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THE BARENBLATT – ZHELTOV – KOCHINA MODEL WITH ADDITIVE WHITE NOISE IN QUASI-SOBOLEV SPACES

G. A. Sviridyuk[1]**,** sviridyuk@susu.ac.ru,
N. A. Manakova[1]**,** manakova@susu.ac.ru,
[1] South Ural State University, Chelyabinsk, Russian Federation.

In order to carry over the theory of linear stochastic Sobolev-type equations to quasi-Banach spaces, we construct a space of differentiable quasi-Sobolev "noises" and establish the existence and uniqueness of a classical solution to the Showalter – Sidorov problem for a stochastic Sobolev-type equation with a relatively p-bounded operator. Basing on the abstract results, we study the Barenblatt – Zheltov – Kochina stochastic model with the Showalter – Sidorov initial condition in quasi-Sobolev spaces with an external action in the form of "white noise".

Keywords: Sobolev-type equations; Wiener process; Nelson – Gliklikh derivative; white noise; quasi-Sobolev spaces; Barenblatt – Zheltov – Kochina stochastic equation.

Introduction

Consider the space l_q of sequences $u = (u_1, u_2, ...)$ of real numbers with the quasi-norm

$$ {}_q\|u\| = \left(\sum_{k=1}^{\infty} |u_k|^q \right)^{\frac{1}{q}}, $$

where $q \in \mathbb{R}_+$. The definition of quasi-norm ${}_{\mathfrak{U}}\| \cdot \|$ on a real subspace $\mathfrak{U}$ differs from the definition of the norm $\| \cdot \|_{\mathfrak{U}}$ only in the triangle inequality axiom

$$ {}_{\mathfrak{U}}\|u + v\| \leq C({}_{\mathfrak{U}}\|u\| + {}_{\mathfrak{U}}\|v\|), $$

with a constant $C \geq 1$. In the case of the space l_q the constant is $C = 2^{\frac{1-q}{q}}$ for $q \in (0, 1)$ and $C = 1$ for $q \in [1, +\infty)$. It is well-known (see Lemma 3.10.1 in [1] for instance) that the quasi-normed space $\mathfrak{U} = (\mathfrak{U}, {}_{\mathfrak{U}}\| \cdot \|)$ is not in general normable although metrizable; that is, on the quasi-normed space $\mathfrak{U}$ there is a metric which agrees with some power of the quasi-norm ${}_{\mathfrak{U}}\| \cdot \|$. Hence, the concepts of fundamental sequence and completion make sense in a quasi-normed space. A complete quasi-normed space is called a quasi-Banach space. Henceforth, for definiteness, we regard the Banach spaces l_q with $q \in [1, +\infty)$ as quasi-Banach spaces.

Given a monotone sequence $\{\lambda_k\} \subset \mathbb{R}_+$ with $\lim\limits_{k \to \infty} \lambda_k = +\infty$, construct the quasi-Sobolev space

$$ l_q^m = \left\{ u = (u_1, u_2, ...) : \sum_{k=1}^{\infty} \left(\lambda_k^{\frac{m}{2}} |u_k| \right)^q < \infty \right\}, \quad m \in \mathbb{R},\ q \in \mathbb{R}_+. $$

This is a quasi-Banach space with the quasi-norm

$$ {}_q^m\|u\| = \left(\sum_{k=1}^{\infty}\left(\lambda_k^{\frac{m}{2}}|u_k|\right)^q\right)^{\frac{1}{q}}; $$

moreover [2], the embedding $l_q^m \hookrightarrow l_q^n$ is dense and continuous for $m \geq n$. On l_q^m define the Laplace quasi-operator $\Lambda u = (\lambda_1 u_1, \lambda_2 u_2, ...)$, which is continuous as $\Lambda : l_q^{m+2} \to l_q^m$ for all $q \in \mathbb{R}_+$ and $m \in \mathbb{R}$. The Barenblatt – Zheltov – Kochina model, describing the filtration of fluid in a medium with cracks and pores, in quasi-Sobolev spaces reads as

$$ (\lambda - \Lambda)u_t = \alpha\Lambda u + f. \tag{0.1} $$

Sufficient conditions were determined in [2] for the existence of a unique solution u in $C([0,\tau); l_q^{m+2}) \cap C^1((0,\tau); l_q^{m+2})$ to the Showalter – Sidorov problem

$$ (\lambda - \Lambda)(u(0) - u_0) = 0 \tag{0.2} $$

for (0.1) with arbitrary $\tau, q \in \mathbb{R}_+$, $m, \lambda \in \mathbb{R}$, $u_0 \in l_q^{m+2}$, and $f \in C^1([0,\tau); l_q^m)$.

