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SYSTEMS ANALYSIS OF THE RESOURCE PROVISION OF AN INDUSTRIAL ENTERPRISE BASED ON MATHEMATICAL MODELING

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In today's highly competitive and economically volatile environment, industrial enterprises are faced with the need to optimize production processes and improve resource efficiency. Mathematical modeling of resource provision allows for both analyzing the current enterprise status and predicting the outcomes of management decisions, making it a crucial tool for making informed strategic decisions. This approach is particularly relevant for industrial enterprises, where the accuracy of resource planning and provision directly impacts financial performance. The advancement of information technologies and the increasing availability of statistical data have facilitated the widespread use of mathematical modeling in to-day's economy. This is particularly relevant for industrial enterprises operating in highly turbulent conditions, largely due to US and EU sanctions pressure, as well as challenges with investment borrowing caused by the high key rate of the Central Bank of Russia. Among various modeling approaches, the use of production functions, regression models, general equilibrium models, as well as econometric and ecological models, is particularly significant. To conduct a systems analysis of industrial enterprises, the authors propose a mathematical model based on the Cobb-Douglas production function, which describes the dependence of output on production resources. To simulate enterprise operations, they developed a proprietary model-building algorithm implemented in the form of a computer program in Java and registered with the State Registry of Computer Programs. The adequacy of the resulting model was verified using the coefficient of determination, which demonstrated its high reliability.

Keywords: systems analysis; resource provision; mathematical modeling; computer software.

Introduction

Systems analysis has recently been used quite frequently to identify and solve problems in the operations of enterprises [1–14]. Systems research affects businesses of various types, including industry [1–3], electric engineering [4], telecommunications [5], information and communications [6, 7], IT [8], measurement [9], pharmaceuticals [10], transport [11, 12], small businesses [13], and raw materials companies [14]. The relevance of the systems approach is explained by the increasing complexity of modern economic systems and the need to take into account many inter-connected factors when making management decisions. Systems analysis allows for both identifying key elements and their interactions, and predicting the consequences of changes to one component for the entire system, which is critical in to-day's fast-paced market and technological innovations. The main sequential stages of systems analysis include: studying the system structure, analyzing its components, identifying the relationships between individual elements; collecting data on the system performance, studying information flows, observing and experimenting with the analyzed system; modeling; model adequacy testing, uncertainty and sensitivity analysis; studying resource capabilities; systems analysis targeting; criteria formation; alternative generation; choosing and decision making; implementing analysis results.

The selection of an appropriate mathematical model is the core of successful systems analysis. The accuracy of forecasts and the effectiveness of developed recommendations depend on model adequacy. Modeling allows abstracting from mi-nor details and focusing on essential relationships, which simplifies the understand-ing of complex processes and the search for optimal solutions. System modeling is based on selecting and constructing a mathematical model of the studied object. Consumer behavior models include models of consumer preferences and utility functions [15, 16] and models of consumer behavior itself [17, 18]. The main challenge in studying consumer behavior is determining the magnitude of consumer demand for purchased goods and services at given prices and income levels.

The process when a consumer decides to purchase a specific set of goods can be represented mathematically as choosing a specific point in the space of goods. If we denote n as the number of considered goods, and $x = (x_i, \ldots, x_n)^i$ as a column vector representing the quantities of goods purchased by the consumer over a certain period at given prices and income, the space of goods is then formed by a set of possible sets of x with non-negative coordinates:

$$C = x : x \geq 0.$$

Consumer choice theory [15, 16] suggests that each consumer has individual preferences at a specific subset of the product space $X \subset x : x \geq 0$. This means that for any pair $x \in X$, $y \in Y$, one of the following three relations may hold:

$x \succ y$ – set x is preferred over set y;

$x \prec y$ – set x is less preferred than set y;

$x \sim y$ – both sets have the same degree of preference for the consumer.

The properties of preference relations include:

– transitivity: if $x \succ y$ and $y \succ z$, then $x \succ z$;

– non-saturation: if $x > y$, then $x \succ y$ (the larger set is always preferred over the smaller one).

Each consumer's preferences can be represented using the utility function $u(x)$, such that $x \succ y$ implies $u(x) > u(y)$, and $x \sim y$ implies $u(x) = u(y)$. This function allows reducing multidimensional preference relations to simple numerical relations: "greater", "less", and "equal". In the consumer behavior model [17]–[18], the consumer aims to maximize utility subject to income constraints:

$$\max_{x \in \delta \cap X} u(x) = \max_{x \in M} u(x).$$

The problem of finding a conditional extremum can be reduced to finding the unconditional extremum of the Lagrange function:

$$L(x) = u(x) + \lambda(M - px).$$

Necessary conditions for a local extremum are as follows:

$$\sum_{j=1}^{n} p_j x_j^* = M,$$

$$\frac{\partial L}{\partial x_i} = \frac{\partial u}{\partial x_i}(x_i^*) - \lambda^* p_i = 0,\ i = 1, \ldots, n.$$

These conditions guarantee a maximum point, provided that the matrix of second derivatives U is negative definite.

Understanding consumer behavior is essential for developing effective marketing strategies and pricing policies. Utility and demand models both explain cur-rent consumer choices and predict their responses to changes in market conditions, which is an important element of systems analysis for market-oriented businesses.

Producer behavior models [19, 20] suggest that a firm produces several types of products with a fixed structure. X denotes the volume of one type of product or the total volume of diversified products.

Each of the three aggregated resource types (labor L, capital K, and materials M) has various combination options. The firm's technology is described by a production function reflecting the relationship between input resources and production output:

$$X = F(x),$$

where $x = (x_1, \ldots, x_n)$ is the column vector of possible costs of different types of resources.

