR ONE STOCHASTIC NON-STATIONARY LINEARIZED HOFF MODEL

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The article investigates the solvability of the non-autonomous linearized Hoff model in the space of stochastic **K**-processes. To do this, firstly the paper provides a result on the solvability of such a model in the deterministic case. Next, stochastic **K**-processes describes and some their specific characteristics are set. Finally, we formulates a theorem on the existence of a solution in the stochastic case and provides a result that illustrates the obtained statements.

Keywords: Sobolev type equations; Nelson – Gliklikh derivative; space of stochastic **K**-*processes.*

Introduction

Let $\Pi \subset \mathbb{R}^n$ is a bounded domain in $\mathbb{R}^n$ with boundary $\partial\Pi$ of C^∞ class. A semilinear Hoff equation

$$(\lambda - \Delta)u_t = \alpha u + u^3 \tag{1}$$

describes buckling of an I-beam under constant load. Here α is a real number, which is characterized material properties of the I-beam; the number $\lambda \in \mathbb{R}_+$ corresponds to the load on the beam [1]. If $\lambda \in \sigma(\Delta)$ then the left part of equation (1) becomes zero. Equations, that are unsolved with respect to the highest time-derivative, are referred to the Sobolev type equations [2]

$$L\dot{u} = Mu + N(u), \qquad \ker L \neq \{0\},$$

where operators L and M are linear bounded operators acting from the Banach space $\mathfrak{U}$ into the Banach space $\mathfrak{F}$, and the operator N is a nonlinear smooth operator acting in the same spaces. Even in the first papers devoted to these equations, their feature was noted in the fact that for them the Cauchy problem

$$u(0) = u_0$$

is fundamentally insolvable for arbitrary initial data u_0, even from dense set in $\mathfrak{U}$. In order for solutions of the Sobolev type equation to exist, it is necessary that the initial data belong to some set of admissible initial values (phase space) for these equations. In order not to check this condition, we shall use the Showalter – Sidorov condition

$$L(u(0) - u_0) = 0.$$

This condition coincides with the Cauchy condition, in the case of a nondegenerate operator L, and in the case of a degenerate operator L, it eliminates the need for matching the initial data. At present, the theory of Sobolev type equations is rapidly developing in various direction. For example, applying the methods of this theory [3, 4] to reconstruct a dynamically distorted sensor signal [5] served as the basis for creating a theory of optimal dynamical measurements [6, 7].

For the first time, the methods of the Sobolev type equations theory [2] were used to study the equation (1) in [8]. Later, within the framework of this theory, the Hoff equation (1) was investigated in various aspects (see, for example, [9, 10, 11, 12, 13]).

In this paper we consider the non-stationary linearized Hoff equation of the form

$$(\lambda - \Delta)u_t = \alpha(t)u + g(t)$$

in stochastic case. In this equation the vector function $g : \mathbb{R} \to \mathfrak{F}$ characterizes the external action on the system, and the scalar function $\alpha : [0, T] \to \mathbb{R}_+$ characterizes the time variation of the parameters of this equation. Non-stationary Sobolev type equations were first considered in [14] and the proposed methods were applied to investigate various problems, for example, in [15, 16, 17]. The article, in addition to the introduction and list of references, contains two parts. The first part provides information on the solvability of problems for non-stationary linearized Hoff equation. The second part describes the space of stochastic **K**-processes and describes the solution of the stochastic Hoff model.

1. Solutions of the Non-Stationary Linearized Hoff Equation

Let $\Pi \subset \mathbb{R}^n$ is a bounded domain in $\mathbb{R}^n$ with boundary $\partial\Pi$ of C^∞ class. In cylinder $\Pi \times \mathbb{R}$ we consider Dirichlet problem

$$u(x,t) = 0, \quad (x,t) \in \partial\Pi \times \mathbb{R} \tag{2}$$

for Hoff equation of the form

$$(\lambda - \Delta)u_t = \alpha(t)u + g(t) \tag{3}$$

with Showalter – Sidorov condition

$$(\lambda - \Delta)(u(0) - u_0) = 0. \tag{4}$$

Here the vector function $g(t)$ characterizes external action on the described system.

Problem (2)–(4) is considered within the framework of Showalter – Sidorov problem

$$P(u(0) - u_0) = 0, \qquad u_0 \in \mathfrak{U} \tag{5}$$

for non-stationary Sobolev type equation

$$L\dot{u}(t) = a(t)Mu(t) + g(t), \tag{6}$$

which set is some Banach spaces $\mathfrak{U}$ and $\mathfrak{F}$ with operators $L, M \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ and $\ker L \neq \{0\}$.

