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MATHEMATICAL METHODS OF MODELING OF IMAGE PROCESSING AND ANALYSIS IN THE MODIFIED DESCRIPTIVE ALGEBRAS OF IMAGES

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The article is devoted to the development of the method of mathematical modeling of image processing and analysis in an apparatus of the modified descriptive algebras of images. A brief chronology of the formation of an algebraic approach in the studies of foreign and domestic scientists is given in materials. The present state of an mathematical apparatus of the modified descriptive algebras of images is described. The results of the latest researches are presented as a technique of the organization of a computing experiment. The combinatorial evaluation of the initial images, which are randomly selected among all the possible variants, is carried at the first step of the computing experiment . At the second step the initial images are used in the development of an algorithm and a mathematical method of their processing and analysis modeling. The second step is to develop an objective function to optimize and select the variable parameters of a mathematical model. At the third step a genetic algorithm should be applied to find the optimal values of variable parameters of the mathematical model.

Keywords: image processing and analysis; modified descriptive algebra of images; mathematical modeling method; combinatorial evaluation sampling; objective function; genetic algorithm; technique of computing experiment.

Introduction

The algebraic approach to the images treatment, analysis and recognition is promising approach. It is developed in the Computational Center of A. A. Dorodnicyn of the Russian Academy of Sciences. A number of studies was carried out in 70-ies of the XIX century. They laid the foundation for the modern works in this area. Thus mathematical morphology was developed by G. Materonom and Zh. Serra [1, 2]. The images theory was created by W. Grenander [3–5]. Academician Yu. I. Zhuravlev developed and studied the algebra of algorithms [6, 7]. Works on the use of category theory in the field of images recognition were carried out by M. Pavel [8]. The image algebras were developed and explored by Zh. Serra and S. Sternberg in the 80s [9]. The standard image algebra was proposed by G. Ritter in 90-ies [10]. The descriptive images algebras (DIA) were developed and researched by I.B. Gurevich at the end of 90-ies [11]. They were based on academic Yu. I. Zhuravlev research. The descriptive image algebra were specialized to the case of a single ring (DIA1R) by V. V. Yashina [12]. Today the works in this field are carried out by I. B. Gurevich and V. V. Yashina. The authors consider the descriptive image algebras to the case of universal algebras with matrix entries. The research results were presented as the individual mathematical apparatus, which is called modified descriptive image algebras (MDIA) [13, 14].

1. Modified Descriptive Image Algebras

MDAI mathematical apparatus conditionally consists of the following 3 parts. The initial data are presented by *rectangular matrices of finite order* (images). The mathematical techniques of processing modeling (image conversion) and data analysis (measurement of features of the observed object in images) are *the main operations of universal algebras.* Thus defined *universal algebras with matrix entries* formally describe the space for image processing and analysis. This apparatus is supplemented by a numerical *multiparametric optimization method* (genetic algorithm) to find the optimal values of the *objective function* (mathematical model of image processing and analysis processes). The subject of research is the formal description of any image processing and their subsequent analysis.

We propose a natural form of image presentation – rectangular matrix of finite order. Note that binary (two-color), in shades of gray (grayscale) and color (full color) images are usually used in the field of digital image processing. Therefore these matrices will be set on the following classical sets $S_{bin} = \{0, 1\}$ and $S_{gray} = \{0, \ldots, 255\}$, as well as on the ternary product $S_{color} = (S_{gray})^3$, respectively. Further they are considered as elements of universal algebras

$$\mathcal{A}_{bin} =< M_{bin}, F_{bin} >, \quad \mathcal{A}_{gray} =< M_{gray}, F_{gray} > \quad \text{and } \mathcal{A}_{color} =< M_{color}, F_{color} >$$

for binary, grayscale and color images, respectively. Methods for image conversion from digital image processing field will be selected as the main operations for sets F_{bin}, F_{gray} and F_{color}. In addition to these specific image conversion, other operations of rectangular finite order matrices, if they are closed on these sets, may also be main operations. The only purpose of this approach to the selection of operations is to connect the elements of universal algebra among themselves by a large number of main algebra operations [13, 14].

All image conversions in DIA were divided into 3 classes: procedural, parametric and generating ones [11, 12]. There are only first two classes in MDIA, because of specificity of the developed image models. The following methods were considered as a procedural conversion variants in the research: methods of image converting, methods of the improving of visual characteristics, methods of image preparation, filtering methods, and methods of morphological operations on the images [13]. These conversions are *parametric unary operations on matrices of universal algebras*

$$\mathcal{A}_{bin}, \quad \mathcal{A}_{gray} \quad \text{and} \quad \mathcal{A}_{color}.$$

The important characteristics of the considered algebras are cardinalities of the subsets

$$N_{bin} \subset M_{bin}, \quad N_{gray} \subset M_{gray} \quad \text{and} \quad N_{color} \subset M_{color},$$

which are called sections of MDIA funnel [14]. The subsets are interconnected by image converting transformations, which, in turn, are not closed and, therefore, are not the main operations of the algebras [13].