It is assumed that the function $F(x)$ is twice continuously differentiable and continuous, and the matrix of its second derivative is negative definite.

If p is the unit price of output and w_j is the unit price of the resource of type j $(j = 1, \ldots, n)$, the profit $P(x)$ for each cost vector x is defined as:

$$P(x) = pF(x) - wx,$$

where $w = (w_1, w_2, \ldots, w_n)$ is the row vector of resource prices.

Assuming non-negative resource volumes involved in production, the profit maximization problem is formulated as follows:

$$\max_{(x \geq 0)} [pF(x) - wx].$$

This is a nonlinear programming problem with non-negativity constraints $x \geq 0$. The necessary conditions for its solution are defined by the Kuhn–Tucker conditions:

$$\frac{\partial P}{\partial x} = p\frac{\partial F}{\partial x} - w \leq 0,$$
$$\frac{\partial P}{\partial x}x = \left(p\frac{\partial F}{\partial x} - w\right)x = 0,$$
$$x \geq 0.$$

If all types of resources are used optimally, i.e. $x^* > 0$, the solution to the problem is:

$$p\frac{\partial F(x^*)}{\partial x} = w \quad \text{or} \quad p\frac{\partial F(x^*)}{\partial x_j} = w_j,\ j = 1, \ldots, n,$$

This means that at the optimal point, the marginal product of each resource is equal to its price. Producer behavior models form the basis for optimizing production processes and increasing the economic efficiency of an enterprise. They allow determining the optimal combination of resources that ensures maximum profit under given constraints, thereby forming a tool for strategic decision-making in production and pricing. In a more general duopoly scenario [19, 20], two competitors produce a similar product using their own production functions:

$$X_i = F_i(x^i), i = 1, 2.$$

The product price depends on the production output of both producers:

$$p = p(X_1, X_2),$$

and decreases as with an increase in production output:

$$\frac{\partial p}{\partial X_1} < 0, \quad \frac{\partial p}{\partial X_2} < 0.$$

Resource prices also depend on the purchase volumes x_j^1, x_j^2 by the first and second firms $w_j = w_j(x_j^1, x_j^2), j = 1, \ldots, n$, whereas prices increase as demand increases:

$$\frac{\partial w_j}{\partial x_j^1} > 0, \frac{\partial w_j}{\partial x_j^2} > 0.$$

Each firm strives to maximize its profits. For example, the first firm might act as follows:

$$\max_{(X_1, x_1^1, \ldots, x_n^1)} \left[p(X_1, X_2)X_1 - \sum_{j=1}^{n} w_1(x_j^1, x_j^2)x_j^2 \right]$$

given that $X_1 = F_1(x_1^1, \ldots, x_n^1)$.

The Lagrange function for solving this problem has the form:

$$L(X_1, X_2, \lambda) = p(X_1, X_2)X_1 - \sum_{j=1}^{n} w_1(x_j^1, x_j^2)x_j^2 + \lambda(F_1(x_1^1, \ldots, x_n^1) - X_1),$$

First-order conditions:

$$\begin{aligned} \frac{\partial L}{\partial X_1} &= p(X_1, X_2) + X_1 \frac{\partial p}{\partial X_1} + X_1 \frac{\partial p}{\partial X_2} \frac{\partial X_2}{\partial X_1} - \lambda = 0, \\ \frac{\partial L}{\partial x_j^{(1)}} &= -w_j(x_j^1, x_j^2) - x_j^1 \frac{\partial w_j}{\partial x_j^1} - x_j^1 \frac{\partial w_j}{\partial x_j^2} \frac{\partial x_j^2}{\partial x_j^1} + \lambda \frac{\partial F_1}{\partial x_j^1} = 0, \; j = 1, \ldots, n. \\ \frac{\partial L}{\partial \lambda} &= F_1(x_1^1, \ldots, x_n^1) - X_1 = 0. \end{aligned}$$

The elimination of λ results in a system of $(n+1)$ equations that determine the strategy of the first firm $X_1, x_1^1, \ldots, x_n^1$

$$\left[p(X_1, X_2) + X_1\frac{\partial p}{\partial X_1} + X_1\frac{\partial p}{\partial X_2}\frac{\partial X_2}{\partial X_1}\right]\frac{\partial F_1}{\partial x_j^1} = w_j + x_j^{(1)}\left(\frac{\partial w_j}{\partial x_j^1} + \frac{\partial w_j}{\partial x_j^2}\frac{\partial x_j^2}{\partial x_j^1}\right),$$

for $X_1, x_1^1, \ldots, x_n^1$, given that $X_1 = F_1(x_1^1, \ldots, x_n^1)$.

The solutions to these equations depend on $\frac{\partial X_2}{\partial X_1}$ and $\frac{\partial x_j^2}{\partial x_j^1}$, $j = 1, \ldots, n$. The last equation represents the response function of the second firm to the strategy of the first firm. Under different assumptions regarding this response, the duopoly model can provide various solutions to the competition problem.

Duopoly modeling is an important area in the study of market behavior, as it allows for the analysis of the strategic interactions between the limited number of large players. If firms understand these interactions, they can formulate their competitive strategies by anticipating the actions of their competitors and optimizing their pricing and production decisions.

Market prices, which shape the relationship between buyers and sellers, are governed by the principle that their dynamics depend on the imbalance between supply and demand. When supply exceeds demand, prices tend to fall. Conversely, when demand exceeds supply, prices rise.