A vector function $u \in C^1(\mathbb{R}; \mathfrak{U})$ is called a *solution* of equation (6), if it satisfies this equation on $\mathbb{R}$. The solution of equation (6) is called a *solution of Showalter – Sidorov problem* (5), (6), if in addition it satisfies (5).

Reduce problem (2)–(4) to problem (5), (6). In order we get the spaces

$$\mathfrak{U} = \{u \in W_q^{m+2}(\Omega) : u(x) = 0,\ x \in \partial\Omega\}, \qquad \mathfrak{F} = W_q^m(\Omega), \tag{7}$$

where $W_q^m(\Omega)$ are Sobolev spaces with $2 \leq q < \infty$ and $m = 0, 1, \ldots$. Operators L and M we define by the next formulas

$$L = \lambda - \Delta, \qquad M = \mathbb{I}. \tag{8}$$

By $\sigma(\Delta)$ we denote a spectrum of homogeneous Dirichlet problem on the domain Π for the Laplace operator Δ. The spectrum of $\sigma(\Delta)$ is negative, discrete, finite and condensed only to $-\infty$. By $\{\lambda_k\}$ we denote the set of eigenvalues, which numbered by nonincreasing order with their multiplicity. And by $\{\psi_k\}$ we define the family of corresponding eigenfunctions orthonormal with respect to the inner product $\langle \cdot, \cdot \rangle$ in space $L_2(\Pi)$, $\psi_k \in C^\infty$, $k \in \mathbb{N}$.

If $\lambda \in \sigma(\Delta)$ has multiplicity $r \in \mathbb{N}$, we put $\ker L = \operatorname{span}\{\psi_1, \psi_2, ..., \psi_r\}$, where ψ_l are the eigenfunctions of the Laplace operator Δ corresponding to the eigenvalue λ, and they can be chosen orthogonal in the sense of the inner product $\langle \cdot, \cdot \rangle$ в $L_2(\Pi)$. Then $\operatorname{im} L = \{g \in \mathfrak{F} : \langle g, \psi_l \rangle = 0,\ l = 1, 2, ..., r\}$.

It is clear that if $\lambda \neq 0$ then the L-spectrum of operator M can be represent as

$$\sigma^L(M) = \{\mu \in \mathbb{C} : \mu_k = \frac{1}{\lambda - \lambda_k}, \lambda_k \neq \lambda\}.$$

If $\lambda = 0$ then the L-spectrum of operator M is such that $\sigma^L(M) = \mathbb{C}$, so everywhere else we take $\lambda \neq 0$. Since the points of the spectrum of the Laplace operator $\{\lambda_k\}$ are real, discrete, had finite multiplicity and condensed only to $-\infty$, then the relative spectrum of $\sigma^L(M)$ is obviously bounded.

Construct projectors P and Q. If $\lambda \notin \sigma(\Delta)$ then the projector $P = \mathbb{I}$, and if $\lambda \in \sigma(\Delta)$ then $P = \mathbb{I} - \sum\limits_{l=1}^{r} \langle \cdot, \psi_l \rangle \psi_l$. The projector Q has the same form, but is defined on the space $\mathfrak{F}$.

Theorem 1. [18] *Let spaces $\mathfrak{U}$ and $\mathfrak{F}$ are from* (7), *and operators L and M are from* (8). *Also let $\lambda \in \mathbb{R}\backslash\{0\}$, $\alpha \in C^1([0, T]; \mathbb{R}_+)$, $g \in C^1([0, T]; \mathfrak{F})$ and*

(i) $\lambda \notin \sigma(\Delta)$. *Then for arbitrary $u_0 \in \mathfrak{U}$ there exists the unique solution* (2)–(4), *which has the form*

$$u(t) = \sum_{k=1}^{\infty} e^{\left(\frac{1}{\lambda - \lambda_k} \int\limits_0^t \alpha(\tau) d\tau\right)} \langle u_0, \psi_k \rangle \psi_k + \sum_{k=1}^{\infty} \int\limits_0^t e^{\left(\frac{1}{\lambda - \lambda_k} \int\limits_s^t \alpha(\tau) d\tau\right)} \frac{\langle g(s), \psi_k \rangle \psi_k}{\lambda - \lambda_k} ds;$$

