ENGINEERING MATHEMATICS

SYNTHESIS OF A DYNAMIC CLUTCH WITH A LINEAR CHARACTERISTIC

E. A. Herreinstein[1], gerenshteinea@susu.ru,
M. G. Novik[1], novikmg@susu.ru
[1]South Ural State University, Chelyabinsk, Russian Federation

This paper examines the synthesis of mechanisms based on given characteristics. It introduces definitions of dynamic clutches and their characteristics and exemplifies the synthesis of mechanisms with the necessary characteristics for practical use in dynamic clutch modes. The first example demonstrates the synthesis of a mechanism based on a given function and the implementation process presenting a diagram of the mechanism, the condition for its transition to the dynamic clutch mode, and its stability region. The second example considers the synthesis of a mechanism with a periodic coefficient. The paper presents the conditions that ensure the maximum simplicity and clarity in the analysis of the vibrations of mechanisms in the dynamic clutch mode, as well as the conditions that ensure a positive torque and stability in this mode. Cartesian coordinates of all clutch parts at the initial moment of time are also obtained. Each example introduces a system of constraints, assumptions, and clarifications that allow for the recording of various aspects of the considered subject and for the practical implementation of mechanisms.

Keywords: inertial mechanism; system of material points; moment of inertia; motion of a rigid body.

Introduction

The design of mechanisms with desired characteristics to reduce or compensate for dynamic loads, vibrations, and impacts has always been a challenging task. Articles [1–3] discuss the use of isometric immersion methods for the synthesis of inertial mechanisms. This paper focuses on the synthesis of a dynamic clutch, which has wide practical applications [4–5]. Notably [1], the kinetic energy of a mechanism can be expressed as follows:

$$T = a\frac{\dot{\Theta}^2}{2} + b\dot{\Theta}\dot{\alpha} + c\frac{\dot{\alpha}^2}{2}, \tag{1}$$

wherein $a = a(\alpha) > 0, b = (\alpha), c = c(\alpha) > 0, ac - b^2 > 0$, where Θ is the rotation angle of the drive shaft, α is the difference in angles of the drive and driven shafts.

1. Basic Definitions

A dynamic clutch is a mechanism, the primary operating mode of which is dynamic clutch mode. The defining characteristic of a dynamic clutch is the following ratio (in the absence of elastic forces):

$$M = -a'(\alpha)\frac{\omega^2}{2}, \tag{2}$$

where $a(\alpha)$ is a function of α to be defined later, ω is the angular velocity of the drive shaft [1]. If the right-hand side is linear in the variable, the characteristic is called linear. Let us consider the problem of synthesizing such a mechanism.

2. Synthesis of a Dynamic Clutch with Constant Coefficients

Let $a' = -2(\lambda - \alpha)$, where $\lambda = \dfrac{L+r}{r}$, L is the length of an inextensible weightless flexible thread (ACB), r is the arc of a circle on the driven disk (AC) (Fig.1).
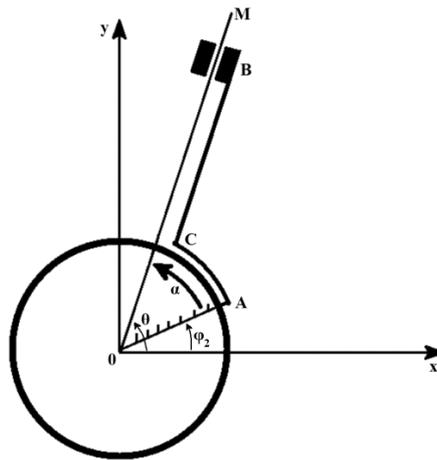

Fig. 1. The dynamic clutch scheme

We choose the integration constant and the coefficients b and c arbitrarily, since no other conditions are specified except (2). However, we do not disregard the positive definiteness of the metric form.

Let $a = (\lambda - \alpha)^2$, $b = 0$, $c = 1$. Notably:

$$1 - \frac{a'^2}{a} = 0. \tag{3}$$

Referring to [1], we obtain the immersion: $x = (\lambda - \alpha)\cos\Theta$, $y = (\lambda - \alpha)\sin\Theta$. At the same time, we assume that $\lambda - \alpha > 0$. In view of (3) we obtain that $z = 0$. If we take into account the fact that the shafts have a certain moment of inertia, assuming the moment of inertia of the drive shaft to be $p - q$ and the driven shaft to be q, we obtain the following metric form:

$$ds^2 = \left(p + (\lambda - \alpha)^2\right) d\Theta^2 - qd\Theta d\alpha + sd\alpha^2,$$

where $s = q + 1$.