The “spider web” model [21] can be used to analyze equilibrium pricing in the market for a specific product. In this model, the aggregate demand function $\Phi(p)$ decreases simultaneously with an increase in the aggregate supply function $\psi(p)$. Both of these functions are continuous and are defined for all positive price values ($p > 0$). They also have certain marginal properties:

As the price approaches zero ($p \to 0$), demand increases infinitely: $\lim\limits_{p\to 0}\Phi(p) = \infty$,

As the price increases infinitely ($p \to \infty$), demand approaches zero: $\lim\limits_{p\to\infty}\Phi(p) = 0$,

As the price approaches zero ($p \to 0$), supply also approaches zero: $\lim\limits_{p\to 0}\psi(p) = 0$,

As the price increases infinitely ($p \to \infty$), supply increases infinitely: $\lim\limits_{p\to\infty}\psi(p) = \infty$.

Market equilibrium occurs when the amount supplied is in balance with the amount demanded, that is, $\Phi(p) = \psi(p)$. Under the assumptions made, this equation has a unique solution p^E, such that $\Phi(p^E) = \psi(p^E) = x^E$.

The “spider web” model allows iteratively approaching the equilibrium price. Suppose that at the initial point in time, the price p_0 is set at a level where demand is lower than supply:

$$\Phi(p_0) < \psi(p_0),$$

then in the model, the price decreases to the p_1 level, at which demand equals supply at the initial price:

$$\Phi(p_1) = \psi(p_1),$$

Then the price rises to p_2, at which

$$\Phi(p_2) = \psi(p_2),$$

and so on. Thus, the process described by the recurrence relation $\Phi(p_i) = \psi(p_i), i = 1, 2, \ldots,$ converges.

The "spider web" model clearly illustrates the dynamic process of reaching market equilibrium. It demonstrates how, under the conditions of delayed producer response to price changes, a stable state is gradually reached, where supply and demand are balanced. This tool is useful for understanding pricing mechanisms in markets characterized by production inertia.

To predict equilibrium prices in the young Russian energy market, a recurrent neural network was developed and tested for seven federal districts based on the data from 2004 to 2017 [22]. The modeling results for the Southern and Siberian Federal Districts are presented in Figures 1 and 2. The forecast accuracy was high, with errors for all regions less than 2%. Highly accurate forecasting allows energy consumers to operate in a balanced market at relatively low tariffs.

The use of modern machine learning methods, such as recurrent neural net-works, significantly enhances forecasting abilities in the economy. These models can capture complex nonlinear relationships and time series, making them particularly effective for analyzing dynamic markets like the energy market. Accurate price forecasts allow market players to optimize their strategies and minimize risks.

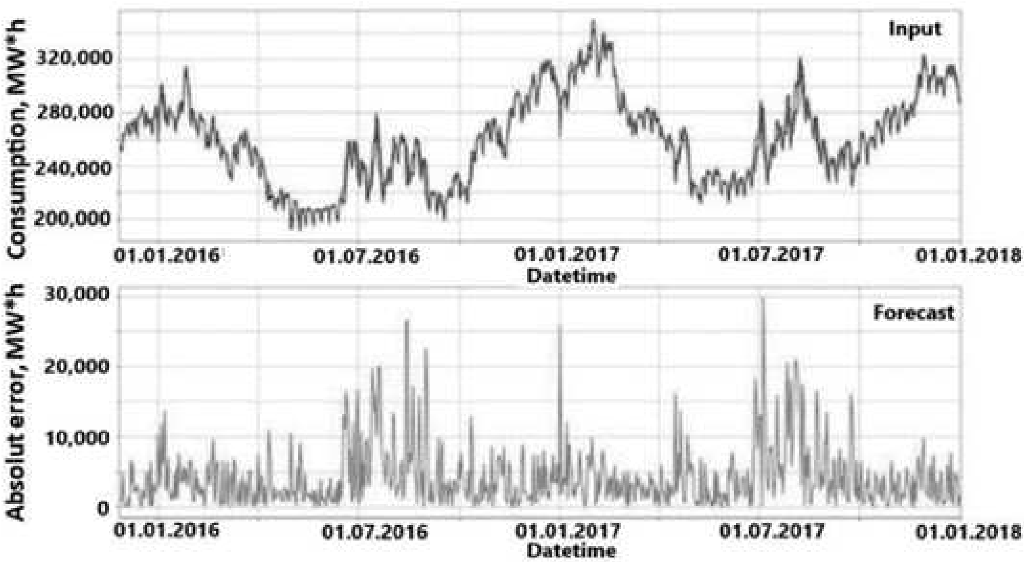

Fig. 1. The results of the run on the test dataset for the Southern Federal District

The Walrasian model [23, 24] is a theoretical framework for analyzing an economic system. It involves I producers (denoted by indices from 1 to I), m different products ($k = 1, \ldots, m$), and n types of raw materials or resources ($j = 1, \ldots, n$). The prices of these resources and products are represented as a row vector $p = (p_1, \ldots, p_n)$, while outputs are represented as a column vector $x = (x_1, \ldots, x_n)$.

Each consuming participant is characterized by a certain level of income $K(p)$ and individual preferences, which are mathematically expressed through the utility function $u(x)$. The set of goods available to the consumer at current prices p forms the set $X(p) =$