(ii) $\lambda \in \sigma(\Delta)$. *Then for arbitrary $u_0 \in \mathfrak{U}$ there exists the unique solution* (2)–(4), *which has the form*

$$u(t) = -\sum_{l \in \mathbb{N} : \lambda_l = \lambda} \frac{\langle g(s), \psi_l \rangle \psi_l}{\alpha(t)} + \sum_{k \in \mathbb{N} \backslash \{l : \lambda_l = \lambda\}}^{\infty} e^{\left(\frac{1}{\lambda - \lambda_k} \int\limits_0^t \alpha(\tau) d\tau\right)} \langle u_0, \psi_k \rangle \psi_k +$$

$$+ \sum_{k \in \mathbb{N} \backslash \{l : \lambda_l = \lambda\}} \int\limits_0^t e^{\left(\frac{1}{\lambda - \lambda_k} \int\limits_s^t \alpha(\tau) d\tau\right)} \frac{\langle g(s), \psi_k \rangle \psi_k}{\lambda - \lambda_k} ds.$$

2. Sobolev type Equations in the Space of Stochastic K-Processes

Let $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ be a complete probability space with probability measure $\mathbf{P}$ associated with the σ-algebra $\mathcal{A}$ of subsets of the set Ω, and $\mathbb{R}$ is a set of real numbers endowed with a Borel σ-algebra. The measurable mapping $\xi : \Omega \to \mathbb{R}$ is called a *random variable.* A set of random variables with zero mathematical expectation and finite variance forms a Hilbert space $\mathbf{L}_2 = \mathbf{L}_2(\Omega;\mathbb{R}) = \{\xi : \mathbf{E}\xi = 0,\ \mathbf{D}\xi < +\infty\}$ with a scalar product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\xi_2$ and the norm $\|\xi\|^2_{\mathbf{L}_2} = \mathbf{D}\xi$.

Let's take the set $\mathfrak{I} \subset \mathbb{R}$ and consider two mappings: $f : \mathfrak{I} \to \mathbf{L}_2$, which matches each $t \in \mathfrak{I}$ with a random variable $\xi \in \mathbf{L}_2$, and $g : \mathbf{L}_2 \times \Omega \to \mathbb{R}$, which each pair (ξ, ω) matches the point $\xi(\omega) \in \mathbb{R}$. Mapping $\eta : \mathfrak{I} \times \Omega \to \mathbb{R}$ (or what is the same $\eta : \mathfrak{I} \to \mathbf{L}_2$), having the form $\eta = \eta(t, \omega) = g(f(t), \omega)$, we call *a stochastic process.* A stochastic process $\eta = \eta(t)$ is continuous on the interval $\mathfrak{I}$ if all its trajectories are continuous (almost surely) (i.e. with a.a. (almost all) $\omega \in \mathcal{A}$ trajectories $\eta(\cdot, \omega)$ are continuous functions). The set of continuous stochastic processes $\eta : \mathfrak{I} \to \mathbf{L}_2$ forms a Banach space with a standard sup-norm, which we denote by the symbol $C(\mathfrak{I}; \mathbf{L}_2)$.

Let $\mathfrak{H}$ be a complex separable Hilbert space with an orthonormal basis $\{\varphi_k\}$, a monotone numerical sequence $\mathbf{K} = \{\lambda_k\} \subset \mathbb{R}_+$ is, what is $\sum\limits_{k=1}^{\infty} \lambda_k^2 < +\infty$, and the sequence $\{\xi_k\} = \xi_k(\omega) \subset \mathbf{L}_2$ of random variables such, what is $\|\xi_k\|_{\mathbf{L}_2} \le \mathrm{C}$ with some constant $\max \mathrm{C} \in \mathbb{R}_+$ and for all $k \in \mathbb{N}$. Let's construct *$\mathfrak{H}$-valued random* $\mathbf{K}$*-value*

$$\xi(\omega) = \sum_{k=1}^{\infty} \lambda_k \xi_k(\omega) \varphi_k.$$

Completion of the linear span of the set $\{\lambda_k \xi_k \varphi_k\}$ according to the norm

$$\|\eta\|_{\mathfrak{H}_\mathbf{K}\mathbf{L}_2} = \left(\sum_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\xi_k \right)^{1/2}$$

is called *space of $\mathfrak{H}$-valued random* $\mathbf{K}$*-value* and are denoted by $\mathfrak{H}_\mathbf{K}\mathbf{L}_2$. That is clear that the space $\mathfrak{H}_\mathbf{K}\mathbf{L}_2$ is a Hilbert space and the random $\mathbf{K}$-value $\xi = \xi(\omega) \in \mathfrak{H}_\mathbf{K}\mathbf{L}_2$.