The diagram of this mechanism is shown in Fig. 1. The driving link is the rod OM rotating in the plane of the figure around point O. The driven link is a circular disk. The load B of mass m moves along the rod. ACB is an inextensible weightless flexible thread of length L. A is the point of the thread attachment to the driven disk, C is a deflection

block attached to the rod. B is the point of the thread attachment to the load. AC is an arc of a circle with radius r on the driven disk where the thread is wrapped. The moment of inertia of the rod relative to O is equal to J_1, the moment of inertia of the driven disk is equal to J_2. The kinetic energy of the system is:

$$T = mr^2 \left(\left(p + (\lambda - |\alpha|)^2 \right) \frac{\dot{\Theta}^2}{2} - q\dot{\Theta}\dot{\alpha} + s\frac{\dot{\alpha}^2}{2} \right),$$

where

$$\lambda = \frac{L + r}{r}, \quad p = \frac{_1 +_2}{mr^2}, \quad q = \frac{_2}{mr^2}, \quad s = q + 1.$$

We see that the coefficients are defined in the strip:

$$|\alpha| \leq \frac{L}{r} = \lambda - 1.$$

Besides, the smoothness is disrupted at point $\alpha = 0$.

At low angular velocity, the load B is at point C, and the mechanism has two degrees of freedom. Upon reaching a certain angular velocity, the load moves away from point O along the rod, and the mechanism enters the dynamic clutch mode. Calculations show that if the following condition is met:

$$m\left(L + r\right)^2 > \frac{J_1 + J_2}{3}$$

(the moment of inertia of the load B relative to point O is large enough), the stability of the dynamic clutch mode is ensured in the region:

$$|\alpha| < \lambda - \sqrt{\frac{p}{3}} = \frac{L + r}{r} - \sqrt{\frac{J_1 + J_2}{3mr^2}}. \tag{4}$$

If condition (4) is not met, the load B will drop to point C, and the mechanism will lose one degree of freedom and can only have a circular motion. This means that instead of solving two equations, we only need to solve one, which reduces the computational complexity of the problem.

The aforementioned example provides one method for synthesizing a mechanism based on a given function:

$$a'(\alpha) = \mu(\alpha).$$

Let the segment OM rotate with the drive shaft perpendicular to its axis. A curve is given in polar coordinates on the disk of the driven shaft (coaxial with the drive shaft [?]):

$$\rho = \sqrt{a(\alpha)}, \quad \left(a(\alpha) = \int_0^\alpha \mu(\xi)\, d\xi \right),$$

where ρ is the polar radius. A point mass of 1 is located at the intersection of this segment and the curve. Its coordinates are:

$$x = \sqrt{a(\alpha)}\cos\Theta, \quad y = \sqrt{a(\alpha)}\sin\Theta.$$

Therefore, its kinetic energy is:

$$T = a\left(\alpha\right) \frac{\dot{\Theta}^2}{2} + \frac{\left(a'\left(\alpha\right)\right)^2}{4a\left(\alpha\right)} \cdot \frac{\dot{\alpha}^2}{2}.$$

If we take into account the inertia of the shafts, the kinetic energy from () will be as follows:

$$T = \left(a_0 + a\left(\alpha\right)\right) \frac{\dot{\Theta}^2}{2} + b_0 \dot{\Theta}\dot{\alpha} + \left(c_0 + \frac{\left(a'\left(\alpha\right)\right)^2}{4a\left(\alpha\right)}\right),$$

where a_0, b_0, c_0 are constants.

Obviously, $\left(a_0 + a\left(\alpha\right)\right)' = a'\left(\alpha\right) = \mu(\alpha)$. This means that a_0 does not affect the clutch dynamics due to its consistency. The synthesis of the mechanism is complete.

3. Synthesis of a Dynamic Clutch with a Periodic Coefficient

Another example will be illustrated by the synthesis of a mechanism with the periodic coefficient $a\left(\alpha\right)$. To eliminate transverse reactions on the axis of the mechanism, the curve $\rho = \rho\left(\alpha\right)$ should be symmetrical with respect to the origin of coordinates. Two loads of equal mass will move along this curve at points symmetrical with respect to the center. Therefore, let the coefficient $a\left(\alpha\right)$ have a period equal to π. The following condition should be met:

$$\left(\frac{1}{a\left(\alpha\right)}\right)' = k\sin(2\alpha), \; k > 0.$$

This ensures the maximum simplicity and clarity in analyzing the vibrations of the mechanism in the dynamic clutch mode. We have:

$$a\left(\alpha\right) = \frac{2}{a_0 - k\cos(2\alpha)},$$

where a_0 is a positive constant higher than the constant k. Next:

$$\rho^2\left(\alpha\right) = \frac{2}{a_0 - k\cos 2\alpha}.$$

We assume that $\xi = \rho\cos(\alpha)$, $\eta = \rho\sin(\alpha)$ in the Cartesian system of coordinates ξ, η rotating together with the driven disk in its plane. Therefore:

$$\left(\xi^2 + \eta^2\right)\left(a_0 - \frac{\xi^2 - \eta^2}{\xi^2 + \eta^2}k\right) = 2.$$

We make transformations and obtain $\left(a_0 - k\right)\xi^2 + \left(a_0 + k\right)\eta^2 = 2$. This is an ellipse with semi-axes: $\sqrt{\dfrac{2}{a_0 - k}}$ and $\sqrt{\dfrac{2}{a_0 + k}}$.