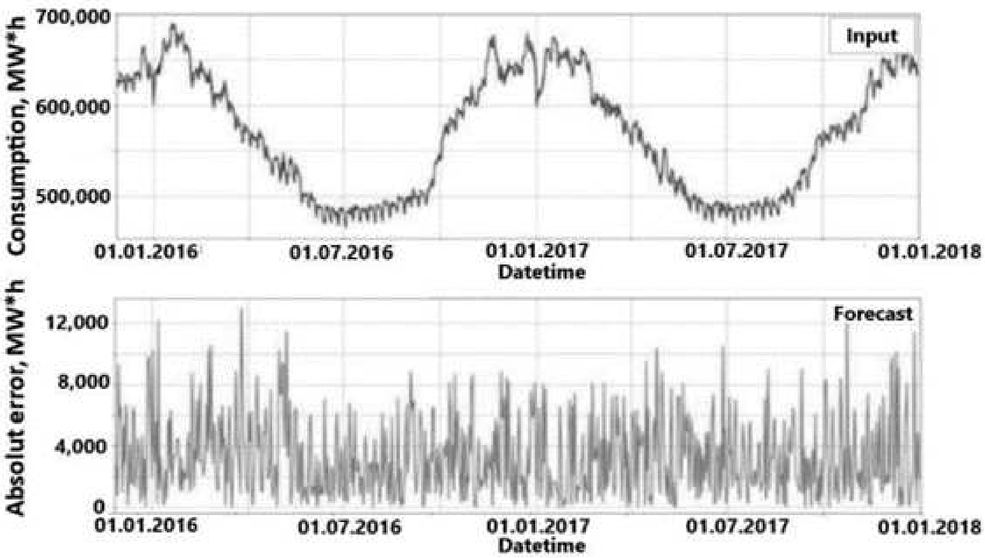

Fig. 2. The results of the run on the test dataset for the Siberian Federal District

$x^* : x \in X, px \leq K(p)$, where X is the domain of the function $u(x)$. The consumer demand function is defined as follows:

$$\Phi(p) = \begin{cases} x^*, u(x^*) = \max\limits_{x \in X(p)} u(p), \\ 0,\ u(x^*) \neq \max\limits_{x \in X(p)} u(p). \end{cases}$$

This function essentially selects the best goods from the available ones that maximize the consumer's utility at a given price level p. It is assumed that each consumer's income comes from two main sources: income from the sale of the initial stock of goods b_i expressed as pb_i and income from participation in production processes, $I_i p$. Thus, the total income of the i-th consumer is equal to $K_i(p) = pb_i + I_i p$.

Each producer (firm) operates within the framework of its unique technological capabilities. The column vector $y = (y_{k1}, \ldots, y_{kn})$ reflects both the costs and output of the k-th producer: positive components correspond to the outputs produced, negative components correspond to the costs incurred. Therefore, the product py_k represents the profit of a given firm. The technological potential of each firm is determined by the set of all possible output vectors Y_k, which is called the production possibility set.

Production activities are distributed by selecting output vectors y_k from the technological set Y_k for each producer from $k = 1$ to m. The overall production process can be summarized as $Y = \sum_{k=1}^{m} y_k$. Consumption is distributed by selecting a consumption vector $x_i \in X_i$ for each producer from $i = 1$ to l. The total aggregate demand vector $x = \sum_{i=1}^{l} x_i$ may include negative components if they represent the supply of resources (e.g., labor).

The joint distribution of production and consumption is described by a set of vectors:

$$(x_1, \ldots, x_i, \ldots, x_l, y_1, \ldots, y_k, \ldots, y_m), x_i \ni X_i, y_k \in Y_k$$

$$x = \sum_{i=1}^{l} x_i = b + \sum_{k=1}^{m} y_k = b + y.$$

The set $(x_1^*, \ldots, x_i^*, \ldots, x_l^*, y_1^*, \ldots, y_k^*, \ldots, y_m^*, p^*)$ defines the competitive equilibrium state in the Walrasian model subject to the following conditions:

$$x_i^* \in \Phi_i(p^*),\, i = 1, \ldots, l,\ y_k^* \ni \psi_k(p^*),\ k = 1, \ldots, m,$$
$$\sum_{k=1}^{m} y_k^* + b \geq \sum_{i=1}^{l} x_i^*,$$
$$p\left(\sum_{k=1}^{m} y_k^* + b\right) = p^* \sum_{i=1}^{l} x_i^*.$$

Here p^* represents the vector of equilibrium prices, and the last two expressions formulate the well-known Walras's Law.

The Walrasian model is the cornerstone of general equilibrium theory, offering a comprehensive insight into economic performance where all markets are interconnected and reach equilibrium simultaneously. Its application allows analyzing the complex interdependencies between production, consumption, and prices, which is crucial for understanding macroeconomic processes and developing policies aimed at economic stabilization.

To conduct a systematic analysis of the activities of industrial enterprises, the authors chose the mathematical model of the Cobb-Douglas production function. This model takes into account autonomous technical progress, which is considered neutral according to Hicks's theory.

1. Theory and Methodology

It is necessary to follow the rules. The choice of the Cobb-Douglas production function for modeling industrial enterprise performance is justified by its versatility and ability to linearize and test the statistical significance of parameters. This model allows both assessing the current status of an enterprise and developing resource optimization strategies. The versatility of the Cobb-Douglas function is demonstrated by its adaptability to various industries and types of production. Its mathematical flexibility allows for the easy interpretation of economic indicators, making it a powerful tool for studying the dependence of output on capital investment, labor, and other production factors. The modified Cobb-Douglas production function, which takes into account Hicks-neutral 'autonomous' technical progress, has the form of a multiplicative dependence of enterprise revenue on production resources (1)

$$CP = A \cdot N^{\alpha} \cdot FA^{\beta} \cdot CA^{\gamma} \cdot e^{\lambda t}, \tag{1}$$

where $e^{\lambda t}$ is a multiplier taking into account Hicks-neutral 'autonomous' technical progress (e is the base of the natural logarithm).

Production function (1) is an economic-mathematical production model that reflects the impact of resource provision on output. The parameters of the production function $A, \alpha, \beta, \gamma, \lambda$ are determined in their most general form based on retrospective data on customer profit CP, number of employees N, fixed assets FA, current assets CA, and the corresponding time t, as a solution to system of equations (2). In the system of equations (2) m, is the number of years over which retrospective data were collected ($m > 4$).

