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AN EFFECT OF THE SLOPE LAYER IN SHEET SAMPLE ON A STRENGTH FOR NEGATIVE COEFFICIENT OF A BIAXIAL LOADING FACTOR OF A SAMPLE

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We investigate a critical state of the slope layer of less durable material in sheet sample by the loadings having opposite signs. It is shown that if there is no contact hardening of layer material, then a destruction occurs on the layer, independent of its slope angle. Using numerical experiments, we investigate a dependence of a sample strength from the slope angle of the layer for different relations between the external loads. We show that if there is a contact hardening of the layer, then a sample, containing less stable layer, may be equal in strength to a homogeneous sample for certain angles of layer slope.

Keywords: slope plastic layer, plastic instability, strain-deformation state, Swift criterion.

Introduction

A loss of a stability of the process of plastic deformation of the construction material is a criterion that sheet constructions and thin-walled shells reach a predestruction state. An efficient method to calculate the critical deformations and strains at the time, when a stability is loss under a biaxial load, is proposed in [1]. An approach of paper [1] is developed and defined more exactly in [2-7]. An explicit analytic dependence of the critical deformations, strains and pressure in sheet constructions and homogeneous thin-walled cylindrical shells are obtained on this basis. Welded envelope and sheet constructions may contain regions of heterogeneity. As a rule, they are layers and interlayers, which are made from a weaker material, i.e. welding seams, fusion zone and the HAZ. The study of a critical state of such combinations is based on the following two theories. The first one is the theory of loss of stability of process of a layer material deformation [3, 5-11]. The second one is the theory of the contact hardening of layer material [5-7, 12-19]. The approaches used in [5-7, 12-19] allow to find a dependence of normal and tangent strains σ_y and τ_{xy} from a coefficient of connection mechanical heterogeneity $K = k^+/k^-$ in neighborhood of contact boundary and, finally, a coefficient of layer contact hardening. Here k^+, k^- are parameters of plasticity of a base material and a layer material, describing a moment of plastic stability loss.

Practical interest is represented by slope layers, i.e. layers, which are arranged at an angle to the directions of the outer mutually orthogonal loads that generate strains σ_1 and σ_2. Welding seams, fusion zone and HAZ of factory seams of spiral-welded tubes are important examples of such layers. In [5, Ch. 4; 6, 7, Ch. 8; 11.08] it is shown that a computing scheme of case, when a layer is orthogonal to one of the external loads, can be used for a slope layer. In this case, more complex parameter K_{incl} (see formula (5) below) is used in this scheme instead of a coefficient K. Note that a parameter K_{incl} depends on mechanical and geometric parameters of connection and on load conditions. The following parameters are given by a problem condition:

1. A coefficient K of mechanical heterogeneity of a connection.
2. An angle of layer slope ν (an angle between a layer direction and a load action direction σ_1, see Fig.1).
3. A relative thickness of a layer χ, i.e. a ratio of its height (thickness) to its width (a thickness of sheet or shell wall).
4. A coefficient of biaxiality of load $m = \sigma_1/\sigma_2$. The following parameters are found

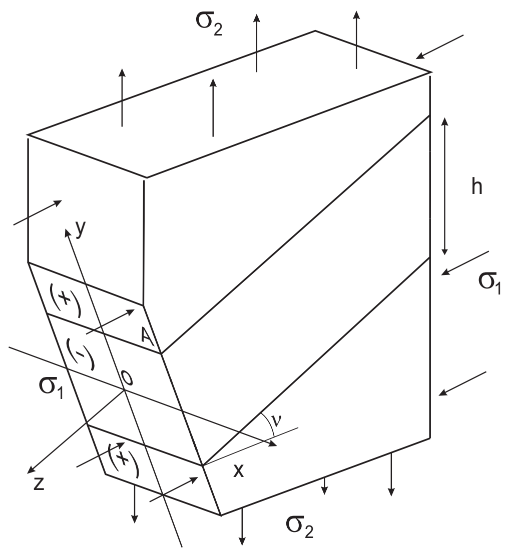

Fig. 1. Sheet sample in a slope layer.

based on data of conditions. They are a coefficient of contact hardening of layer material g , $g \geq 1$, which depends on χ, K_{incl} and ν, and condition coefficient of mechanical heterogeneity K_{incl}, which also depends on K and ν. A search for the conditions under which a less strong area does not reduce a strength of a compound is interesting. Such conditions in the form of relations between the parameters and loading conditions. Consider such conditions as relations between parameters and load conditions.