Combining the above mentioned algebras into a single mathematical object with converting operations, one can get *a space to convert the images* (image condition space, the ICS). Formally, the ICS is written as

$$GS_I = \left\langle \{\mathcal{A}_{bin}, \mathcal{A}_{gray}, \mathcal{A}_{color}\}, \{O_T^{color \to gray}, O_T^{gray \to bin}\} \right\rangle.$$

The results of research of universal algebras with the matrix entries are *definitions, propositions, theorems, lemmas, statements and corollaries about semigroups, monoids and abelian groups of universal algebras with rectangular matrices of finite order*. Three definitions and three corollaries were formulated, and 15 statements were proved for MDIA on the set M_{bin} and with the main binary operations (Boolean operations on matrices). One statement was proved, and one corollary was formulated for MDIA on the set M_{gray} and with the unary parametric main operations (morphological operations on matrices). Three definitions and one corollary were formulated, and 3 statements were proved for MDIA on the set M_{color} and witn the binary main operations (Boolean operations on matrices with vector elements). One statement was proved, and one definition was formulated for MDIA on the set M_{gray} and with the main operations on matrices (filtering operation on matrices). One statement was proved, and one definition was formulated for ICS [13].

Methods to calculate quantitative characteristics of the matrices of the considered universal algebras (parametric conversions), in contrast to the procedural conversions, are not their main operations, but are used in the field of digital image processing for matrix measurement (image analysis). In researches they were presented by classes of methods to calculate deterministic, probabilistic and structural linguistic features of the matrices of universal algebras (objects of observation in images). [13].

The superposition of the main operations of universal algebras with the matrix entries describes the mathematical methods of image processing modelling in ICS and is called in MDIA (DIA and DIA1R) asT-DAICS $\mathcal{R}_T$, descriptive algebraic image conversion scheme. *A composition of parametric transformations* formally describes the process of measuring of quantitative characteristics of the considered matrices and is called in MDIA (DIA and DIA1R) as P-DAICS $\mathcal{R}_P$. A consistent application of mathematical methods $\mathcal{R}_T$ and $\mathcal{R}_P$ models the image processing with its subsequent analysis. Such combined functions are usually performed by systems of computer and technical vision. Thus, mathematical methods of processing modeling with the subsequent image analysis formally describe the measurement functions of systems of technical and computer vision [13].

2. Method of Computing Experiment

Mathematical modeling of image processing and analysis in MDAI involves the following steps such that the listed actions are carried out in strict compliance:

STEP 1. *Form the initial data sample.*

1. Define a size (width m and height n in pixels) of the initial images $I_{color} = \| < r_{ij}, g_{ij}, b_{ij} > \|$, $< r_{ij}, g_{ij}, b_{ij} > \in S_{color}$, computational accuracy ε, image resolution $dpcm$.

2. Calculate a number of pixels δ for accuracy ε in accordance with (1):

$$\delta = \left\lceil \frac{\varepsilon}{dpcm} \right\rceil \tag{1}$$

3. Calculate a sectional size of a funnel K_{bin} in MDIA $\mathcal{A}_{bin} =< M_{bin}, F_{bin} >$ of binary images $I_{bin} = \|z_{ij}\|, z_{ij} \in S_{bin}$ [13, 14]. Calculate p by formula (2). Substitute the values of δ and p in (3) and calculate coordinates of a vector $\overrightarrow{k_{bin}}$. The sum of

coordinates of $\overrightarrow{k_{bin}}$, according to (4), is an evaluation of K_{bin}.

$$p = \sum_{i=1}^{n}\sum_{j=1}^{m} z_{ij}, \quad I_{bin} = \|z_{ij}\|, z_{ij} \in S_{bin}, \tag{2}$$

$$\overrightarrow{k_{bin}} = (k_{p-\delta}^{bin}, \ldots, k_{p+\delta}^{bin}) = (C_{nm}^{p-\delta}, \ldots, C_{nm}^{p+\delta}), \tag{3}$$

$$K_{bin} = \sum_{l=p-\delta}^{p+\delta} k_l^{bin} = \sum_{l=p-\delta}^{p+\delta} C_{nm}^{l}. \tag{4}$$