The use of the power production function method in analyzing production economics is often hampered by the lack of a solution to the system of equations (2). This is explained by the relationship between statistical data, which is not so much due to a functional relationship, but rather to the temporal proximity of the sets of exogenous variables, where all quantities change proportionally. This leads to the appearance of multicollinearity between independent variables, also known as the Mandershausen effect.

$$\begin{cases} \sum_{i=1}^{m} \ln CP_i = m \ln A + \alpha \sum_{i=1}^{m} \ln N_i + \beta \sum_{i=1}^{m} \ln FA_i + \gamma \sum_{i=1}^{m} \ln CA_i + \lambda \sum_{i=1}^{m} t_i, \\ \sum_{i=1}^{m} (\ln CP_i \cdot \ln N_i) = \ln A \sum_{i=1}^{m} \ln N_i + \alpha \sum_{i=1}^{m} (\ln N_i)^2 + \beta \sum_{i=1}^{m} (\ln FA_i \cdot \ln N_i) + \\ + \gamma \sum_{i=1}^{m} (\ln CA_i \cdot \ln N_i) + \lambda \sum_{i=1}^{m} (t_i \cdot \ln N_i), \\ \sum_{i=1}^{m} (\ln CP_i \cdot \ln FA_i) = \ln A \sum_{i=1}^{m} \ln FA_i + \alpha \sum_{i=1}^{m} (\ln FA_i \cdot \ln N_i) + \beta \sum_{i=1}^{m} (\ln FA_i)^2 + \\ + \gamma \sum_{i=1}^{m} (\ln FA_i \cdot \ln CA_i) + \lambda \sum_{i=1}^{m} (t_i \cdot \ln FA_i), \\ \sum_{i=1}^{m} (\ln CP_i \cdot \ln CA_i) = \ln A \sum_{i=1}^{m} \ln CA_i + \alpha \sum_{i=1}^{m} (\ln CA_i \cdot \ln N_i) + \\ + \beta \sum_{i=1}^{m} (\ln CA_i \cdot \ln FA_i) + \gamma \sum_{i=1}^{m} (\ln CA_i)^2 + \lambda \sum_{i=1}^{m} (t_i \cdot \ln CA_i), \\ \sum_{i=1}^{m} (\ln CP_i \cdot t_i) = \ln A \sum_{i=1}^{m} t_i + \alpha \sum_{i=1}^{m} (t_i \cdot \ln N_i) + \beta \sum_{i=1}^{m} (t_i \cdot \ln FA_i) + \\ + \gamma \sum_{i=1}^{m} (t_i \cdot \ln CA_i)^2 + \lambda \sum_{i=1}^{m} (t_i)^2. \end{cases} \tag{2}$$

The phenomenon of multicollinearity identified by Mendershausen poses a serious methodological challenge in estimating the parameters of econometric models. It leads to instability in coefficient estimates, increases their standard errors, and complicates the adequate interpretation of the influence of individual factors. Therefore, methods capable of mitigating this effect are needed to obtain reliable results.

The following transformations should be performed to address the problem of multicollinearity. We divide the total differential of function (1) by the function itself:

$$dCP/CP = \alpha \cdot dN/N + \beta \cdot dFA/FA + \gamma \cdot dCA/CA + \lambda \cdot dt. \tag{3}$$

We introduce the following notations:

$$\frac{dCP}{CP} = 2 \cdot \frac{CP_{i+1} - CP_i}{CP_{i+1} + CP_i} = z, \quad \frac{dN}{N} = 2 \cdot \frac{N_{i+1} - N_i}{N_{i+1} + N_i} = x, \quad dt = t_{i+1} - t_i = 1,$$
$$\frac{dFA}{FA} = 2 \cdot \frac{FA_{i+1} - FA_i}{FA_{i+1} + FA_i} = y, \quad \frac{dCA}{CA} = 2 \cdot \frac{CA_{i+1} - CA_i}{CA_{i+1} + CA_i} = w.$$

Then expression (3) is transformed into equation (4):

$$z = \alpha \cdot x + \beta \cdot y + \gamma \cdot w + \lambda. \tag{4}$$

Based on the transformed initial data from the system of equations (5), we determine the elasticity coefficients $\alpha, \beta, \gamma, \lambda$.

$$\begin{cases} \sum_{i=1}^{m} z_i = \lambda \cdot m + \alpha \sum_{i=1}^{m} x_i + \beta \sum_{i=1}^{m} y_i + \gamma \sum_{i=1}^{m} w_i, \\ \sum_{i=1}^{m} (x_i \cdot z_i) = \lambda \cdot \sum_{i=1}^{m} x_i + \alpha \sum_{i=1}^{m} (x_i)^2 + \beta \sum_{i=1}^{m} (x_i \cdot y_i) + \gamma \sum_{i=1}^{m} (x_i \cdot w_i), \\ \sum_{i=1}^{m} (y_i \cdot z_i) = \lambda \cdot \sum_{i=1}^{m} y_i + \alpha \sum_{i=1}^{m} (x_i \cdot y_i) + \beta \sum_{i=1}^{m} (y_i)^2 + \gamma \sum_{i=1}^{m} (y_i \cdot w_i), \\ \sum_{i=1}^{m} (w_i \cdot z_i) = \lambda \cdot \sum_{i=1}^{m} w_i + \alpha \sum_{i=1}^{m} (x_i \cdot w_i) + \beta \sum_{i=1}^{m} (y_i \cdot w_i) + \gamma \sum_{i=1}^{m} (w_i)^2. \end{cases} \tag{5}$$

The conjugation coefficient A (6) is determined based on the obtained numerical values of the elasticity coefficients:

$$A = \frac{\sum_{i=1}^{m} (z_i \cdot x_i^{\alpha} \cdot y_i^{\beta} \cdot w_i^{\gamma} \cdot e^{\lambda t})}{\sum_{i=1}^{m} (x_i^{\alpha} \cdot y_i^{\beta} \cdot w_i^{\gamma} \cdot e^{\lambda t})^2}. \tag{6}$$

The developed algorithm for mathematical modeling of enterprise resource provision was formalized as a computer program in Java and registered with the Federal Intellectual Property Service in the State Register of Computer Programs [25].