Let

$$B = cos^2(\nu) + msin^2(\nu); \quad C = (1-m)sin(2\nu). \tag{1}$$

Denote τ_{yz} as τ. In [7, p. 231-232] it is noted that

$$\tau = 0.5C\sigma_2 \ ; \ \sigma_{yav} = B\sigma_2. \tag{2}$$

Here $\sigma_{y\ av} = \int_0^1 \sigma_y(x,\chi)dx$ is an average value of strains σ_y on a contact surface. A formula for coefficient g of contact hardening is also given there:

$$\sigma_{y\ av} = 2g\sqrt{(k^-)^2 - \tau^2}; \quad \tau \neq k^-. \tag{3}$$

For given load conditions and an angle τ of strain layer slope, the right-hand side of equation (3) is the same for all points of a layer. In this case, an analogue of coefficient K, that is condition coefficient of mechanical heterogeneity can be calculated by the formula [6, 7, p. 233, 8-11]:

$$K_{incl} = \sqrt{\frac{(k^+)^2 - \tau^2}{(k^-)^2 - \tau^2}} = K\sqrt{1 + \frac{K^2-1}{K^2}\frac{g^2C^2}{B^2}}; \quad \tau \neq k^-. \tag{4}$$

A limit $m > 0$ was imposed in [5, Ch. 4; 6, 7, Ch. 8; 8-11]. For example, if there are no axial loads on a thin-walled cylindrical shell, which is exposed to the internal pressure, then $m = 0,5$. In any case, if $m > 0$, then $B \neq 0$. If $m \leq 0$, then the computational scheme of these works may be unusable. In particular, (1) shows that for $m = -cos^2\nu/sin^2\nu$ a coefficient $K_{incl} = \infty$, while a computational scheme g in these works is carried out for $K < 1,5$. Moreover, papers [6, 7, pp. 79-83; 16-18] shows, that for $K \geq K_{cr}$, $K_{cr} \approx 1,98$, a connection works otherwise. Namely, more strong base material does not anywhere go into a state of plastic flow during the process of load. Note that full contact hardening is realised in a less stable layer, when tangent strains τ_{xy} near a free surface reach their maximum theoretically possible value. At this case, they are $\tau_{xy} = \sqrt{K^2 - \tau^2}$. A scheme to calculate the critical values of strains and deformations for $K_{incl} > 1,5$ is constructed in [20]. Purpose of the paper is to investigate the dependence of a strength of the sheet sample, having a less stable slope layer, from an angle ν of layer slope, load conditions (coefficient m for $m < 0$) and a coefficient K of mechanical heterogeneity.

1. Conditions of Critical State of Layer and Uniform Strength

Consider conditions under which a less stable layer does not reduce a strength of the connection. This occurs if a base material reaches a critical state at the same time with the layer. A critical state is reached in the layer earlier than in a base material, if and only if critical values of strains σ_1 and σ_2 satisfy at least one of the following limitations:

$$| \sigma_1 | < 2k^+; \quad | \sigma_2 | < 2k^+ \tag{5}$$

Suppose $B < 0$, then $m < ctg^2\nu$ by (2); $\sigma_{y\ av} = B\sigma_2$. By definition, $\sigma_2 = m\sigma_1$, and by condition, $m < 0$. Therefore, $\sigma_{y\ av}$ has the same sign as σ_1. Consequently, if there is a distention in the direction of σ_2, and there is a compression in the direction of σ_1, then a layer is exposed to the compressive load. Suppose $B > 0$, i.e. $0 > m > -ctg^2\nu$. Then, $\sigma_{y\ av}$ has the same sign as σ_2, and $\sigma_{y\ av}$ and σ_1 have different signs. Therefore, if there is a distention in the direction of action σ_2, and there is a compression in the direction of σ_1, then a layer is exposed to the tensile load. Note that (see [7, p. 232])

$$\sigma_2 = \frac{2gk^-}{\sqrt{B^2 + g^2C^2}}, \tag{6}$$

then condition (5) is satisfied if and only if

$$\frac{g}{\sqrt{B^2 + g^2C^2}} < K \vee \frac{g \mid m \mid}{\sqrt{B^2 + g^2C^2}} < K. \tag{7}$$

Consequently, if $0 <| m |\leq 1$, then a layer reaches its critical state, when a second inequality (7) holds, and if $| m |\geq 1$, then a layer reaches its critical state, when a first inequality holds. If both inequalities (7) do not hold, then base material reaches its critical state during load process, and a layer does not reduce a connection strength.