Positive torque transmission and stability in the dynamic clutch mode are ensured if the following conditions are met simultaneously:

$$\sin(2\alpha) > 0, \quad \cos(2\alpha) > 0.$$

Thus, we have obtained the Cartesian coordinates of all clutch parts at the initial moment of time. Since the relative position of the mechanism parts is maintained even with a constant difference in the shaft rotation angles, the synthesis is complete.

Conclusion

This paper presents two examples of synthesizing mechanisms with characteristics necessary for practical use in the dynamic clutch mode. Each example defines a system of constraints, assumptions, and clarifications that ensure the practical implementation of mechanisms with the necessary parameters.

References

1. Herrenstein A.V., Herrenstein E.A., Novik M.G. [Synthesis of Inertial rotation Mechanisms]. *Science of SUSU. Sections of Natural Sciences*, 2022, pp. 45–49. (in Russian)

2. Herrenstein A.V., Herrenstein E.A., Novik M.G. [The Connection Between Isometric Dives and the Problems of Synthesis of Inertial Mechanisms]. *Science of SUSU. Sections of Natural Sciences*, 2023, pp. 91–96. (in Russian)

3. Herrenstein A.V., Herrenstein E.A., Novik M.G. [A System of Material Points Equivalent to a Rigid Body]. *Science of SUSU. Sections of Natural Sciences*, 2024, pp. 124–127. (in Russian)

4. Koperchuk A.V., Murin A.V. Improving the Lockable Hydrodynamic Clutch for Mining Machine Drives. *Mining Informational and Analytical Bulletin (scientific and technical journal)*, 2012, no. 3, pp. 300–305. (in Russian)

5. Vlasenko D.A., Melezhyk R.S. Research of Dynamics of a Finger Coupling With an Elastic Disk-Type Element Under Shock Loads. *Vestnik Donetsk National Technical University*, 2022, no. 1(27), pp. 12–20. (in Russian)

Evgenia A. Herreinstein, PhD (Engineering), Associate Professor of the Department of Applied Mathematics and Software Development, South Ural State University (Chelyabinsk, Russian Federation), gerenshteinea@susu.ru

Mariya G. Novik, Senior Lecturer, Department of Computational Mechanics, South Ural State University (Chelyabinsk, Russian Federation), novikmg@susu.ru

СИНТЕЗ ДИНАМИЧЕСКОЙ МУФТЫ С ЛИНЕЙНОЙ ХАРАКТЕРИСТИКОЙ

Е. А. Гереншейн[1]**, М. Г. Новик**[1]
[1]Южно-Уральский государственный университет, г. Челябинск, Российская Федерация

Данная статья посвящена вопросам синтеза механизмов по заданным характеристикам. Вводятся определения динамической муфты и ее характеристики. Приведено

два примера синтеза механизмов, имеющих на режиме динамической муфты необходимые для практического использования характеристики. Первый пример демонстрирует синтез механизма по заданной функции и процесс его реализации. Приведены схема механизма, условие его перехода в режим динамической муфты и область его устойчивости. Во втором примере рассматривается синтез механизма с периодическим коэффициентом. Приведены условия, которые обеспечивают наибольшую простоту и наглядность при анализе колебаний механизма на режиме динамической муфты и условия, обеспечивающие положительный силовой момент и устойчивость на режиме динамической муфты. Получены декартовы координаты всех частей муфты в начальный момент времени. В каждом примере вводится система ограничений, допущений и уточнений, позволяющая зафиксировать многие аспекты рассматриваемой темы и реализовать механизмы на практике.

Ключевые слова: инерционный механизм; система материальных точек; момент инерции; движение твердого тела.

Литература

1. Геренштейн, А.В. Синтез инерционных механизмов вращения / А.В. Геренштейн, Е.А. Геренштейн, М.Г. Новик // Наука ЮУрГУ. Секции естественных наук. Материалы 75-й научной конференции, 2022. – С. 45–49.

2. Геренштейн, А.В. Связь изометрических погружений с задачами синтеза инерционных механизмов / А.В. Геренштейн, Е.А. Геренштейн, М.Г. Новик // Наука ЮУрГУ. Секция естественных наук. Материалы 75-й научной конференции, 2023. – С. 91–96.

3. Геренштейн, А.В. Система материальных точек, эквивалентная твердому телу / А.В. Геренштейн, Е.А. Геренштейн, М.Г. Новик // Наука ЮУрГУ. Секции естественных наук. Материалы 76-й научной конференции, 2024. – С. 124–127.

4. Коперчук, А.В. Совершенствование блокируемой гидродинамической муфты для приводов горных машин / А.В. Коперчук, А.В. Мурин // Горный информационно-аналитический бюллетень (научно-технический журнал). – 2012. – №3. – С. 300–305.

5. Власенко, Д.А. Исследование динамики муфты пальцевой с эластичным элементом дискового типа в условиях ударных нагрузок / Д.А. Власенко, Р.С. Мележик // Вестник Донецкого национального технического университета. – 2022. – №1(27). – С. 12–20.

Геренштейн Евгения Аркадьевна, кандидат технических наук, доцент кафедры прикладной математики и программирования, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), gerenshteinea@susu.ru

Новик Мария Геннадьевна, старший преподаватель кафедры вычислительной механики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), noviktmg@susu.ru