Automation of calculations and modeling using specialized software in Java significantly improves analysis efficiency and accuracy. The development of such tools enables the rapid processing of large volumes of data, scenario modeling, and the generation of reliable results to support decision-making, minimize human error, and improve research reproducibility.

The following criteria can be used to assess the reliability of the constructed mathematical model (1):

— Mean absolute percentage error, MAPE:

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|y_i - \hat{y}_i|}{y_i} \cdot 100\%, \tag{7}$$

where y_i is the actual value, $\hat{y}_i$ is the predicted value.

— The average elasticity coefficient indicates the average change in the y-factor when the x-factor deviates by 1% from its nominal value. It is calculated using formula (8):

$$\bar{L} = f'(\bar{x}) \cdot \frac{\bar{x}}{\bar{y}}. \tag{8}$$

— The correlation coefficient is a measure of the linear relationship between two random variables. The sample correlation coefficient r_{xy} ranges from -1 to 1. The closer the value of $|r_{xy}|$ to one, the closer the linear relationship and the better the linear relationship fits the observed data. If $|r_{xy}| = 1$, the relationship becomes functional, i.e., the ratio $\hat{y}_i = a + bx_i$ holds for all observations. If $r_{xy} > 0$, the relationship is direct; if $r_{xy} < 0$ it is inverse. This coefficient is calculated using formula (9):

$$r_{xy} = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{\sqrt{[n\sum x_i^2 - (\sum x_i)^2] \cdot [n\sum y_i^2 - (\sum y_i)^2]}}. \tag{9}$$

— The determination coefficient is the ratio of the explained part $D(\hat{y})$ of the variance of the variable y to the total variance $D(y)$. It is used to assess the quality (accuracy) of the constructed regression model. The higher this indicator, the better the model describes the initial data and takes values in the range from 0 to 1. This coefficient is calculated using formula (10):

$$R^2 = 1 - \frac{\sum e_i^2}{\sum (y_i - \bar{y})^2}. \tag{10}$$

— Fisher's F-test. The core of the method is to test the "null" hypothesis H_0 about the statistical insignificance of the regression equation (i.e., about the statistically insignificant difference of the F value from zero). This hypothesis is rejected if the condition $F_p > F_k$ is met, where F_k is determined from F-test tables with the number of degrees of freedom $k_1 = k$ (the number of independent variables in the regression equation), $k_1 = k$ and a given significance level α ($\alpha = 0.05$ is assumed for calculations). It is calculated using formula (11):

$$F_p = (n-2)\frac{R^2}{1 - R^2}. \tag{11}$$

The resulting production function (1), after validity verification, allows analytically evaluating the elasticity of output and production, material, labor, and capital intensity of production processes, as well as calculate the marginal rates of technological substitution of resources. The convex form of the production function (1) implies a decrease in the marginal products of all factors of production. This means that we can substitute between production factors without changing the output.

To determine the marginal products of production factors, the first derivative of the production function (1) should be calculated for each factor. The marginal product of the production factor reflects the change in output when a given factor increases by one additional unit. For example, the marginal product of labor for function (1) in its most general form is given by:

$$\frac{\partial CP}{\partial N} = A\alpha N^{\alpha-1} FA^{\beta} CA^{\gamma} e^{\lambda t} \tag{12}$$

Since $(\alpha - 1) < 0$, the marginal product of labor (like any other production factor) decreases as its input increases. Equation (12) can be reduced to form (13) by multiplying and dividing the right-hand side by N:

$$Э_N = \frac{\partial CP/\partial N}{CP/N} = \alpha. \tag{13}$$

Thus, α represents the labor elasticity of output, which shows the percentage change in marketable output with a 1% job growth. Similar transformations apply to other production resources. Notably, α characterizes the elasticity of output with respect to fixed assets, and γ – with respect to current assets. The highest elasticity coefficient determines the specific nature of the production process: whether it is labor-intensive, asset-intensive, or material-intensive.

The indicator $h = \alpha + \beta + \gamma$ is called the elasticity of production and it reflects the impact of scale on output. If $h = 1$, function (1) assumes a constant effect of scale in production. If $h > 1$, increasing effects predominate, implying increasing returns on inputs in this production process. If $h < 1$, decreasing effects of scale are observed, implying the output of resources (inputs) decreases.

A careful verification of model reliability and adequacy is a compulsory step in systems analysis. The use of various statistical criteria, such as the MAPE, the coefficient of determination, and the F-test, allows confirming the reliability of the obtained results and their applicability to real-world practice. This ensures a high level of confidence in the model and its ability to serve as a basis for informed management decisions.

The developed mathematical model (1) and resource forecasting can serve as a basis for obtaining various production scenarios. The forecast values of the arguments N, FA, CA are needed to extrapolate trend (1). They can be obtained through autonomous forecasting using a quadratic polynomial relationship:

$$y = a_0 + a_1 t + a_2 t^2, \tag{14}$$

where y is the N, FA, CA indicators sequentially forecasted independently of each other, and t is time normalized relative to the base year.