Mapping $\eta : \mathfrak{I} \to \mathfrak{H}_\mathbf{K}\mathbf{L}_2$, which is defined as

$$\eta(t) = \sum_{k=1}^{\infty} \lambda_k \eta_k(t) \varphi_k,$$

where $\{\eta_k\} \subset C(\mathfrak{I}; \mathbf{L}_2)$, are called *a continuous $\mathfrak{H}$-valued stochastic* $\mathbf{K}$*-process*, if the series in this equality converges uniformly according to the norm $\|\cdot\|_{\mathfrak{H}_\mathbf{K}\mathbf{L}_2}$ on any compact in $\mathfrak{I}$ and process trajectories $\eta = \eta(t)$ are continuous almost surely. The $\mathfrak{H}$-valued stochastic $\mathbf{K}$-process is *differentiating by Nelson – Gliklikh* [19, 20], if the series in

$$\overset{\circ}{\eta}(t) = \sum_{k=1}^{\infty} \lambda_k \overset{\circ}{\eta}_k(t) \varphi_k$$

converges uniformly according to the norm $\|\cdot\|_{\mathfrak{H}_\mathbf{K}\mathbf{L}_2}$ on any compact in $\mathfrak{I}$ and process trajectories $\overset{\circ}{\eta} = \overset{\circ}{\eta}(t)$ are continuous almost surely. Here $\overset{\circ}{\eta}_k$ is a Nelson – Gliklikh derivation of stochastic process $\eta_k : \mathfrak{I} \to \mathbf{L}_2$. By the symbol $C(\mathfrak{I};\mathfrak{H}_{\mathbf{K}}\mathbf{L}_2)$ we denote the space of continuous $\mathfrak{H}$-valued stochastic $\mathbf{K}$-processes and analogously by the symbol $C^{\ell}(\mathfrak{I};\mathfrak{H}_{\mathbf{K}}\mathbf{L}_2)$ we denote the space $\mathfrak{H}$-valued stochastic $\mathbf{K}$-processes, which are continuous differentiable in Nelson – Gliklikh sense up to and including the order of $\ell \in \mathbb{N}$.

Now let's $\mathfrak{U}$ and $\mathfrak{F}$ are complex separable Hilbert spaces with an orthonormal basis $\{\varphi_k\}$ and $\{\psi_k\}$ correspondingly. By symbols $\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ and $\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$ we denote the Hilbert spaces, which are completion of linear span of *random* $\mathbf{K}$*-values*

$$\xi = \sum_{k=1}^{\infty} \lambda_k \xi_k \varphi_k \ (\xi_k \in \mathbf{L}_2) \ \text{ and } \ \zeta = \sum_{k=1}^{\infty} \mu_k \zeta_k \psi_k \ (\zeta_k \in \mathbf{L}_2)$$

according to the norm

$$\|\eta\|^2_{\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2} = \sum_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\xi_k \ \text{ and } \ \|\omega\|^2_{\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2} = \sum_{k=1}^{\infty} \mu_k^2 \mathbf{D}\zeta_k$$

correspondingly. Note that in different spaces ($\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ and $\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$) the sequence $\mathbf{K}$ can be different ($\mathbf{K} = \{\lambda_k\}$ in $\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ and $\mathbf{K} = \{\mu_k\}$ in $\mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$), however, all sequences marked with the symbol $\mathbf{K}$ must be monotonic and summable with a square. All results, generally speaking, will be true for different sequences of $\{\lambda_k\}$ and $\{\mu_k\}$, but for simplicity's sake we will limit ourselves to the case $\lambda_k = \mu_k$.

Consider a linear evolutionary stochastic equation

$$L \overset{\circ}{\eta} = a(t) M \eta + \varpi. \tag{9}$$

The process $\eta \in \mathbf{C}^1(\mathbb{R};\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2)$ we call *solution of equation* (9) for some process $\varpi : [0,T] \to \mathfrak{F}_{\mathbf{K}}\mathbf{L}_2$, if when substituting it in (9) it turns this equation into an identity almost surely. Solution $\eta = \eta(t)$ of equation (9) we call *solution of the Showalter – Sidorov problem*

$$\lim_{t \to \tau+} P(\eta(t) - \eta_0) = 0 \tag{10}$$

if it is fulfilled for this function and some random $\mathbf{K}$-value $\eta_\tau \in \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$.