4. Calculate a sectional size of a funnel K_{gray} in MDIA $\mathcal{A}_{gray} =< M_{gray}, F_{gray} >$ of the halftone images $I_{gray} = \|y_{ij}\|, y_{ij} \in S_{gray}$ for each of image variants $I_{bin} = \|z_{ij}\|$ with number of pixels $l = \overline{p-\delta, p+\delta}$, which is allowable by accuracy ε [13, 14]. Let $\|z_{ij}\| = O_T^{gray\to bin}(\|y_{ij}\|, \theta)$, where θ is a converting threshold. Substitute the values of δ, p and θ in (5) and calculate coordinates of a vector $\overrightarrow{\theta_{gray}}$. A value K_{gray} is calculated as a scalar product of vectors $\overrightarrow{k_{bin}}$ and $\overrightarrow{\theta_{gray}}$, according to (6).

$$\overrightarrow{\theta_{gray}} = (\theta_{p-\delta}^{gray}, \ldots, \theta_{p+\delta}^{gray}) = (\theta^{nm-(p-\delta)}(256-\theta)^{p-\delta}, \ldots, \theta^{nm-(p+\delta)}(256-\theta)^{p+\delta}), \tag{5}$$

$$K_{gray} = \overrightarrow{k_{bin}}\overrightarrow{\theta_{gray}} = \sum_{l=p-\delta}^{p+\delta} k_l^{bin}\theta_l^{bin} = \sum_{l=p-\delta}^{p+\delta} C_{nm}^{l}\theta^{nm-l}(256-\theta)^{l}. \tag{6}$$

5. Calculate a sectional size of a funnel K_{color} in MDIA $\mathcal{A}_{color} =< M_{color}, F_{color} >$ of the color images $I_{color} = \| < r_{ij}, g_{ij}, b_{ij} > \|, < r_{ij}, g_{ij}, b_{ij} > \in S_{color}$, according to (7) [13, 14].

$$K_{color} = (K_{gray})^3. \tag{7}$$

6. Calculate the geometrical probability that the selected images are in the funnel section in MDIA of color images in accordance with (8).

$$P^* = P\left(I_{color} \in N_{color}\right) = \frac{K_{color}}{256^{3nm}}. \tag{8}$$

7. Form a set of initial images $\widetilde{I} = \{I_{color}\}$ with probability P^* and with any distribution law of random numbers.

STEP 2. *Develop a mathematical model and image processing and analysis algorithm.*

1. Develop an algorithm of the three-tier architecture lower level [13, 14].
2. Develop an algorithm of the three-tier architecture middle level [13, 14].
3. Develop an algorithm of the three-tier architecture upper level [13, 14].
4. Develop an algorithm of image processing and analysis, based on the algorithms of the lower, middle and upper levels.
5. Develop a mathematical model of image processing and analysis $\widetilde{I} = \{I_{color}\}$ in MDIA [13, 14].

6. Choose optimized parameters of the mathematical model and formulate the objective function [14, 15].

STEP 3. *Optimize the objective function and clarify the mathematical model.*

1. Calculate optimum values of the parameters of the mathematical model [14, 15].
2. Form a test set of the images $\widetilde{I} = \{I_{color}\}$ for analysis in accordance with step 7 of STEP 1.
3. Evaluate a quality of the mathematical model.

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МАТЕМАТИЧЕСКИЕ МЕТОДЫ МОДЕЛИРОВАНИЯ ОБРАБОТКИ И АНАЛИЗА ИЗОБРАЖЕНИЙ В МОДИФИЦИРОВАННЫХ ДЕСКРИПТИВНЫХ АЛГЕБРАХ ИЗОБРАЖЕНИЙ

А.Р. Исхаков

Статья посвящена разработке метода математического моделирования обработки и анализа изображений в аппарате модифицированных дескриптивных алгебр изображений. В материалах приведена краткая хронология становления алгебраического подхода в исследованиях зарубежных и отечественных ученых. Описано современное состояние математического аппарата модифицированных дескриптивных алгебр изображений. Результаты последних исследований представлены в виде методики организации вычислительного эксперимента. На первом этапе вычислительного эксперимента проводится комбинаторная оценка начальных изображений, выбранных случайно из числа всех возможных вариантов. Начальные изображения на втором этапе используются в разработке алгоритма и математического метода моделирования их обработки и анализа. На втором этапе необходимо разработать целевую функцию для оптимизации и выбрать варьируемые параметры математической модели. На третьем этапе для поиска оптимальных значений варьируемых параметров математической модели должен быть применен генетический алгоритм.

Ключевые слова: обработка и анализ изображений, модифицированная дескриптивная алгебра изображений, метод математического моделирования, комбинаторная оценка выборки, целевая функция, генетический алгоритм, методика вычислительного эксперимента.

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