2. A Place of Sample Destruction, if There is No Contact Hardening of a Layer

Suppose a layer is so wide that there is no contact hardening of it: $g = 1$. Note that there is no contact hardening, if a thickness (a width) of layer is greater then a sample thickness. Let us show that for $g = 1$ a critical state occurs always in the layer, but not in the main material of sheet sample. Consider case for $\mid m \mid \geq 1$. Condition of destruction by the layer, as follows from (7), takes the form:

$$B^2 + C^2 > \frac{1}{K}. \tag{8}$$

Let us find relations between m, an angle of layer slope ν and coefficient of heterogeneity K such that a critical state in a layer is archived earlier than in a base material, i.e. an inequality (8) holds. An inequality (8) can be presented in the following form:

$$P_1(m) = a_1 m^2 + b_1 m + c_1 \geq 0, \tag{9}$$

where

$$a_1 = s(4 - 3s); \quad b_1 = -6s(1 - s); \quad c_1 = (1 - s)(1 + 3s) - \frac{1}{K^2}; \quad s = sin^2\nu. \tag{10}$$

It follows from (10) that discriminant of a trinomial (9)

$$-8s(s - 1)(s^2 + 6s - 12) + \frac{4s(4 - 3s)}{K^2} < 0,$$

if $0 < s < 1$. This implies

Proposition 1. *A critical state in a layer is archived earlier than in a base material, for* $\mid m \mid \geq 1$ *and for any ratio between the parameters* m, K *and* ν.

Consider case for $\mid m \mid \leq 1$. It follows from (7) that a condition of destruction of a layer takes the form:

$$B^2 + C^2 > \frac{m^2}{K^2}. \tag{11}$$

Inequality (11) can be presented as

$$P_2(m) = a_2 m^2 + b_2 m + c_2 \geq 0, \tag{12}$$

where

$$a_2 = s(4 - 3s) - \frac{1}{K^2}; \quad b_2 = -6s(1 - s); \quad c_2 = (1 - s)(1 + 3s); \quad s = sin^2\nu. \tag{13}$$

Proposition 2. *A critical state in a layer is archived earlier than in a base material, for* $-1 \leq m < 0$ *and for any ratio between the parameters* m, K *and* υ.

Proof.

It follows from (12) and (13) that

$$P_2(0) = c_2 = (1 - s)(1 + 3s) > 0. \tag{14}$$

$$P_2^{'}(0) = b_2 = -6s(1-s) < 0. \tag{15}$$

$$P_2(0) = P_1(-1) = 12s((1-s) + 1 - \frac{1}{K^2} > 0.$$

for any values K, where $K > 1$, and s, where $0 < s < 1$. Therefore $P_2(m) > 0$ for $-1 \leq m < 0$.

□

Proposition 3. *A critical state in a layer is archived earlier than in a base material, for any m such that $0 \leq m < 1$, if and only the following condition holds:*

$$K sin\nu < 1. \tag{16}$$

Proof.

It is follows from (13) that

$$P_2^{'}(1) = 1 - \frac{1}{K^2}; \quad P_2^{'}(1) = 2(s - \frac{1}{K^2}).$$

Therefore $P_2(1) > 0$ for any admissible values of the parameters, because always $K > 1$, and $P_2^{'} < 0$ due to condition (16). It is follows from these inequalities and inequalities (14) and (15), that $P_2(m) > 0$ for all m such that $0 \leq m < 1$. Therefore, inequality (12) holds. This completes the proof.