Theorem 2. *Let the scalar function $a \in C^{p+1}([\tau_0, \tau_n];\mathbb{R}_+)$, the operator M is an (L,p)-bounded ($p \in \mathbb{N}_0$). Then for arbitrary random $\mathbf{K}$-value $\eta_0 \in \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ that are independent from $\mathfrak{U}$-valued $\mathbf{K}$-process $L_1^{-1}Q\varpi : (0,T) \to \mathfrak{U}_{\mathbf{K}}\mathbf{L}_2$ such that $Q\varpi \in C((0,T), \mathfrak{F}^{\mathbf{1}}_{\mathbf{K}}\mathbf{L}_2)$ and $(\mathbb{I}_{\mathfrak{F}} - Q)\varpi \in C^{p+1}((0,T), \mathfrak{F}^{\mathbf{0}}_{\mathbf{K}}\mathbf{L}_2)$, there exists almost surely the unique solution $\eta \in C([0,T];\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2) \cap C^1((0,T];\mathfrak{U}_{\mathbf{K}}\mathbf{L}_2)$ of problem* (9), (10)*, which has the form*

$$\eta(t) = -\sum_{k=0}^{p} H^k M_0^{-1} \left(\frac{1}{a(t)} \frac{\circledcirc\!\!\!\!d}{dt} \right)^k \frac{\varpi^0(t)}{a(t)} + U(t,0)P\eta_0 + \int_0^t U(t,s) P L_1^{-1} \varpi^1(s) ds.$$

Here the symbol $\frac{\circledcirc\!\!\!\!d}{dt}$ denote the Nelson – Gliklikh derivation, and operator $U(t,\tau)$ is a flow of resolve operators (more see in [17]).

Now let $\Pi = [0, L]$. Consider the stochastic equation

$$(\lambda - \Delta)u_t = \alpha(t)u + \sigma f(t)$$

with $\lambda \neq 0$ and boundary Dirichlet conditions. Highlight that σ is random variable, which is uniform distribution on $[-1, 1]$.

Let us find a numerical solution of this problem with parameters $\lambda = 2$, $\alpha(t) = 5t/2$. Set the initial data u_0 in form $u_0 = 0.3 \sin\left(\frac{\pi x}{L}\right) \cdot \exp(-10(x-0.5)^2)$, and the vector funtion $f(s)$ in form $f(s) = 5\exp(-20(x-0.5)^2) \cdot \sin\left(\frac{3\pi x}{L}\right)$ we have next result.

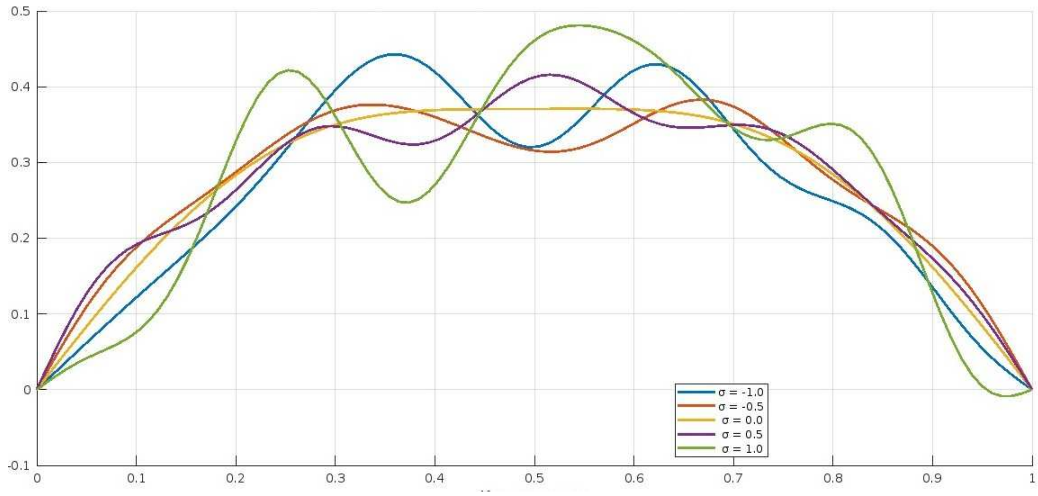

Fig. 1. The cross sections of the solution at the time $t = 0.95$

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РЕШЕНИЕ ОДНОЙ СТОХАСТИЧЕСКОЙ НЕАВТОНОМНОЙ ЛИНЕАРИЗОВАННОЙ МОДЕЛИ ХОФФА

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В статье исследуется разрешимость неавтономной линеаризованной модели Хоффа в пространстве стохастических **K**-процессов. Для этого сначала приводится результат о разрешимости такой модели в детерминированном случае. Далее приводится описание стохастических **K**-процессов и их некоторые особенности. После чего формулируется теорема о существовании решения в стохастическом случае и приводится результат, иллюстрирующий полученные утверждения.

Ключевые слова: уравнения соболевского типа; производная Нельсона – Гликлиха; пространство стохастических ***K****-процессов.*

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