The goal of this note is, firstly, to extend the concept of white noise [3] to the spaces l_q^m, and secondly, to consider the stochastic version [4] of problem (0.1), (0.2) in these spaces.

1. White Noise in Quasi-Sobolev Spaces

The spaces $\mathbf{C}^l\mathbf{L}_2$ of random processes ($\mathbf{C}^l\mathbf{L}_2(\varepsilon,\tau)$ with intervals $(\varepsilon,\tau) \subset \mathbb{R}$) whose Nelson – Gliklikh derivatives through order $l \in \{0\} \cup \mathbb{N}$ are almost surely (a.s.) continuous on (ε,τ) (that is, a.s. all trajectories of these derivatives are continuous on (ε,τ)) were considered for the first time in [4]. An example is the Wiener process

$$ \beta(t) = \sum_{k=0}^{\infty}\xi_k \sin\frac{\pi}{2}(2k+1)t \tag{1.1} $$

modeling Brownian motion on a line in the Einstein – Smoluchowski theory because

$$ \overset{o}{\beta}{}^{(k)}(t) = (-1)^{k+1}\prod_{i=1}^{k-1}(2i-1)(2t)^{-k}\beta(t) \text{ for all } t \in \mathbb{R}_+ \text{ and } k \in \mathbb{N} \tag{1.2} $$

according to Gliklikh's theorem ([4], Theorem 1.2). Recall that ξ_k are independent Gaussian variables with expectation $\mathbf{E}\xi_k = 0$ and variance $\mathbf{D}\xi_k = [\frac{\pi}{2}(2k+1)]^{-2}$, for $k \in \{0\} \cup \mathbb{N}$.

Introduce now the space $\mathbf{l}_q^m\mathbf{L}_2$ of sequences of random variables $\omega = (\omega_1, \omega_2, ...)$ with the quasi-norm

$$ {}_q^m|\|\omega\|| = \Big(\sum_{k=1}^{\infty}(\lambda_k^m\mathbf{D}\omega_k)^{\frac{q}{2}}\Big)^{\frac{1}{q}}, \quad q \in \mathbb{R}_+, \ m \in \mathbb{R}. $$

These $\mathbf{l}_q^m\mathbf{L}_2$ are obviously quasi-Banach spaces, and by analogy with quasi-Sobolev spaces we call them *quasi-Sobolev stochastic spaces*. Indeed, the embedding $\mathbf{l}_q^m\mathbf{L}_2 \hookrightarrow \mathbf{l}_q^n\mathbf{L}_2$ is dense and continuous for all $m \geq n$ and $q \in \mathbb{R}_+$, and in addition, the Laplace quasi-operator $\Lambda : \mathbf{l}_q^{m+2}\mathbf{L}_2 \to \mathbf{l}_q^m\mathbf{L}_2$ is linear, continuous, and even continuously invertible for all $m \in \mathbb{R}$ and $q \in \mathbb{R}_+$.

Furthermore, introduce the spaces $\mathbf{C}^l\mathbf{l}_q^m\mathbf{L}_2$ (meaning $\mathbf{C}^l\mathbf{l}_q^m\mathbf{L}_2(\varepsilon, \tau)$, where $(\varepsilon, \tau) \subset \mathbb{R}$) of random processes $\eta = (\eta_1, \eta_2, ...)$ with $\eta_k = \eta_k(t)$ for $t \in (\varepsilon, \tau)$ and $k \in \mathbb{N}$, whose Nelson – Gliklikh derivatives through order $l \in \{0\} \cup \mathbb{N}$ are a.s. continuous on (ε, τ). An example is the Wiener process

$$W_{qS} = (\beta_1, \beta_2, ...), \tag{1.3}$$

where $\beta_k = \beta_k(t)$ for $t \in \mathbb{R}_+$ are Brownian motions of the form (1.1). By Gliklikh's theorem, $W_{qS} \in \mathbf{C}^l\mathbf{l}_q^m\mathbf{L}_2$ for all $l \in \{0\} \cup \mathbb{N}$ provided that the series

$$\sum_{k=1}^{\infty} \lambda_k^{\frac{mq}{2}} \tag{1.4}$$

converges. To find conditions for this convergence is the subject of future research. Here we observe that for $m = -2q^{-1}$ and $\lambda_k = k^2$ the series (1.4) converges. Following [3, 4], we refer to the Nelson – Gliklikh derivative $\overset{o}{W}_{qS}(t) = (2t)^{-1}W_{qS}(t)$ of the Wiener process $W_{qS}(t)$ as white noise.