□

3. Numerical Analysis

The results of 1 and 2 propositions are illustrated in Fig. 2 and 3. They show how normalized critical strains $\mid \sigma_1 \mid /2k^-$ and $\mid \sigma_2 \mid /2k^-$ depend on an angle of layer slope for different values of a load parameter m. A coefficient of a contact hardening is set to be $g = 1$. It is follows from 1 and 2 propositions that all curves are situated under 1. One can see that a strength of the sample containing layer, and a homogeneous sample of the same material are equal at $m = -1$, and only in the case where the external forces are parallel and orthogonal to the layer.

If $\nu \neq 0$ and $\nu \neq \pi/2$, then layer strength is always lower than a strength of the base sample material. Consequently, the layer material earlier reaches a critical state. A strength of layer and all construction depends on an angle ν. A minimum value of the critical strain depends on parameters ν , m and g on load conditions. It decreases with a decrease of load coefficient m. It follows from formula (6) that for $m < 0,5$ and for $\mid \sigma_1 \mid /2k^-$, and for $\mid \sigma_2 \mid /2k^-$.

$$\nu_{min} = arctg\sqrt{\frac{3-2m}{2-3m}}. \tag{17}$$

The minimal value of strains $\mid \sigma_1 \mid$ and $\mid \sigma_2 \mid$ can be obtained in an explicit form. Let

$$t = \frac{1 + 2g^2(m-1)}{2g^2(m-1) - m}. \tag{18}$$

then, the lowest strain value σ_2 is

$$\frac{\mid \sigma_2 \mid}{2k^-} = \frac{g(1+t)}{\sqrt{(1+4gt) + (2t - 8gt)m + (t^2 + 4gt)m^2}}. \tag{19}$$

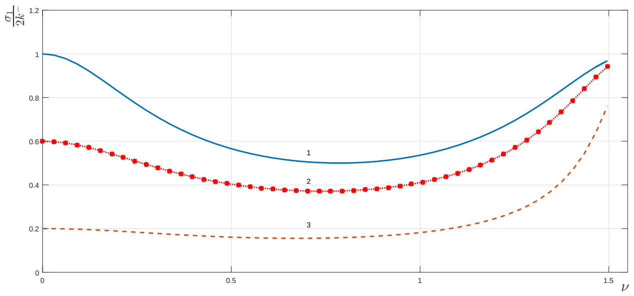

Fig. 2. A dependence of the critical normalized strains $\frac{|\sigma_1|}{2K^-}$ on an angle of layer slope under different load conditions: 1) $m = -1$; 2) $m = -0,6$; 3) $m = -0,2$. Contact hardening coefficient $g = 1$.

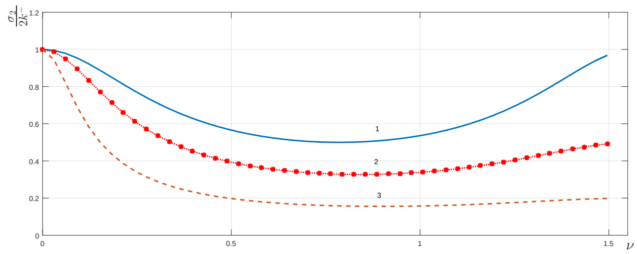

Fig. 3. A dependence of the critical normalized strains $\frac{|\sigma_2|}{2K^-}$ n an angle of layer slope under different load conditions: 1) $m = -1$; 2) $m = -2$; 3) $m = -5$. Contact hardening coefficient $g = 1$

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ВЛИЯНИЕ НАКЛОННОГО СЛОЯ В ЛИСТОВОМ ОБРАЗЦЕ НА ПРОЧНОСТЬ ПРИ ОТРИЦАТЕЛЬНОМ КОЭФФИЦИЕНТЕ ДВУХОСТНОСТИ НАГРУЖЕНИЯ ОБРАЗЦА

А.Н. Дияб

Исследуется критическое состояние наклонного слоя из менее прочного материала в листовом образце под действием нагрузок противоположных знаков. Показано, что при отсутствии контактного упрочнения материала слоя разрушение происходит по слою независимо от его угла наклона. На основе численных экспериментов исследована зависимость прочности образца от угла наклона слоя при различных отношениях между внешними нагрузками. Показано, что при наличии контактного упрочнения слоя образец, содержащий менее прочный слой, может быть равным по прочности однородному образцу при некоторых углах наклона слоя.

Ключевые слова: наклонный пластический слой, пластическая неустойчивость, напряженно-деформированное состояние, критерий Свифта.

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