Initially, a trend for the N, FA, CA indicators is constructed based on historical data. The unknown parameters a_0, a_1, a_2 of function (14) are determined by solving the following system of linear algebraic equations:

$$\begin{cases} \sum_{i=1}^{m} y_i = m a_0 + a_1 \sum_{i=1}^{m} t_i + a_2 \sum_{i=1}^{m} t_i^2, \\ \sum_{i=1}^{m} (y_i t_i) = a_0 \sum_{i=1}^{m} t_i + a_1 \sum_{i=1}^{m} t_i^2 + a_2 \sum_{i=1}^{m} t_i^3, \\ \sum_{i=1}^{m} (y_i t_i) = a_0 \sum_{i=1}^{m} t_i^2 + a_1 \sum_{i=1}^{m} t_i^3 + a_2 \sum_{i=1}^{m} t_i^4. \end{cases}$$

After constructing models (14) and verifying their adequacy using the good-ness-of-fit criteria, forecast values for labor and material resources are calculated using the trend extrapolation method for the t argument. Their use in model (1) allows for output forecasting. Notably, according to the fundamental rule of forecasting, the depth of retrospection should be 2–2.5 times greater than the forecast horizon.

In addition to the direct problem of output forecasting, model (1) also allows for solving the inverse problem of calculating the required resource supply for a given output. This leads to the construction of an isoquant map demonstrating a variety of ways to achieve a given output level.

The ability of the model to solve both direct (output forecasting based on resources) and inverse (determining resources for a given output) problems is a significant advantage. Constructing isoquants, which illustrate different combinations of production factors to achieve the same output, provides valuable information for managers when planning and optimizing production capacity, especially in the face of changing resource availability or costs.

Table 1

Accounting data of Prokatmontazh JSC, Magnitogorsk

Year	Quarter	CP, ths. rub.	FA, ths. rub.	CA, ths. rub.	FOT, ths. rub.
2020	1	1,112,396	489,926	514,412	317,704
2020	2	1,405,132	489,926	603,875	344,180
2020	3	1,873,509	489,926	626,241	357,417
2020	4	1,463,679	489,926	492,046	304,467
2021	1	1,400,545	539,621	699,751	354,720
2021	2	1,534,930	539, 21	821,447	384,280
2021	3	2,067,472	539,621	851,871	399,060
2021	4	1,666,317	539,621	669,327	339,940
2022	1	1,950,587	824,981	738,325	418,670
2022	2	2,926,881	824,981	866,729	453,559
2022	3	3,359,345	824,981	898,831	471,004
2022	4	2,599,784	824,981	706,224	401,226
2023	1	1,709,072	1,223,173	779,424	468,472
2023	2	2,115,993	1,223,173	914,975	507,512
2023	3	2,278,762	1,223,173	948,863	527,031
2023	4	2,034,609	1,223,173	745,536	448,953
2024	1	2,332,026	1,330,054	1,181,134	568,394
2024	2	2,650,029	1,330,054	1,386,549	615,760
2024	3	3,074,034	1,330,054	1,437,902	639,443
2024	4	2,544,027	1,330,054	1,129,781	544,711

2. Practical Testing of the Systems Analysis Method

The developed systems analysis method for resource provision was tested at three industrial enterprises in the Chelyabinsk region. To test this methodology in macroeconomic instability conditions, the annual time interval was replaced by a quarterly one, and the inflation component was removed from the databases. Initial modeling data were obtained from the accounting statements of the enterprises.

Based on the data in Table 1, we constructed the production function for the operating activities of Prokatmontazh JSC, Magnitogorsk (15), using the described algorithm.

$$CP = 95.581 \cdot FOT^{6.49512} \cdot FA^{-2.65476} \cdot CA^{-2.88816} \cdot e^{0.11576 \cdot t}. \quad (15)$$

The validity of the constructed mathematical model was confirmed using the coefficient of determination of $R^2 = 0.89$. Consequently, in 89% of cases, changes in marketable output are associated with changes in labor resources. The significance of the coefficient of determination was verified using the F-test, the calculated value of which was $F_e = 24.2$. The tabular value of the F-test $F_{tab}(\alpha, \nu_1, \nu_2)$, where α is the significance level, $\nu_1 = m$ is the number of factors in the model, $\nu_2 = n - m - 1$ is the number of degrees of freedom, and, n is the number of observations, was $F_{tab}(0.05; 4; 19) = 2.90$. Since $F_e > F_tab$, the constructed mathematical model for Prokatmontazh JSC is adequate and reliable.

A mathematical model of the operating activities of Alias JSC (16) was constructed based on the retrospective data of this enterprise (Table 2).

Table 2

Accounting data for Alias JSC, Chelyabinsk

Year	Quarter	CP, ths. rub.	FA, ths. rub.	CA, ths. rub.	FOT, ths. rub.
2020	1	29,175	1,320	79,450	8,850
2020	2	28,137	1,320	82,740	9,031
2020	3	37,754	1,320	81,450	9,121
2020	4	58,241	1,320	80,545	9,304
2021	1	50,660	1,025	75,340	8,850
2021	2	42,679	1,025	73,420	9,031
2021	3	40,923	1,025	71,350	9,121
2021	4	48,994	1,025	69,934	10,751
2022	1	49,981	764	72,450	11,827
2022	2	127,736	764	88,430	12,068
2022	3	37,667	764	97,320	13,275
2022	4	48,362	764	107,720	14,602
2023	1	34,073	3,236	106,519	15,710
2023	2	59,528	3,236	110,312	16,537
2023	3	62,120	3,236	115,325	17,364
2023	4	66,733	3,236	117,995	18,232
2024	1	47,482	10,751	116,325	18,546
2024	2	41,503	10,751	118,340	18,924
2024	3	60,693	10,751	122,980	19,733
2024	4	97,016	10,751	124, 994	20, 20

$$CP = 26.658 \cdot FOT^{-1.32515} \cdot FA^{-0.15314} \cdot CA^{0.35774} \cdot e^{0.10885 \cdot t}. \quad (16)$$

The coefficient of determination for the constructed model is 0.76. Consequently, in 76% of cases, changes in marketable output are associated with changes in resource indicators, primarily current assets. The calculated F-test value was $F_e = 18.3$. Since it exceeds the tabular value of $F_{tab}(0.05; 4; 19) = 2.90$, the constructed mathematical model for Alias JSC is adequate and reliable.