2. The Barenblatt – Zheltov – Kochina Stochastic Model

Take $\mathfrak{U} = \mathbf{l}_q^{m+2}\mathbf{L}_2$ and $\mathfrak{F} = \mathbf{l}_q^m\mathbf{L}_2$ with $m \in \mathbb{R}$ and $q \in \mathbb{R}_+$. Consider the Barenblatt – Zheltov – Kochina stochastic model with the Showalter – Sidorov condition (0.1), (0.2). Fixing $\alpha, \lambda \in \mathbb{R}$, construct the operators $L = \lambda - \Lambda$ and $M = \alpha\Lambda$, where Λ is the Laplace quasi-operator. Define the operator $\Lambda^{-1}u = \{\lambda_k^{-1}u_k\}$ and call it the Green quasi-operator. Consider $L, M \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ (see [2]); moreover, L is a Fredholm operator for all $\lambda \in \mathbb{R}$. Therefore, we can reduce the Barenblatt – Zheltov – Kochina stochastic equation (0.1) to the linear stochastic Sobolev-type equation

$$L \overset{o}{\eta} = M\eta + Nw, \tag{2.1}$$

where $\eta = \eta(t)$ is the required random process, while $w = w(t)$ is a given one, on the interval $(0, \tau)$. The operator $N \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ is to be specified below.

Introduce the L-resolvent set $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{F}; \mathfrak{U})\}$ and the L-spectrum $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ of the operator M. If the L-spectrum $\sigma^L(M)$ of M is bounded then M is called an (L, σ)-bounded operator. In this case there exist projections

$$P = \frac{1}{2\pi i}\int_{\gamma} R_\mu^L(M)d\mu \in \mathcal{L}(\mathfrak{U}), \quad Q = \frac{1}{2\pi i}\int_{\gamma} L_\mu^L(M)d\mu \in \mathcal{L}(\mathfrak{F}).$$

Here $R_\mu^L(M) = (\mu L - M)^{-1}L$ is the *right* and $L_\mu^L(M) = L(\mu L - M)^{-1}$ is the *left L-resolution* of M, while the closed contour $\gamma \subset \mathbb{C}$ bounds a region including $\sigma^L(M)$. Put $\mathfrak{U}^0 = \ker P$, $\mathfrak{U}^1 = \mathrm{im} P$, $\mathfrak{F}^0 = \ker Q$, and $\mathfrak{F}^1 = \mathrm{im} Q$, and denote by L_k and M_k the restrictions of L and M to $\mathfrak{U}^k$ for $k = 0, 1$.

Theorem 1. (Splitting theorem [5]) *If M is an (L, σ)-bounded operator then*

(i) *we have $L_k(M_k) \in \mathcal{L}(\mathfrak{U}^k; \mathfrak{F}^k)$ for $k = 0, 1$;*

(ii) *there exist operators* $M_0^{-1} \in \mathcal{L}(\mathfrak{F}^0; \mathfrak{U}^0)$ *and* $L_1^{-1} \in \mathcal{L}(\mathfrak{F}^1; \mathfrak{U}^1)$.

Construct the operators $H = M_0^{-1} L_0 \in \mathcal{L}(\mathfrak{U}^0)$ and $S = L_1^{-1} M_1 \in \mathcal{L}(\mathfrak{U}^1)$. An operator M is called (L,p)-*bounded*, with $p \in \{0\} \cup \mathbb{N}$, whenever ∞ is a removable singular point (that is, $H \equiv \mathbb{O}$ when $p = 0$) or a pole of order $p \in \mathbb{N}$ (that is, $H^p \neq \mathbb{O}$ and $H^{p+1} \equiv \mathbb{O}$) of the L-*resolution* $(\mu L - M)^{-1}$ of M.

Take an (L,p)-bounded operator M with $p \in \{0\} \cup \mathbb{N}$. Impose on (2.1) the Showalter – Sidorov initial condition

$$\left[R^L_\alpha(M)\right]^{p+1} (\eta(0) - \xi_0) = 0. \tag{2.2}$$

Below we consider also the *weak Showalter – Sidorov condition* (in the sense of Krein):

$$\lim_{t \to 0+} \left[R^L_\alpha(M)\right]^{p+1} (\eta(t) - \xi_0) = 0. \tag{2.3}$$

Definition 1. *Refer to a random process* $\eta \in \mathbf{C}^1 \mathbf{l}^m_q \mathbf{L}_2(0,\tau)$ *as a (classical) solution to (2.1) whenever almost surely all its trajectories satisfy (2.1) for all* $t \in (0,\tau)$. *Refer to a solution* $\eta = \eta(t)$ *to (2.1) as a (classical) solution to problem (2.1), (2.2) whenever it also satisfies (2.2).*

Remark 1. In the case that M is $(L,0)$-bounded, conditions (2.2) and (2.3) are equivalent to the following conditions respectively:

$$L(\eta(0) - \xi_0) = 0 \text{ and } \lim_{t \to 0+} L(\eta(t) - \xi_0) = 0. \tag{2.4}$$

Theorem 2. *Given an* (L,p)*-bounded operator* M *with* $p \in \{0\} \cup \mathbb{N}$, *for every* $N \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$, *every random processes* $w = w(t)$ *satisfying* $(\mathbb{I} - Q)Nw \in \mathbf{C}^{p+1} \mathbf{l}^m_q \mathbf{L}_2$ *and* $QNw \in \mathbf{C} \mathbf{l}^m_q \mathbf{L}_2$, *and every random quantity* $\xi_0 \in \mathbf{l}^m_q \mathbf{L}_2$ *independent of* w *for all fixed* $t \in (0,\tau)$ *there exists a unique solution* $\eta \in \mathbf{C}^1 \mathbf{l}^m_q \mathbf{L}_2$ *to problem (2.1), (2.2), which, moreover, is of the form*

$$\eta(t) = U^t \xi_0 + \int_0^t U^{t-s} L_1^{-1} Q N w(s) ds - \sum_{n=0}^{p} H^q M_0^{-1} (\mathbb{I} - Q) N \overset{o}{w}{}^{(n)}(t). \tag{2.5}$$

Remark 2. We can prove Theorem 2 by analogy with the deterministic case [5]. However, as the white noise $w(t) = (2t)^{-1} W_{qS}(t)$ is not differentiable at $t = 0$, it cannot appear in the right-hand side of (2.1). A way around this obstacle, proposed in [4, 6, 7], relies on limit passage. To use this approach, rearrange the second term in the right-hand side of (2.5) as

$$\begin{gathered} \int_\varepsilon^t U^{t-s} L_1^{-1} Q N \overset{o}{W}_{qS}(s) ds = \\ L_1^{-1} Q N W_{qS}(t) - U^{t-\varepsilon} L_1^{-1} Q N W_{qS}(\varepsilon) + SP \int_\varepsilon^t U^{t-s} L_1^{-1} Q N W_{qS}(s) ds. \end{gathered} \tag{2.6}$$

Integration by parts makes sense for arbitrary $\varepsilon \in (0,t)$, with $t \in \mathbb{R}_+$, by the definition of Nelson – Gliklikh derivative. Passing in (2.6) to the limit as $\varepsilon \to 0$, we obtain

$$\int_0^t U^{t-s} L_1^{-1} Q N \overset{o}{W}_{qS}(s) ds = L_1^{-1} Q N W_{qS}(t) + SP \int_0^t U^{t-s} L_1^{-1} Q N W_{qS}(s) ds.$$

Proceed to problem (2.3) for the stochastic Barenblatt – Zheltov – Kochina equation on $\mathbb{R}_+$,

$$L \overset{o}{\eta} = M\eta + N \overset{o}{W}_{qS}, \tag{2.7}$$

where $W_{qS} = W_{qS}(t)$ is a Wiener process. Then the following statement holds.

Lemma 1. *For all $\lambda \in \mathbb{R}$ and $\alpha \in \mathbb{R} \setminus \{0\}$ the operator M is $(L, 0)$-bounded.*

Theorem 3. *For all $\lambda \in \mathbb{R}$, $\alpha \in \mathbb{R} \setminus \{0\}$, $N \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$, and $\xi_0 \in \mathbf{l}_q^m\mathbf{L}_2$ independent of W_{qS} there exists a unique solution $\eta = \eta(t)$ to problem (2.2), (2.7), which, moreover, is of the form*

$$\eta(t) = U^t\xi_0 + L_1^{-1}[QNW_{qS}(t) + M_1 \int_0^t U^{t-s}L_1^{-1}QNW_{qS}(s)ds] - M_0^{-1}(\mathbb{I} - Q)N \overset{o}{W}_{qS}(t).$$