A similar algorithm was used to construct a mathematical model of the operating activities of Artprofil LLC (17) based on the retrospective data of this enterprise (Table 3).

Table 3

Accounting data for Artprofil LLC, Chelyabinsk

Year	Quarter	CP, ths. rub.	FA, ths. rub.	CA, ths. rub.	FOT, ths. rub.
2020	1	11,985	2,996	1,533	375
2020	2	13,782	3,445	1,763	375
2020	3	16,179	4,044	2,070	375
2020	4	17,977	4,494	2,299	375
2021	1	12,172	3,408	1,974	475
2021	2	13,998	3,919	2,270	475
2021	3	16,431	4,601	2,666	475
2021	4	18,257	5,112	2,959	475
2022	1	17,069	5,120	2,873	575
2022	2	19,630	5,889	3,388	575
2022	3	23,043	6,913	3,882	575
2022	4	25,605	7,682	4,349	575
2023	1	15,136	4,843	2,298	675
2023	2	17,406	5,570	2,643	675
2023	3	20,434	6,539	3,009	675
2023	4	22,705	7,266	3,207	675
2024	1	17,670	6,018	4,367	775
2024	2	20,321	6,909	4,745	775
2024	3	23,855	8,111	5,241	775
2024	4	26,494	9,538	4,949	875

$$P = 38.667 \cdot FOT^{-0.40307} \cdot FA^{1.02821} \cdot CA^{-0.01515} \cdot e^{-0.00126 \cdot t}. \tag{17}$$

The coefficient of determination for the constructed model is 0.977. Consequently, in 97.7% of cases, changes in marketable output are associated with changes in resource indicators, primarily fixed assets. The calculated F-test value was $F_e = 283.19$. Since it exceeds significantly the tabular value $F_{tab}(0.05; 4; 19) = 2.90$, the constructed mathematical model of Artprofil LLC is adequate and reliable.

Our practical tests at the selected enterprises confirm the applicability and significance of the developed methodology for constructing mathematical models of operating activities carried out by industrial enterprises as a basis for systems analysis.

Conclusions

In an idealized theoretical model of the production function of an enterprise, production elasticity indicators fall within the range of [0, 1] and their sum equals one, as it reflects the entire set of resources used. However, in practice, these assumptions are not met.

The systems analysis of the resource provision of an industrial enterprise led to a new scientific result: the sum of production elasticity indicators is not always equal to one (17). This indicates underutilization of available resources within an enterprise. The fact that some elasticity indicators may have negative values (15–17) is of particular interest. Mathematically, a negative exponent indicates that an increase in the base of the exponent results in a decrease in its value, since $x^{-a} = (1/x)^a$. Economically, a negative production elasticity indicator leads to a decrease in the production performance indicator with the additional involvement of a given resource. According to the authors' research, negative elasticity indicators of production resources are not isolated incidents but rather relatively common among small businesses in Russia.

The identified characteristics of production elasticity, including their negative values, highlight the complexity of the relationship between resources and output in the real economy. The obtained results both enrich our theoretical understanding of production functions, and provide valuable practical advice for enterprises facing inefficient resource use and the need to revise their production strategies.

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СИСТЕМНЫЙ АНАЛИЗ РЕСУРСНОГО ОБЕСПЕЧЕНИЯ ПРОМЫШЛЕННОГО ПРЕДПРИЯТИЯ НА ОСНОВЕ МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ

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В условиях современной рыночной конкуренции и экономической нестабильности, промышленные предприятия сталкиваются с необходимостью оптимизации производственных процессов и повышения эффективности использования имеющихся ресурсов.

Математическое моделирование ресурсного обеспечения является незаменимым инструментом для анализа текущего состояния предприятия и прогнозирования результатов управленческих решений, что способствует принятию обоснованных стратегических решений. Данный подход особенно важен для промышленных предприятий, где точность планирования и распределения ресурсов напрямую влияет на их финансовые показатели. Расширение возможностей информационных технологий и увеличение доступности статистических данных стимулируют широкое применение математического моделирования в современной экономике. Это особенно актуально для промышленных предприятий, функционирующих в условиях высокой неопределенности, обусловленной, в частности, санкционным давлением со стороны США и стран Евросоюза, а также трудностями с привлечением инвестиций из-за высокой ключевой ставки Центрального банка России. В числе различных подходов к моделированию особое место занимают производственные функции, регрессионные модели, модели общего равновесия, а также эконометрические и экологические модели. Для системного анализа деятельности промышленных предприятий авторы предлагают математическую модель, базирующуюся на производственной функции Кобба-Дугласа, которая отражает зависимость объема выпуска продукции от используемых ресурсов. Разработан оригинальный алгоритм построения модели, который был реализован в виде компьютерной программы на языке Java и зарегистрирован в государственном Реестре программ для ЭВМ. Достоверность построенной модели подтверждена высоким значением коэффициента детерминации.

Ключевые слова: системный анализ; ресурсное обеспечение; математическое моделирование; компьютерное обеспечение.

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