Here

$$U^t = \begin{cases} \sum\limits_{k=1}^{\infty} e^{\mu_k t} < \cdot, e_k > e_k, & \text{if } \lambda \neq \lambda_k,\ k \in \mathbb{N}; \\ \sum\limits_{k \neq l} e^{\mu_k t} < \cdot, e_k > e_k, & \text{if } \exists\, l \in \mathbb{N} : \lambda = \lambda_l, \end{cases}$$

with the points $\mu_k = \dfrac{\alpha\lambda_k}{\lambda - \lambda_k}$ of the L-spectrum of M, the sequence $\{\xi_{0k}\} = \xi_0 \in \mathbf{l}_q^m\mathbf{L}_2$, and the vector $e_k = (0, \ldots, 0, 1, 0, \ldots)$ in which the unity appears in slot k. The operators L_1^{-1} and M_1^{-1} are defined as

$$L_1^{-1}\zeta = \begin{cases} \{(\lambda - \lambda_k)^{-1}\zeta_k\}, \text{ if } \lambda \neq \lambda_k \text{ for all } k \in \mathbb{N}; \\ ((\lambda - \lambda_1)^{-1}\zeta_1, \ldots, (\lambda - \lambda_{l-1})^{-1}\zeta_{l-1}, 0, (\lambda - \lambda_{l+1})^{-1}\zeta_{l+1}, \ldots), \text{ if } \exists\, l \in \mathbb{N} : \lambda = \lambda_l; \end{cases}$$

$$M_1\eta = \begin{cases} \{\alpha\lambda_k\eta_k\}, \text{ if } \lambda \neq \lambda_k \text{ for all } k \in \mathbb{N}; \\ (\alpha_1\lambda_1\eta_1, \ldots, \alpha_{l-1}\lambda_{l-1}\eta_{l-1}, 0, \alpha_{l+1}\lambda_{l+1}\eta_{l+1}, \ldots), \text{ if } \exists\, l \in \mathbb{N} : \lambda = \lambda_l. \end{cases}$$

$$M_0^{-1}\zeta = \begin{cases} \{0\}, \text{ if } \lambda \neq \lambda_k \text{ for all } k \in \mathbb{N}; \\ (0, \ldots, 0, (\alpha_l\lambda_l)^{-1}\zeta_l, 0, \ldots), \text{ if } \exists\, l \in \mathbb{N} : \lambda = \lambda_l. \end{cases}$$

The projection is

$$Q = \begin{cases} \sum\limits_{k=1}^{\infty} < \cdot, e_k > e_k, \text{ if } \lambda \neq \lambda_k \text{ for all } k \in \mathbb{N}; \\ \sum\limits_{k=1,\ k \neq l}^{\infty} < \cdot, e_k > e_k, \text{ if } \exists\, l \in \mathbb{N} : \lambda = \lambda_l. \end{cases}$$

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Georgy A. Sviridyuk, Doctor Physico-Mathematical Sciences, Professor, Department of Equations of Mathematical Physics, South Ural State University (Chelyabinsk, Russian Federation), sviridyuk@susu.ac.ru

Natalia A. Manakova, Candidate of Physico-Mathematical Sciences, Docent, Department of Equations of Mathematical Physics, South Ural State University (Chelyabinsk, Russian Federation), manakova@susu.ac.ru

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МОДЕЛЬ БАРЕНБЛАТТА – ЖЕЛТОВА – КОЧИНОЙ С АДДИТИВНЫМ "БЕЛЫМ ШУМОМ" В КВАЗИСОБОЛЕВЫХ ПРОСТРАНСТВАХ

Г.А. Свиридюк, Н.А. Манакова

В статье рассматривается перенос теории линейных стохастических уравнений соболевского типа на квазибанаховы пространства. Для этого строятся пространства дифференцируемых квазисоболевых "шумов" и доказываются существование и единственность классического решения задачи Шоуолтера – Сидорова для стохастического уравнения соболевского типа с относительно p-ограниченным оператором. На основе абстрактных результатов производится исследование стохастической модели Баренблатта – Желтова – Кочиной с начальным условием Шоуолтера – Сидорова в квазисоболевых пространствах с внешним воздействием в виде "белого шума".

Ключевые слова: уравнения соболевского типа, винеровский процесс, производная Нельсона – Гликлиха, "белый шум"; квазисоболевы пространства, стохастическое уравнение Баренблатта – Желтова – Кочиной.

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Свиридюк Георгий Анатольевич, доктор физико-математических наук, профессор, заведующий кафедрой уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), sviridyuk@susu.ac.ru

Манакова Наталья Александровна, кандидат физико-математических наук, доцент, кафедра уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), manakova@susu.ac.ru

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